Diagnosis of Planar Phased Arrays Through a Probabilistic Compressive Sensing Approach

M. Salucci, A. Gelmini, G. Oliveri, and A. Massa

Abstract

This work deals with the detection of faulty elements in planar phased antenna arrays starting from far-field pattern measurements. Owing to the intrinsically *sparse* nature of the problem unknowns at hand, the diagnosis problem is formulated as a probabilistic Compressive Sensing (*CS*) one and it is effectively and efficiently solved through a customized Bayesian *CS* (*BCS*) solution approach. Some representative synthetic benchmarks are shown in order to verify the potentialities as well as the current limitations of the proposed *BCS*-based diagnosis tool, as well as to assess its flexibility in dealing with arbitrary excitation taperings of the *AUT.*

Contents

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1 Numerical Validation

1.1 Bayliss Array, $N = 316$, Isotropic Sources

Parameters

- Gold Array
  - Total number of elements: $N = 316$;
  - Type of elements: isotropic/ideal [1];
  - Spacing along x and y: $d_x = d_y = 0.5$ [λ];
  - Excitation tapering: Bayliss;
    - Radius: $R = 5$ [λ];
    - Transition index: $t = 3$;
    - Peak sidelobe level: $PSL = 25$ [dB];

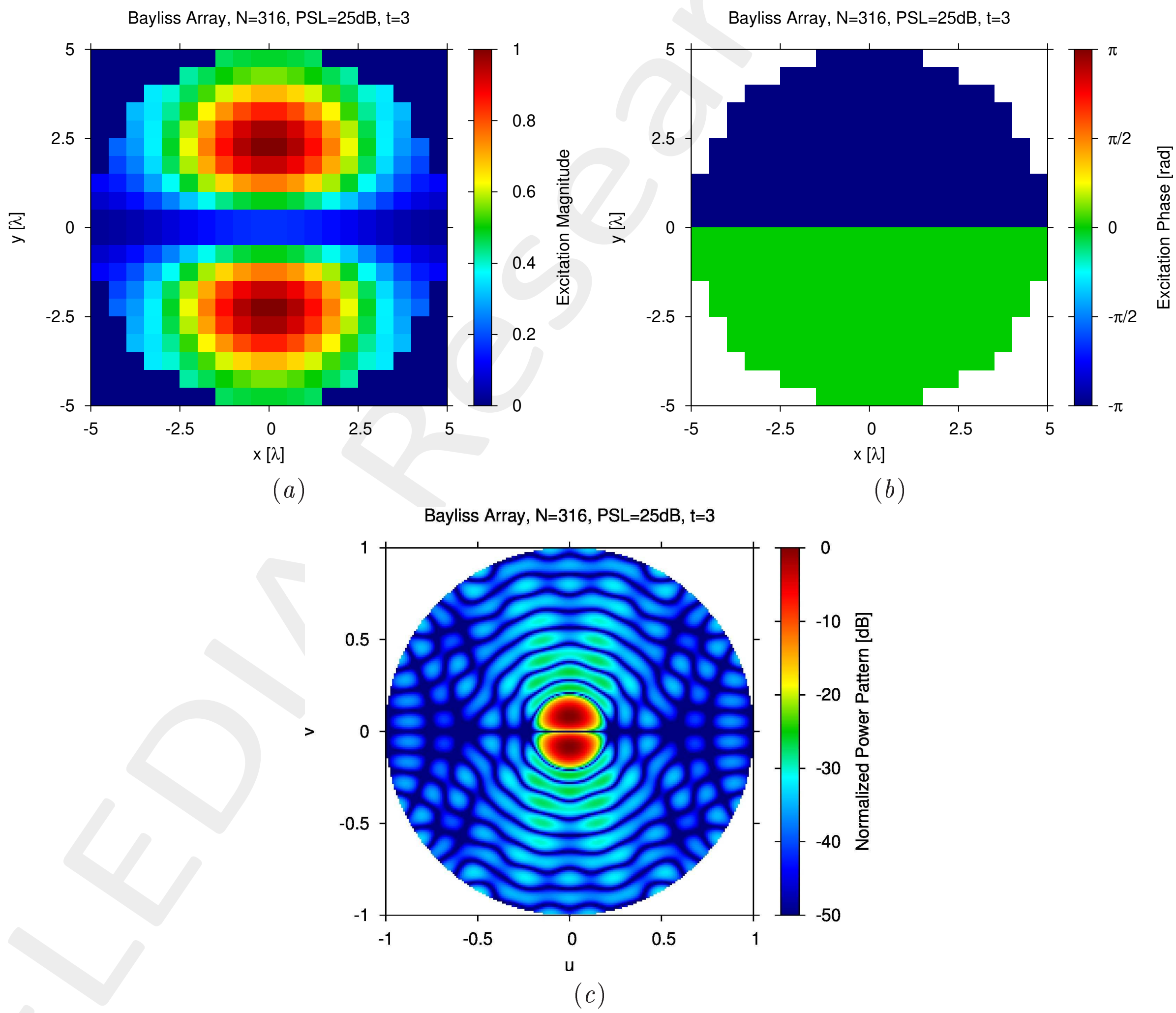

Figure 1: (a) magnitude and (b) phase of the array excitations; (c) normalized power pattern.

[1] In order to model *isotropic* radiators, let us assume that the embedded elements patterns are equal to $F_{\theta}^{(n)}(u, v) = 1$ and $F_{\varphi}^{(n)}(u, v) = 0$, for $n = 1, ..., N$.

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- Failed Array

  - Failure factor: $\kappa = 0$ (total failures);

  - Failure rate: see table below;

N_f	$\Phi = \frac{N_f}{N}$
3	1%
6	2%
13	4%
25	8%
51	16%

Table 1: Number of failures (N_f) and corresponding failure rate ($\Phi = \frac{N_f}{N}$).

- Measurement set-up

  - Type of sampling: uniform sampling in the (u, v) plane;

  - Number of points in the visible range: $K = 317$;

  - Ratio between measurements and number of elements: $\nu = \frac{K}{N} \simeq 1.0$ ($\nu^{(opt)}$);

- *BCS* solver

  - Noise variance: $\eta = 5 \times 10^{-1}$ ($\eta^{(opt)}$);

  - Tolerance factor: $\iota = 10^{-8}$;

- Signal-to-Noise-Ratio: $SNR = \{10; 20; ...; 100\}$.

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Results

$\Phi = \frac{N_f}{N} = 1\%$ ($N_f = 3$) - Best and Worst BCS Reconstructions

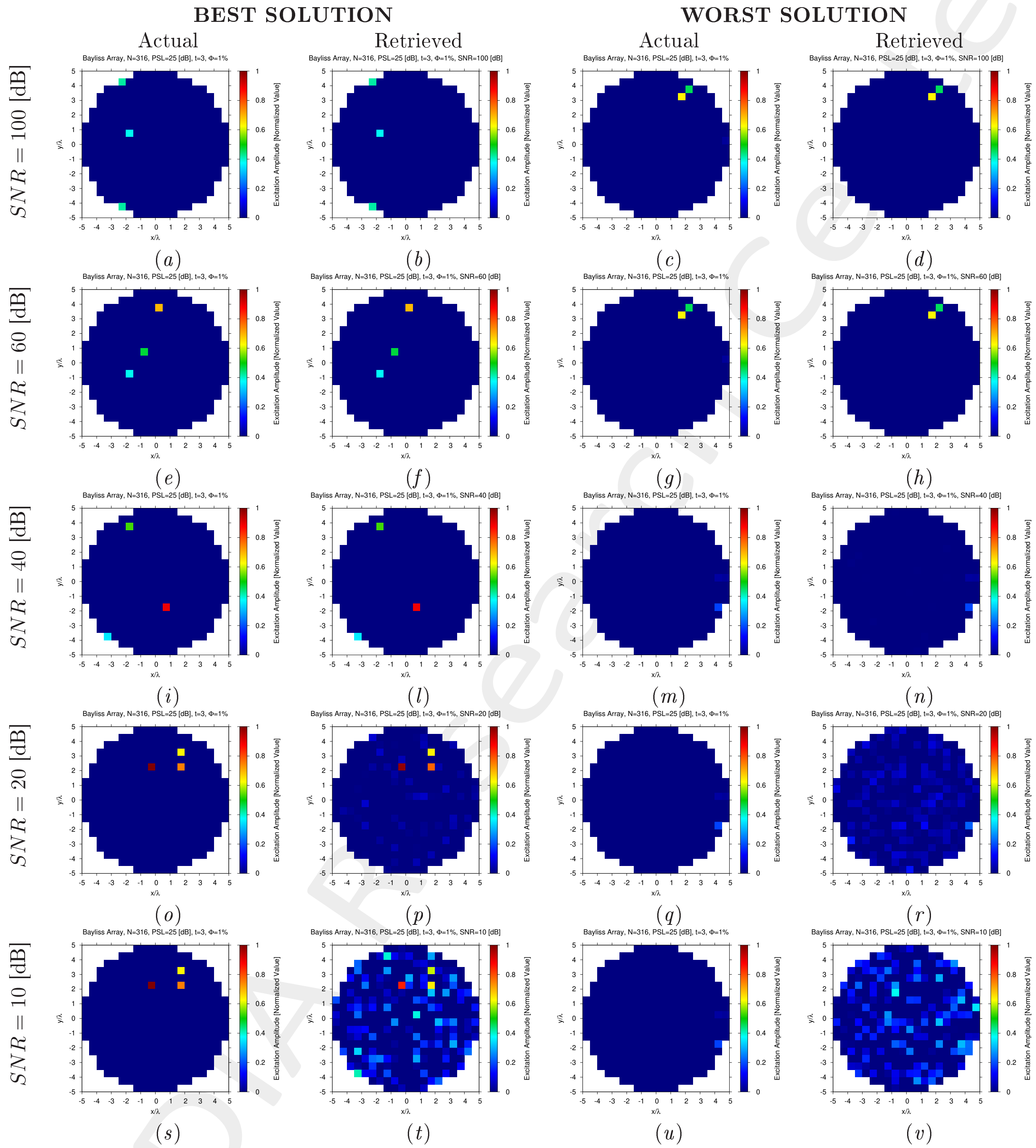

Figure 2: Bayliss Array ($N = 316$, $PSL = 25$ [dB], $t = 3$, $\Phi = 1\%$) - Best and worst reconstructions by BCS under several SNR values.

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$\Phi = \frac{N_f}{N} = 2\%$ $(N_f = 6)$ **- Best and Worst** BCS **Reconstructions**

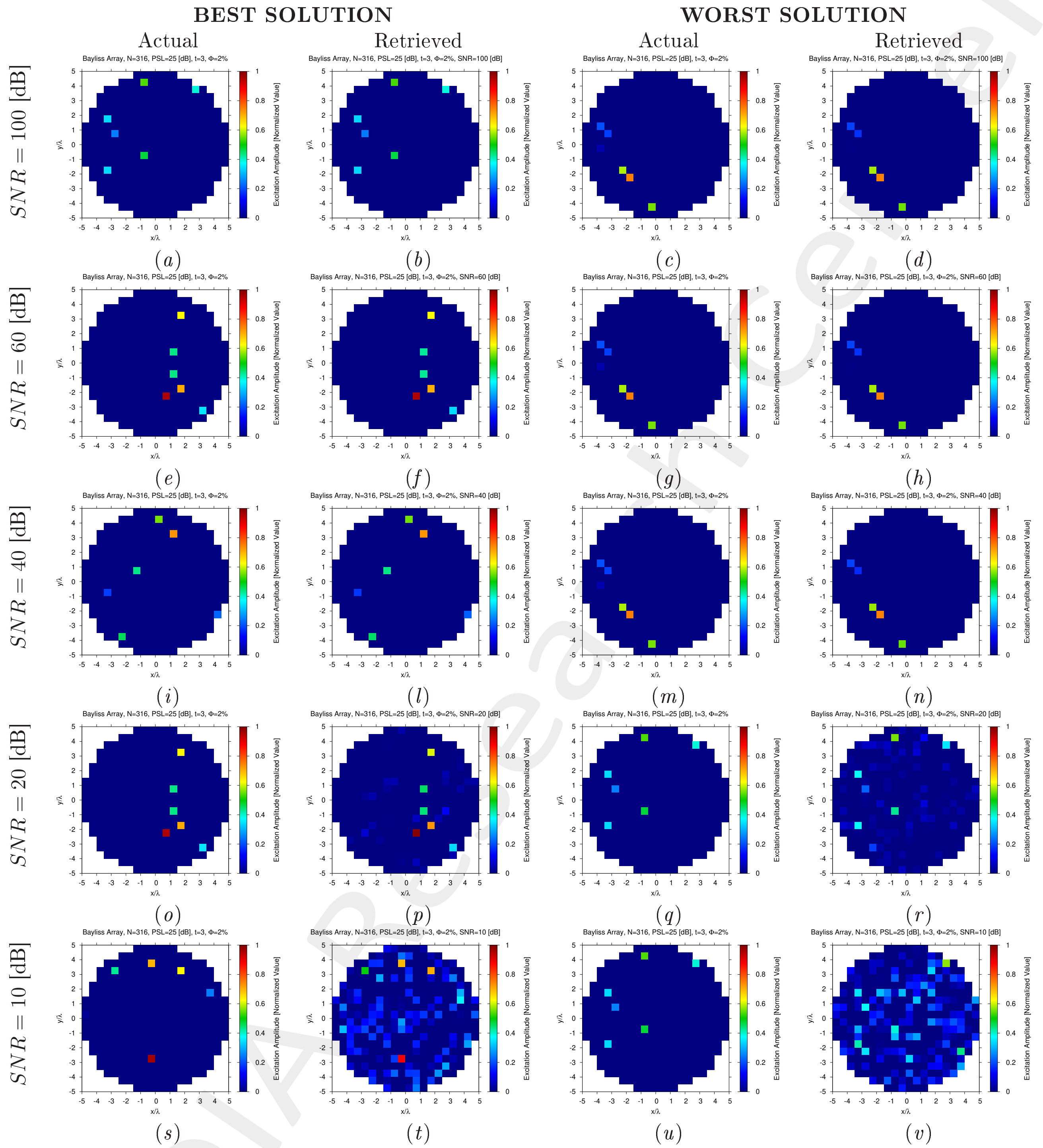

Figure 3: Bayliss Array ($N = 316$, $PSL = 25$ [dB], $t = 3$, $\Phi = 2\%$) - Best and worst reconstructions by BCS under several SNR values.

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$\Phi = \frac{N_f}{N} = 4\%$ $(N_f = 13)$ **- Best and Worst** ***BCS*** **Reconstructions**

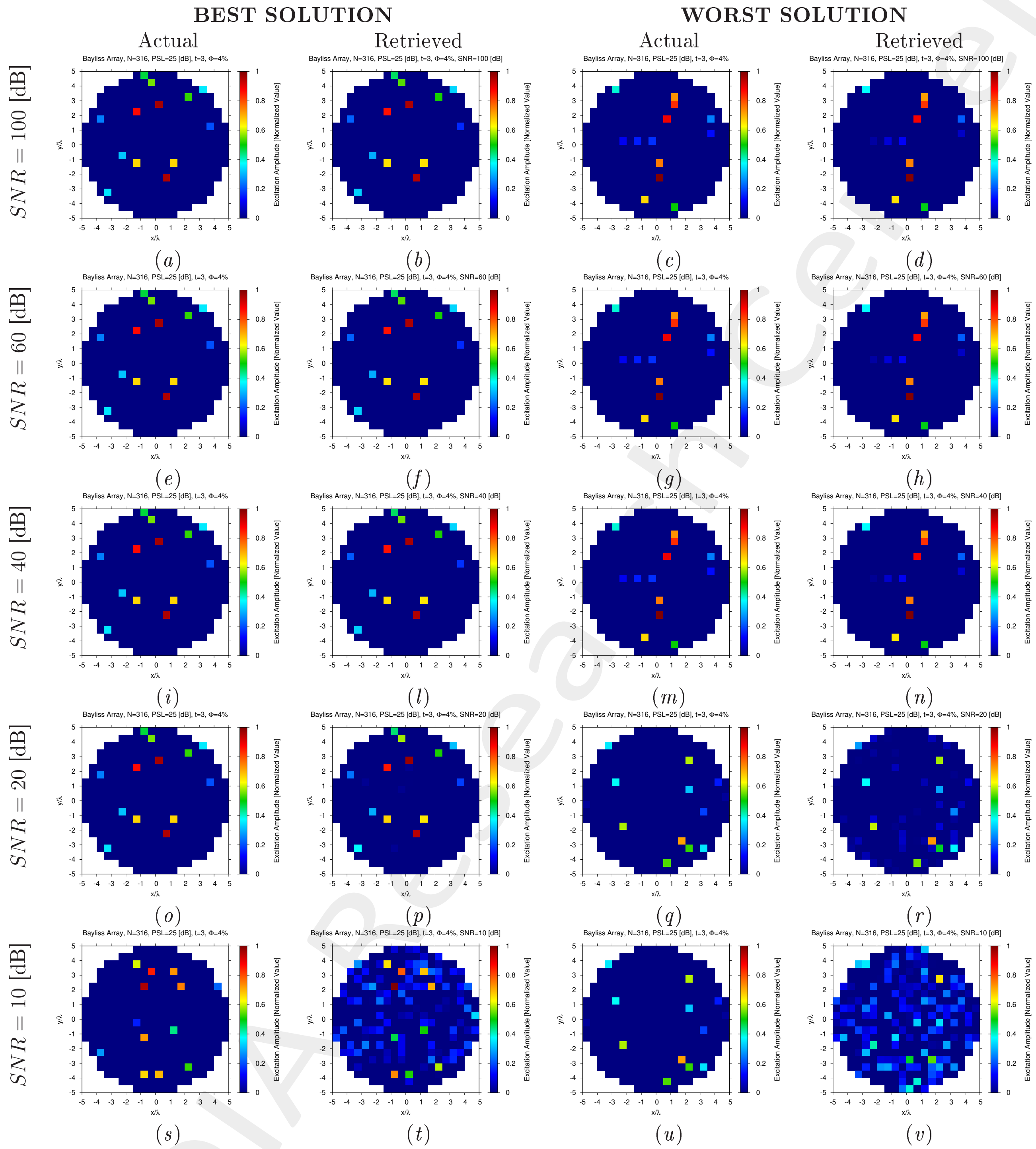

Figure 4: Bayliss Array ($N = 316$, $PSL = 25$ [dB], $t = 3$, $\Phi = 4\%$) - Best and worst reconstructions by *BCS* under several *SNR* values.

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$\Phi = \frac{N_f}{N} = 8\%$ ($N_f = 25$) **- Best and Worst** *BCS* **Reconstructions**

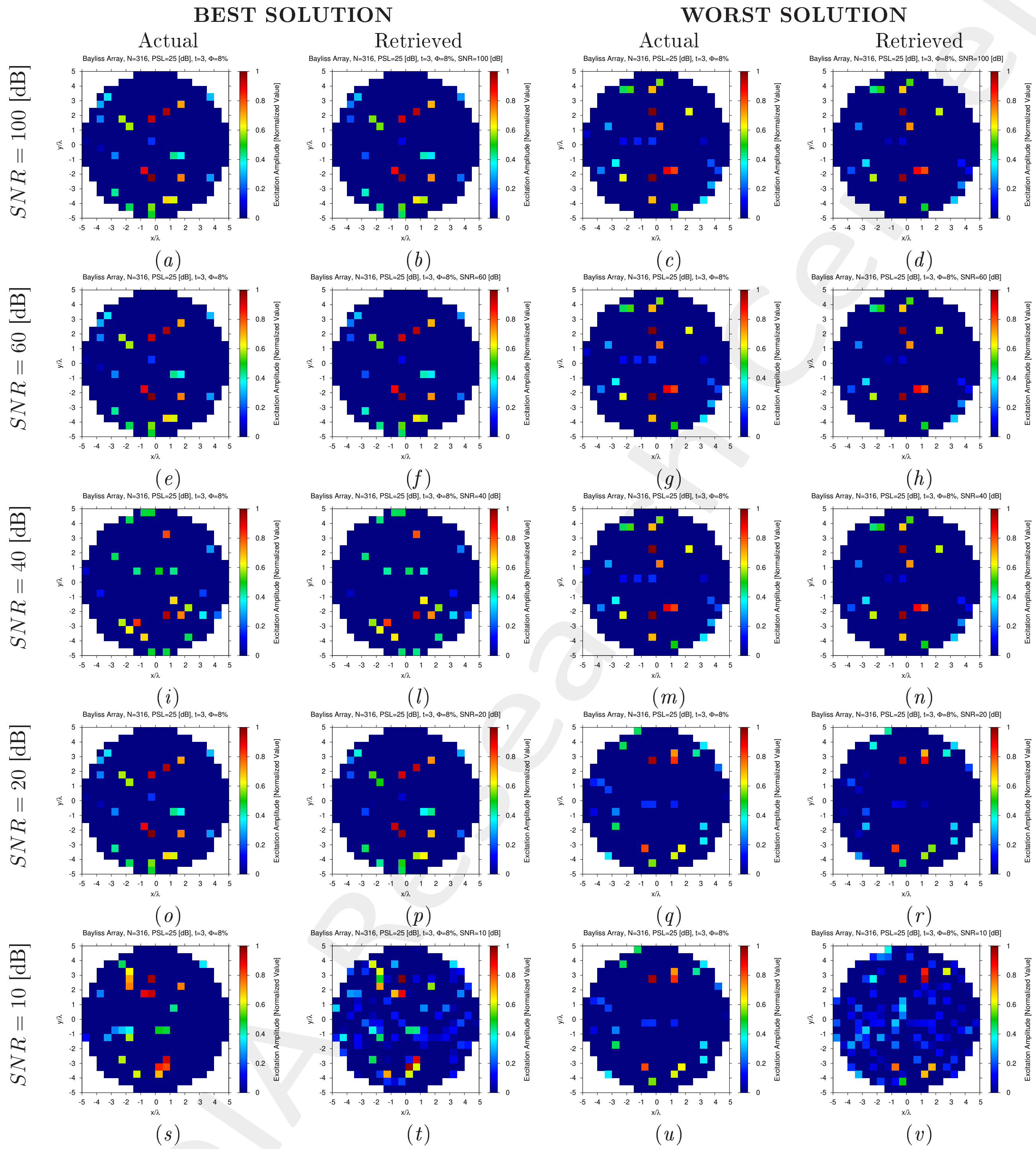

Figure 5: Bayliss Array ($N = 316$, $PSL = 25$ [dB], $t = 3$, $\Phi = 8\%$) - Best and worst reconstructions by *BCS* under several *SNR* values.

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$\Phi = \frac{N_f}{N} = 16\%$ $(N_f = 51)$ - **Best and Worst** ***BCS*** **Reconstructions**

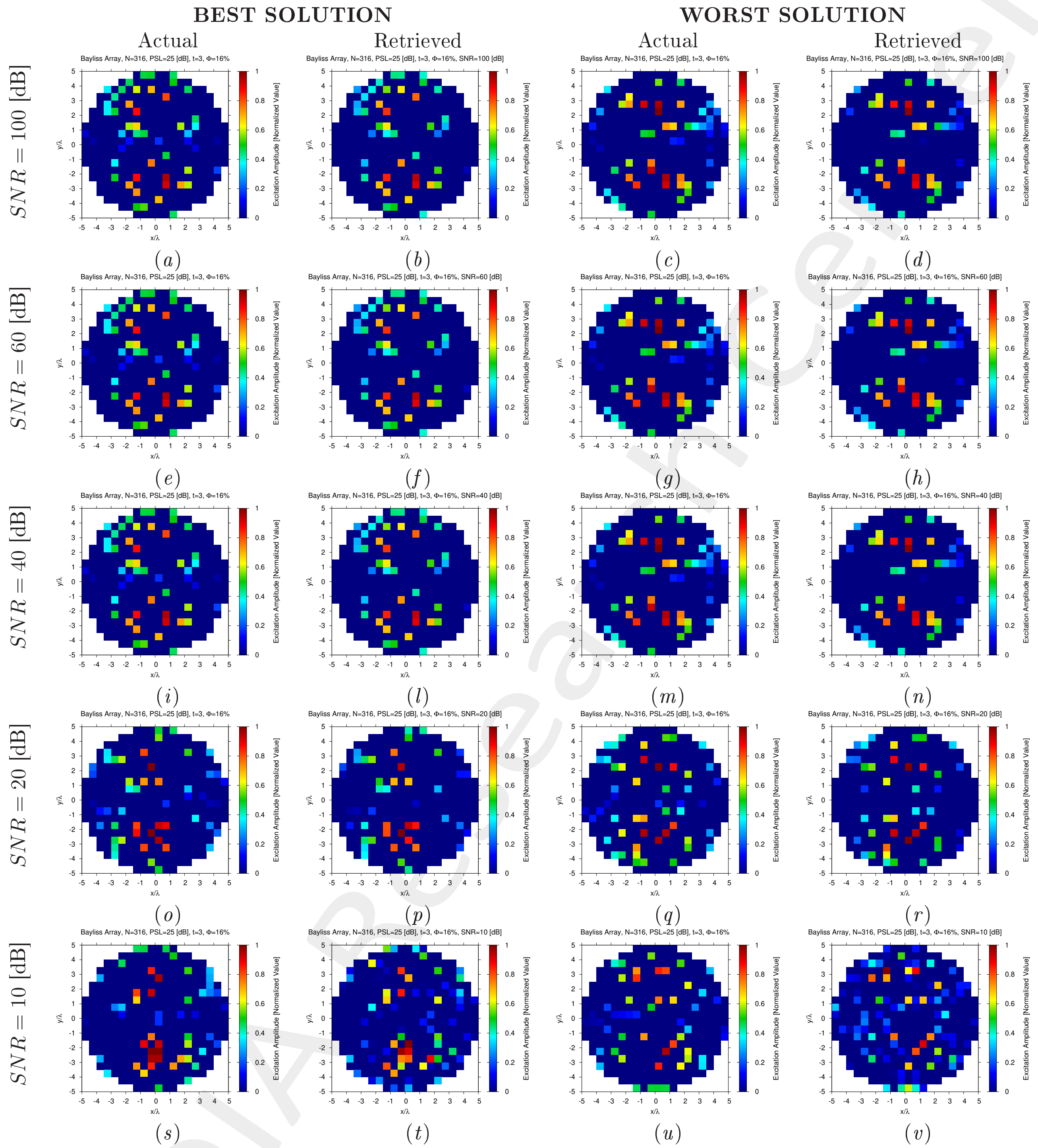

Figure 6: Bayliss Array ($N = 316$, $PSL = 25$ [dB], $t = 3$, $\Phi = 16\%$) - Best and worst reconstructions by BCS under several SNR values.

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Diagnosis Error and Confidence Level

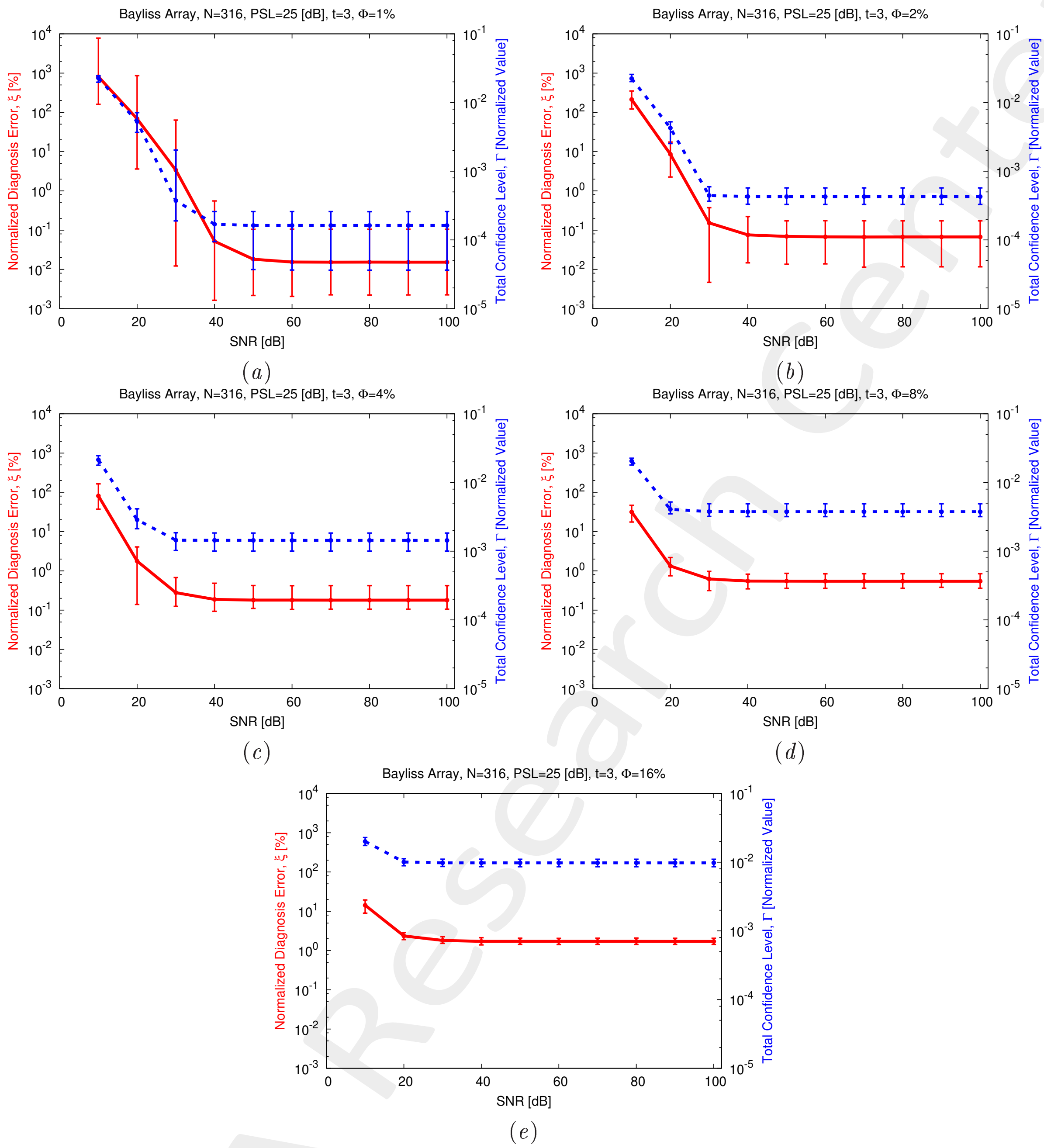

Figure 7: Bayliss Array ($N = 316$, $PSL = 25$ [dB], $t = 3$) - Behavior of the average, minimum and maximum diagnosis error (ξ) and total confidence level (Γ) versus the SNR, for (a) $\Phi = 1\%$, (b) $\Phi = 2\%$, (c) $\Phi = 4\%$, (d) $\Phi = 8\%$, and (e) $\Phi = 16\%$.

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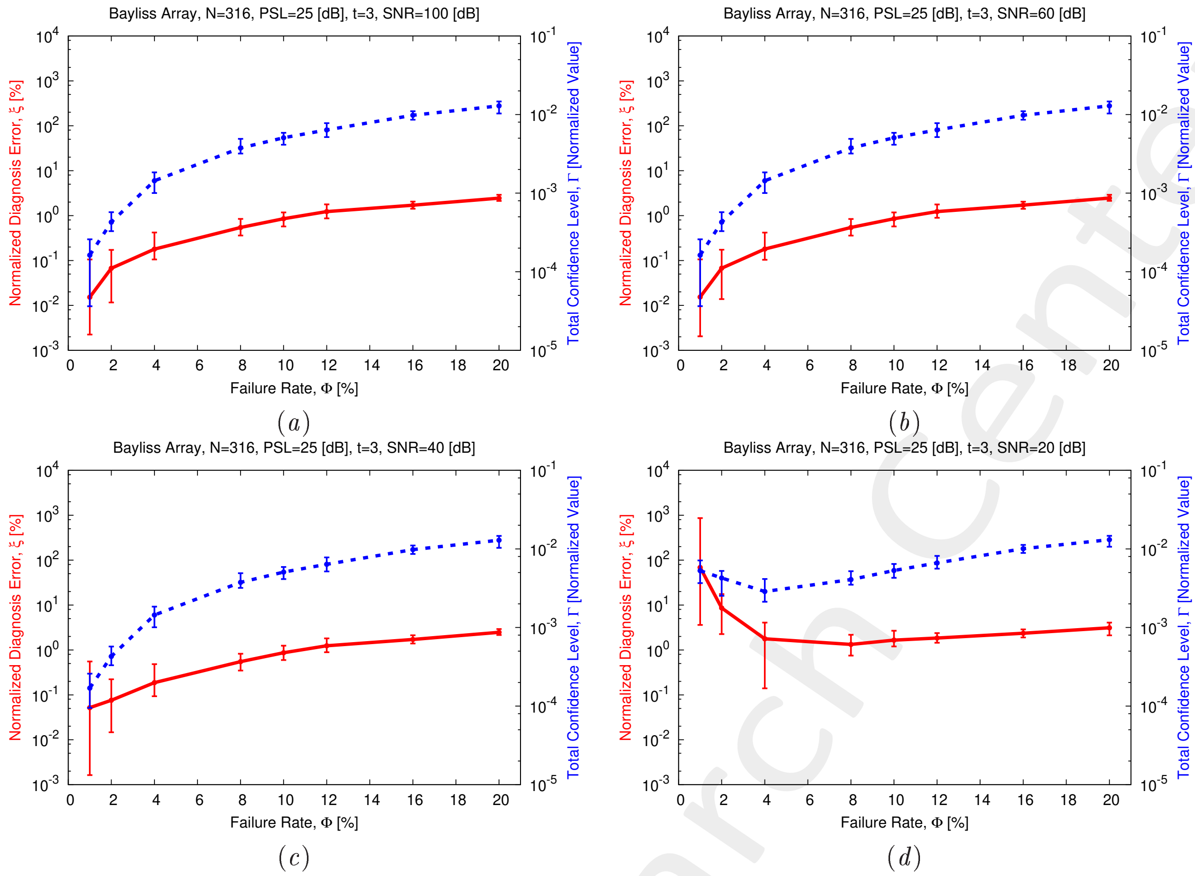

Figure 8: Bayliss Array ($N = 316$, $PSL = 25$ [dB], $t = 3$) - Behavior of the average, minimum and maximum diagnosis error (ξ) and total confidence level (Γ) versus the failure rate (Φ), for (a) $SNR = 100$ [dB], (b) $SNR = 60$ [dB], (c) $SNR = 40$ [dB], and (d) $SNR = 20$ [dB].

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1.2 Slepian Array, $N = 400$, Isotropic Sources

Parameters

- Gold Array
  - Total number of elements: $N = 400$;
  - Type of elements: isotropic/ideal [2]
  - Spacing along x and y: $d_x = d_y = 0.5$ $[\lambda]$;
  - Excitation tapering: Slepian;
    - Angular region at the receiver: $\Psi = \{(u, v) : -u_0 \leq u \leq u_0, -v_0 \leq v \leq v_0\}$, with $u_0 = v_0 = 0.1$.

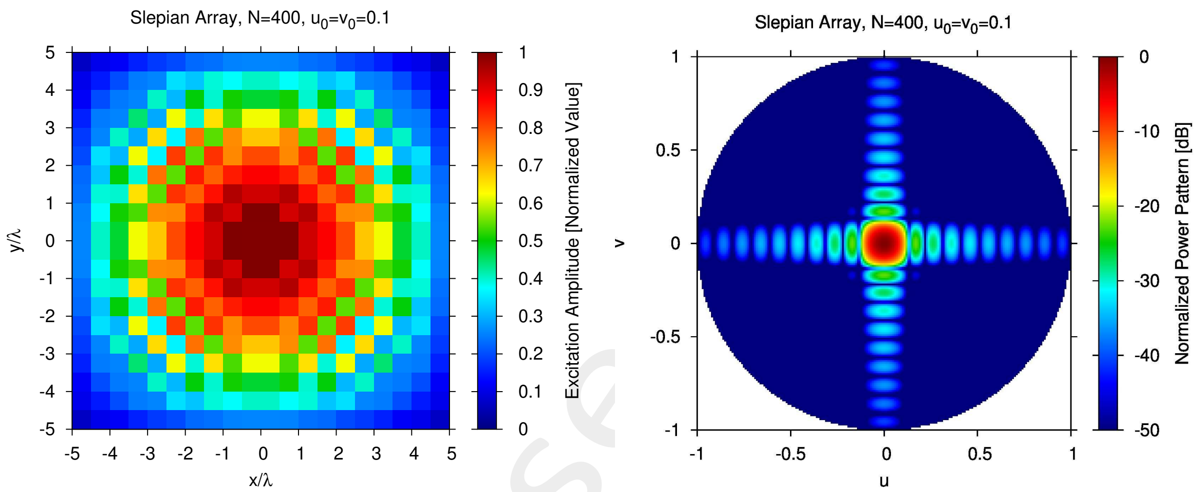

Figure 9: (a) Array excitations and (b) normalized power pattern of the gold array.

- Failed Array
  - Failure factor: $\kappa = 0$ (total failures);
  - Failure rate: see table below;

N_f	$\Phi = \frac{N_f}{N}$
4	1%
8	2%
16	4%
32	8%
64	16%

Table 2: Number of failures (N_f) and corresponding failure rate ($\Phi = \frac{N_f}{N}$).

- Measurement set-up

[2]In order to model *isotropic* radiators, let us assume that the embedded elements patterns are equal to $F_\theta^{(n)}(u, v) = 1$ and $F_\varphi^{(n)}(u, v) = 0$, for $n = 1, ..., N$.

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- Type of sampling: uniform sampling in the $(u,\, v)$ plane;
- Number of points in the visible range: $K = 408$;
- Ratio between measurements and number of elements: $\nu = \frac{K}{N} \simeq 1.0$ ($\nu^{(opt)}$);

- BCS solver
  - Noise variance: $\eta = 5 \times 10^{-1}$ ($\eta^{(opt)}$);
  - Tolerance factor: $\iota = 10^{-8}$;

- Signal-to-Noise-Ratio: $SNR = \{10;\, 20;\, ...;\, 100\}$.

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$\Phi = \frac{N_f}{N} = 1\%$ $(N_f = 4)$ **- Best and Worst BCS Reconstructions**

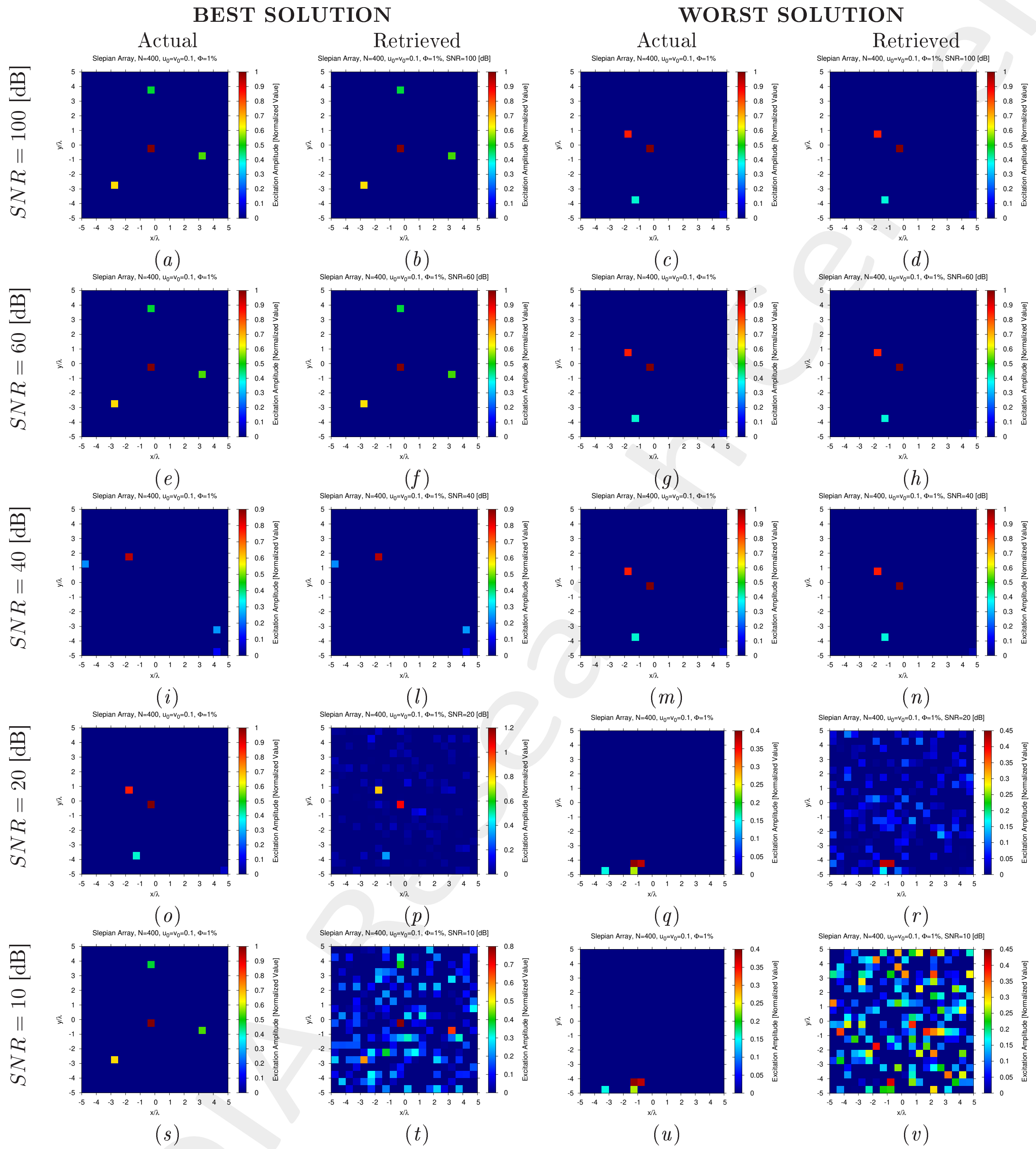

Figure 10: Slepian Array ($N = 400$, $u_0 = v_0 = 0.1$, $\Phi = 1\%$) - Best and worst reconstructions by BCS under several SNR values.

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$\Phi = \frac{N_f}{N} = 2\%$ $(N_f = 8)$ - **Best and Worst** BCS **Reconstructions**

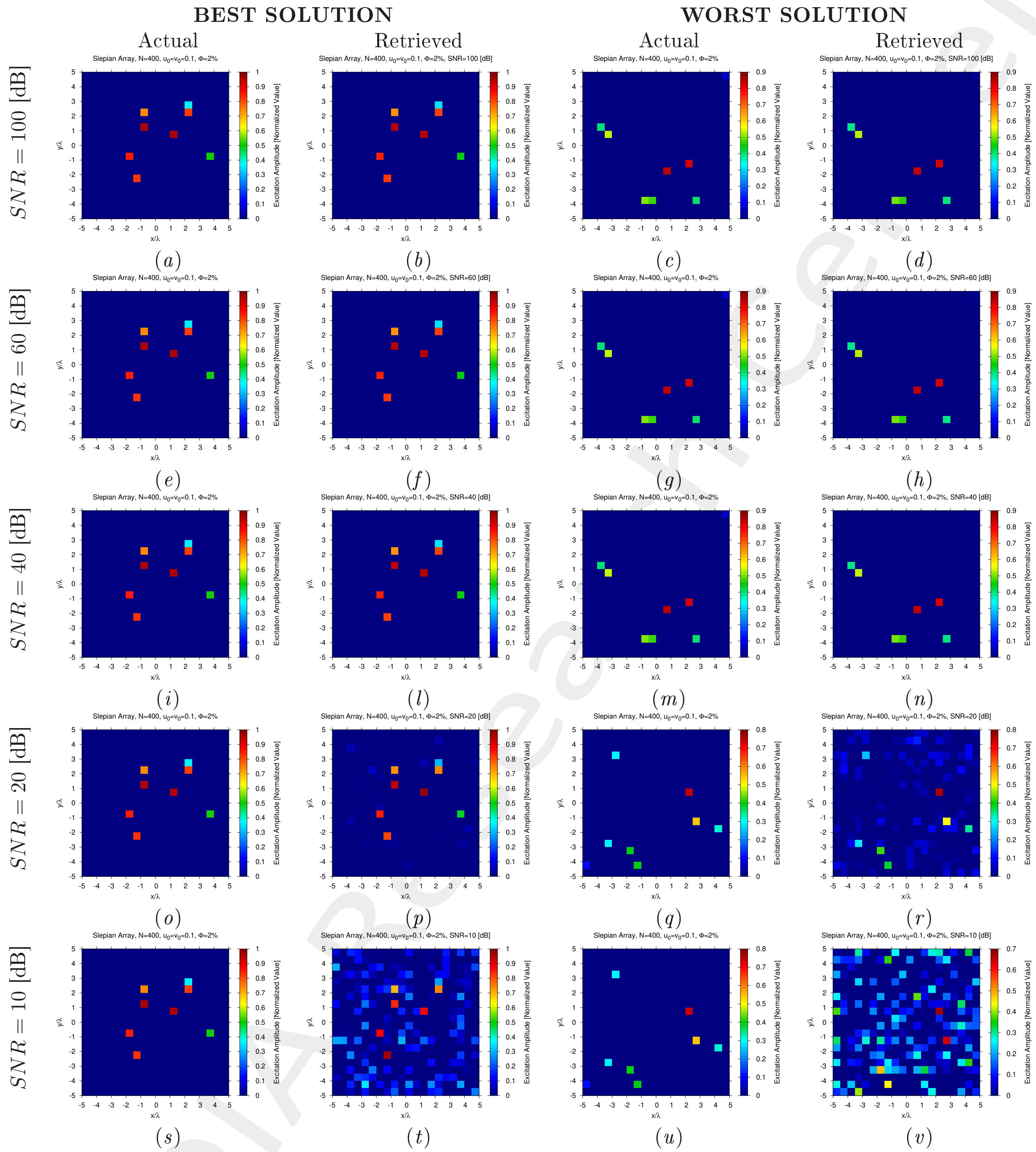

Figure 11: Slepian Array ($N = 400$, $u_0 = v_0 = 0.1$, $\Phi = 2\%$) - Best and worst reconstructions by BCS under several SNR values.

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$\Phi = \frac{N_f}{N} = 4\%$ $(N_f = 16)$ - Best and Worst *BCS* Reconstructions

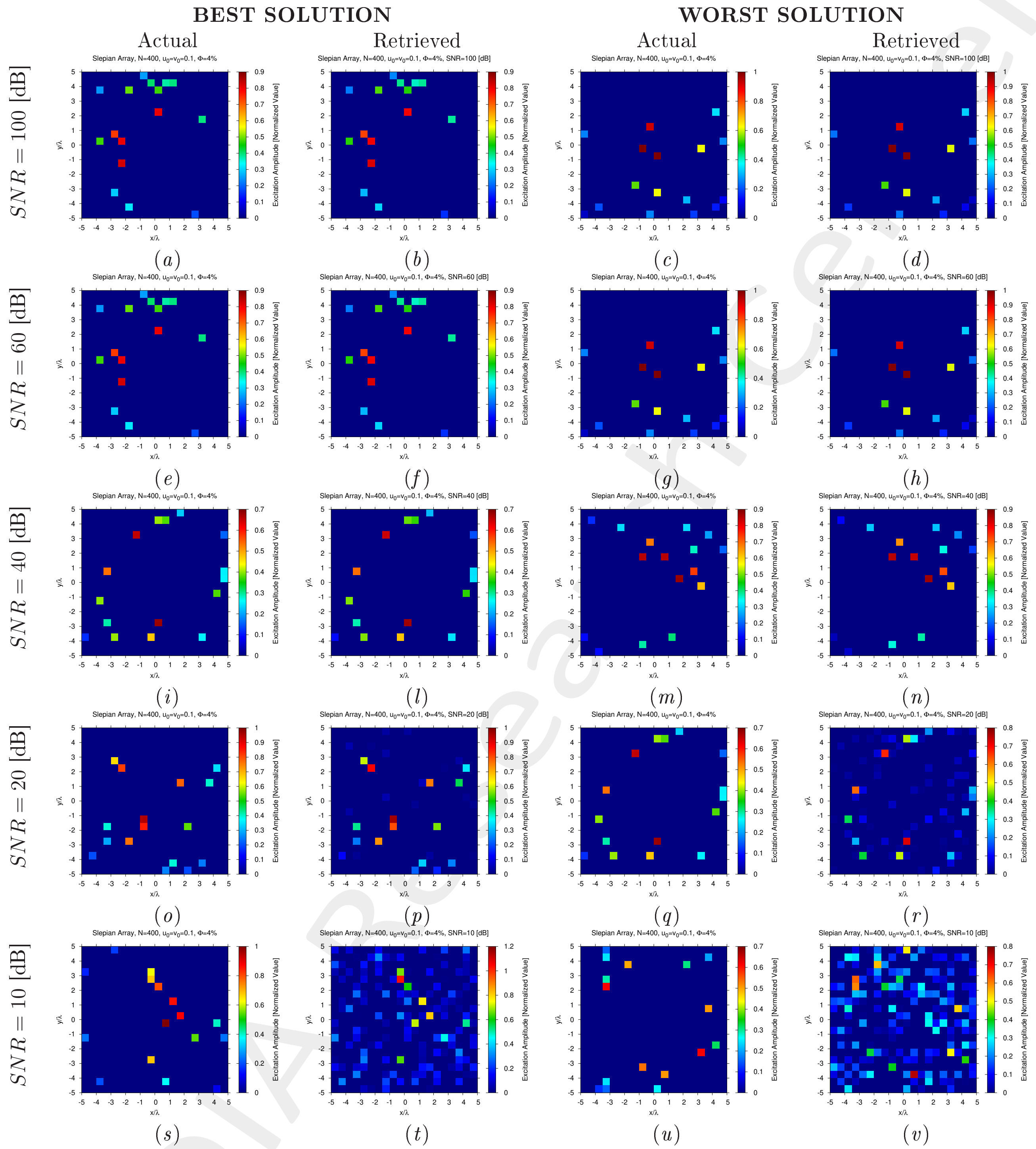

Figure 12: Slepian Array ($N = 400$, $u_0 = v_0 = 0.1$, $\Phi = 4\%$) - Best and worst reconstructions by *BCS* under several *SNR* values.

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$\Phi = \frac{N_f}{N} = 8\%$ ($N_f = 32$) **- Best and Worst** *BCS* **Reconstructions**

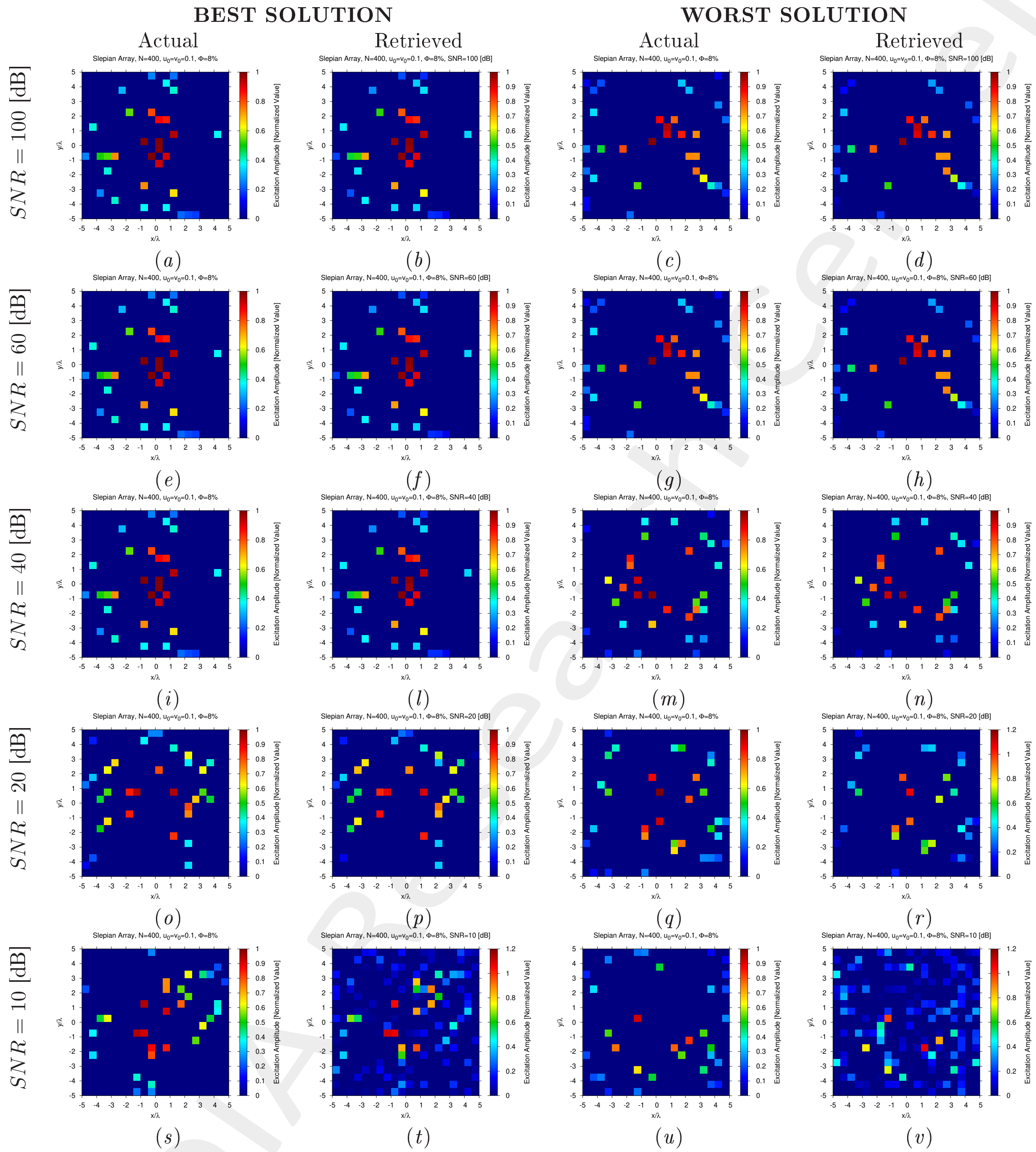

Figure 13: Slepian Array ($N = 400$, $u_0 = v_0 = 0.1$, $\Phi = 8\%$) - Best and worst reconstructions by *BCS* under several *SNR* values.

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$\Phi = \frac{N_f}{N} = 16\%$ $(N_f = 64)$ **- Best and Worst** BCS **Reconstructions**

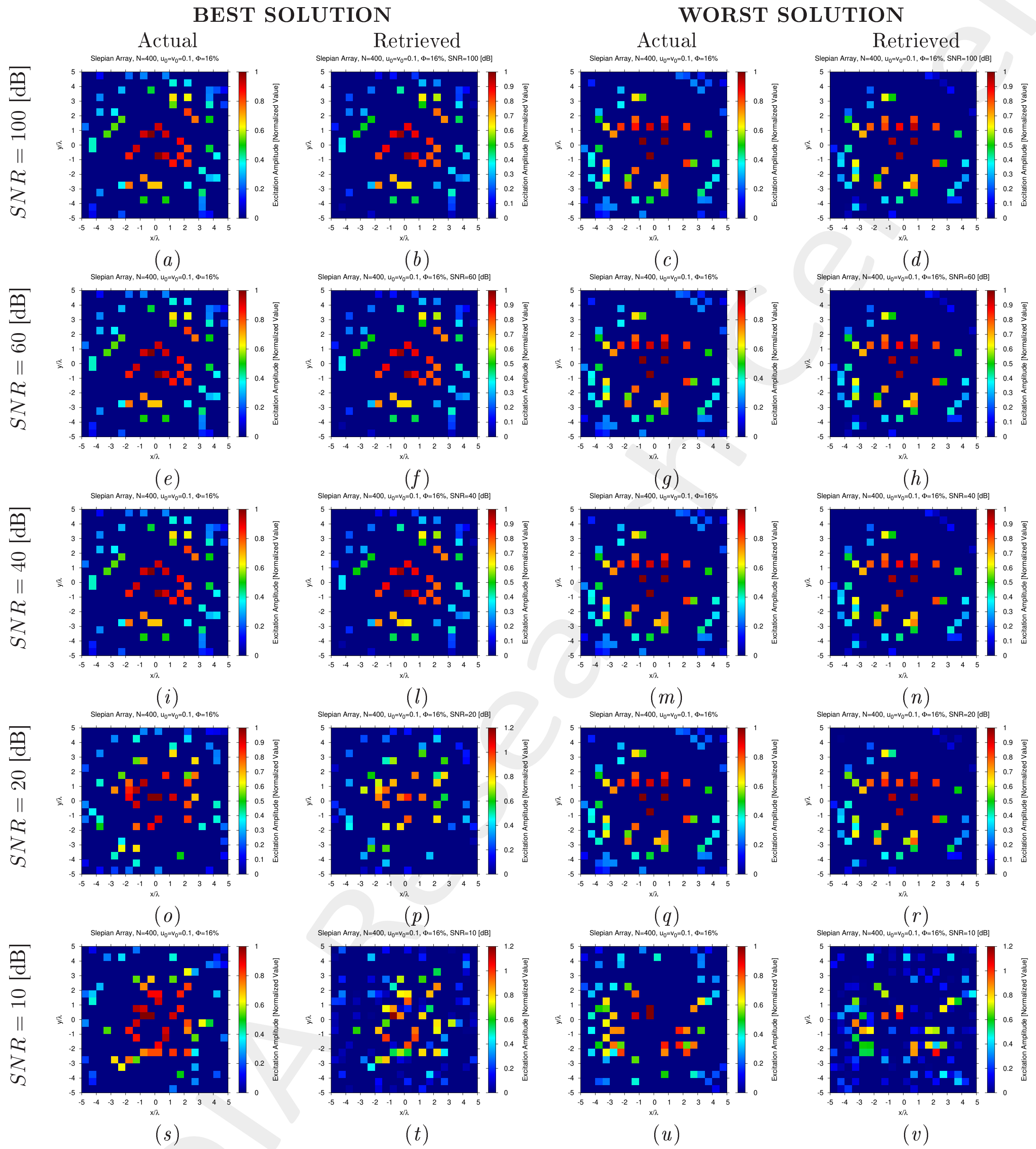

Figure 14: Slepian Array ($N = 400$, $u_0 = v_0 = 0.1$, $\Phi = 16\%$) - Best and worst reconstructions by BCS under several SNR values.

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Diagnosis Error and Confidence Level

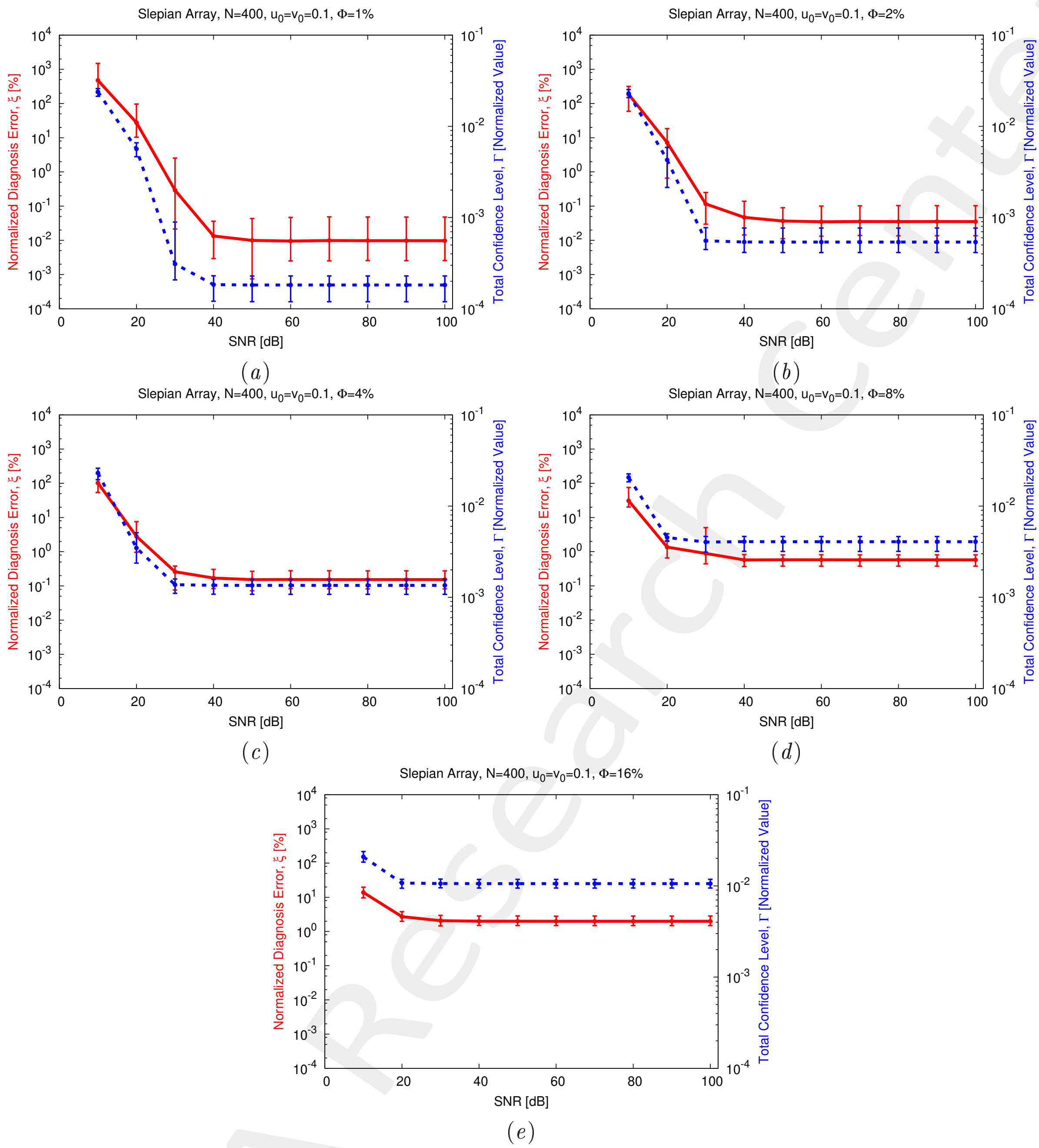

Figure 15: Slepian Array ($N = 400$, $u_0 = v_0 = 0.1$) - Behavior of the average, minimum and maximum diagnosis error (ξ) and total confidence level (Γ) versus the SNR, for (a) $\Phi = 1\%$, (b) $\Phi = 2\%$, (c) $\Phi = 4\%$, (d) $\Phi = 8\%$, (e) $\Phi = 16\%$.

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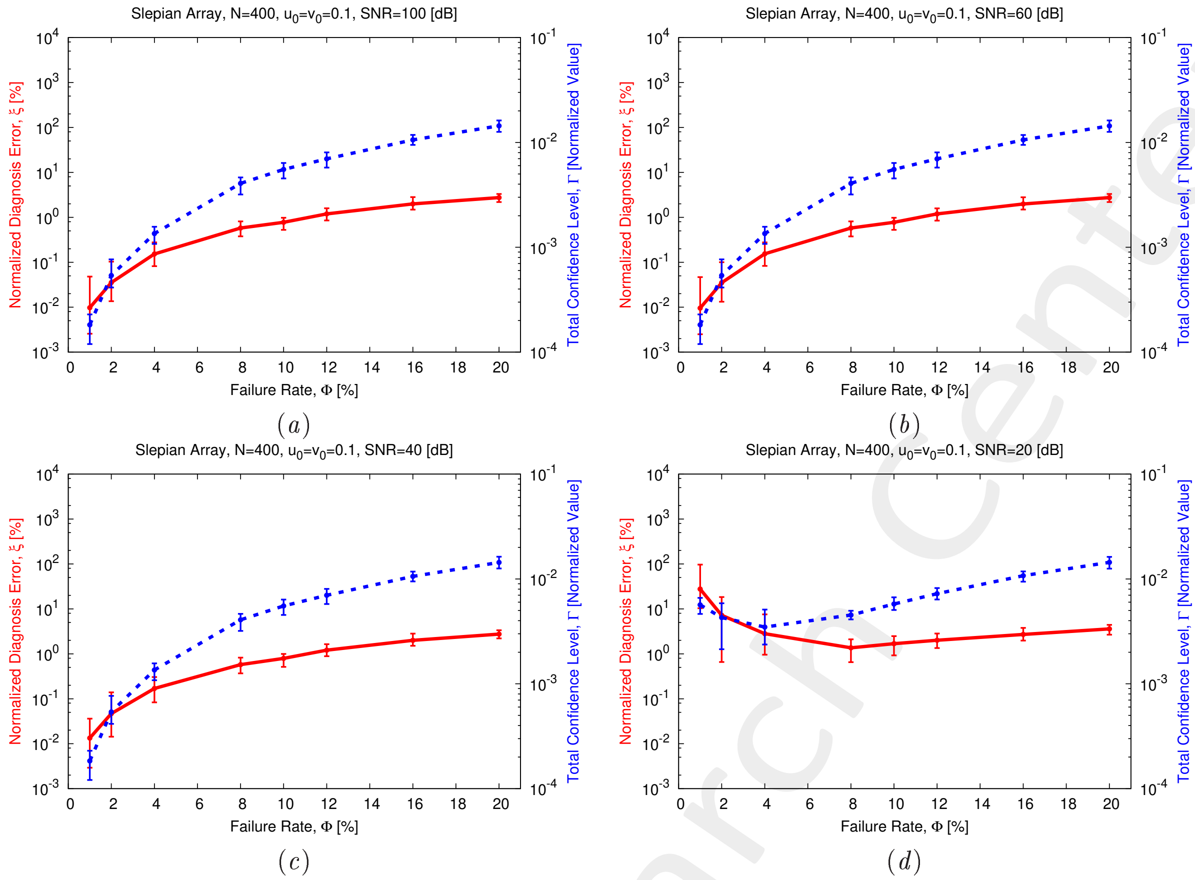

Figure 16: Slepian Array ($N = 400$, $u_0 = v_0 = 0.1$) - Behavior of the average, minimum and maximum diagnosis error (ξ) and total confidence level (Γ) versus the failure rate (Φ), for (a) $SNR = 100$ [dB], (b) $SNR = 60$ [dB], (c) $SNR = 40$ [dB], and (d) $SNR = 20$ [dB].

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More information on the topics of this document can be found in the following list of references.

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