
Sparse Target Reconstruction via Multi-Resolution Bayesian Compressive Sensing

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1 Comparison with IMSA-BCS based on First Order Born Approximation

1.1 E-shaped Object, $\ell = 1.5\lambda$

Test Case Description

Direct solver:

- Side of the investigation domain: $L = 6.0\lambda$
- Cubic domain divided in $\sqrt{D} \times \sqrt{D}$ cells
- Number of cells for the direct solver: $D = 1600$ (discretization = $\lambda/10$)

Investigation domain:

- Cubic domain divided in $\sqrt{N} \times \sqrt{N}$ cells
- Number of cells for the inversion:
 - First Step IMSA: $N^{(1)} = 100$ (discretization = $\lambda/10$)
 - Following Steps IMSA: $N^{(i)}$ not fixed, defined according to the estimated $RoI \mathcal{D}^{(i)}$

Measurement domain:

- Total number of measurements: $M = 60$
- Measurement points placed on circles of radius $\rho = 4.5\lambda$

Sources:

- Plane waves
- Number of views: $V = 60$; $\theta_{inc}^v = 0 + (v - 1) \times (360/V)$
- Amplitude: $A = 1.0$
- Frequency: $F = 300$ MHz ($\lambda = 1$)

Background:

- $\varepsilon_r = 1.0$
- $\sigma = 0$ [S/m]

Scatterer

- E-shaped object, $\ell = 1.5\lambda$
- $\varepsilon_r \in \{1.05, 1.10, 1.20, 1.50, 2.00\}$
- $\sigma = 0$ [S/m]

1.1.1 E-shaped Object, $\ell = 1.5\lambda$, $\tau = 0.10$ - (IMSA-BCS) CSI vs. BORN reconstructed profiles

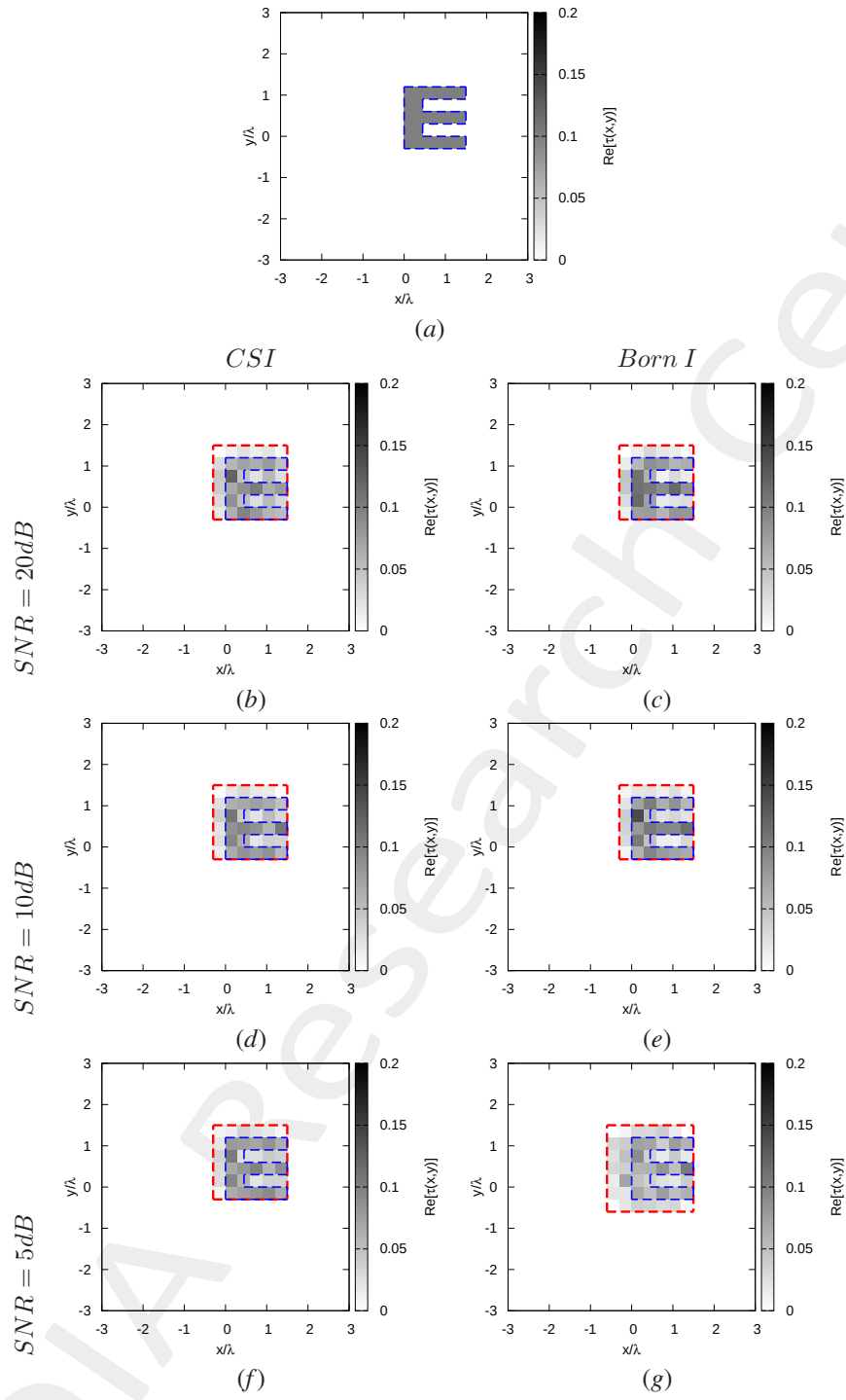


Figure 1: *E-shaped Object*, $\ell = 1.5\lambda$, $\tau = 0.10$ - *IMSA-BCS CSI vs. Born I* - (a) Actual profile, (b)(d)(f) *IMSA – BCS CSI* and (c)(e)(g) *IMSA – BCS Born* reconstructed profiles for (b)(c) $SNR = 20$ [dB], (d)(e) $SNR = 10$ [dB] and (f)(g) $SNR = 5$ [dB].

	$SNR = 50dB$	
	CSI	$BORN$
ξ_{tot}	2.38×10^{-3}	2.35×10^{-3}
ξ_{int}	2.57×10^{-2}	2.41×10^{-2}
ξ_{ext}	1.20×10^{-3}	1.10×10^{-3}
	$SNR = 20dB$	
	CSI	$BORN$
ξ_{tot}	2.50×10^{-3}	2.18×10^{-3}
ξ_{int}	2.61×10^{-2}	2.02×10^{-2}
ξ_{ext}	1.31×10^{-3}	1.11×10^{-3}
	$SNR = 10dB$	
	CSI	$BORN$
ξ_{tot}	2.27×10^{-3}	2.20×10^{-3}
ξ_{int}	2.30×10^{-2}	2.00×10^{-2}
ξ_{ext}	1.20×10^{-3}	1.11×10^{-3}
	$SNR = 5dB$	
	CSI	$BORN$
ξ_{tot}	2.22×10^{-3}	3.79×10^{-3}
ξ_{int}	2.15×10^{-2}	3.29×10^{-2}
ξ_{ext}	1.21×10^{-3}	2.17×10^{-3}

Table I: *E-shaped Object*, $\ell = 1.5\lambda$, $\tau = 0.10$ - *CSI* vs. *BORN* - Reconstruction errors: total (ξ_{tot}), internal (ξ_{int}) and external (ξ_{ext}) errors.

1.1.2 E-shaped Object, $\ell = 1.5\lambda$, $\tau = 0.20$ - (IMSA-BCS) CSI vs. BORN reconstructed profiles

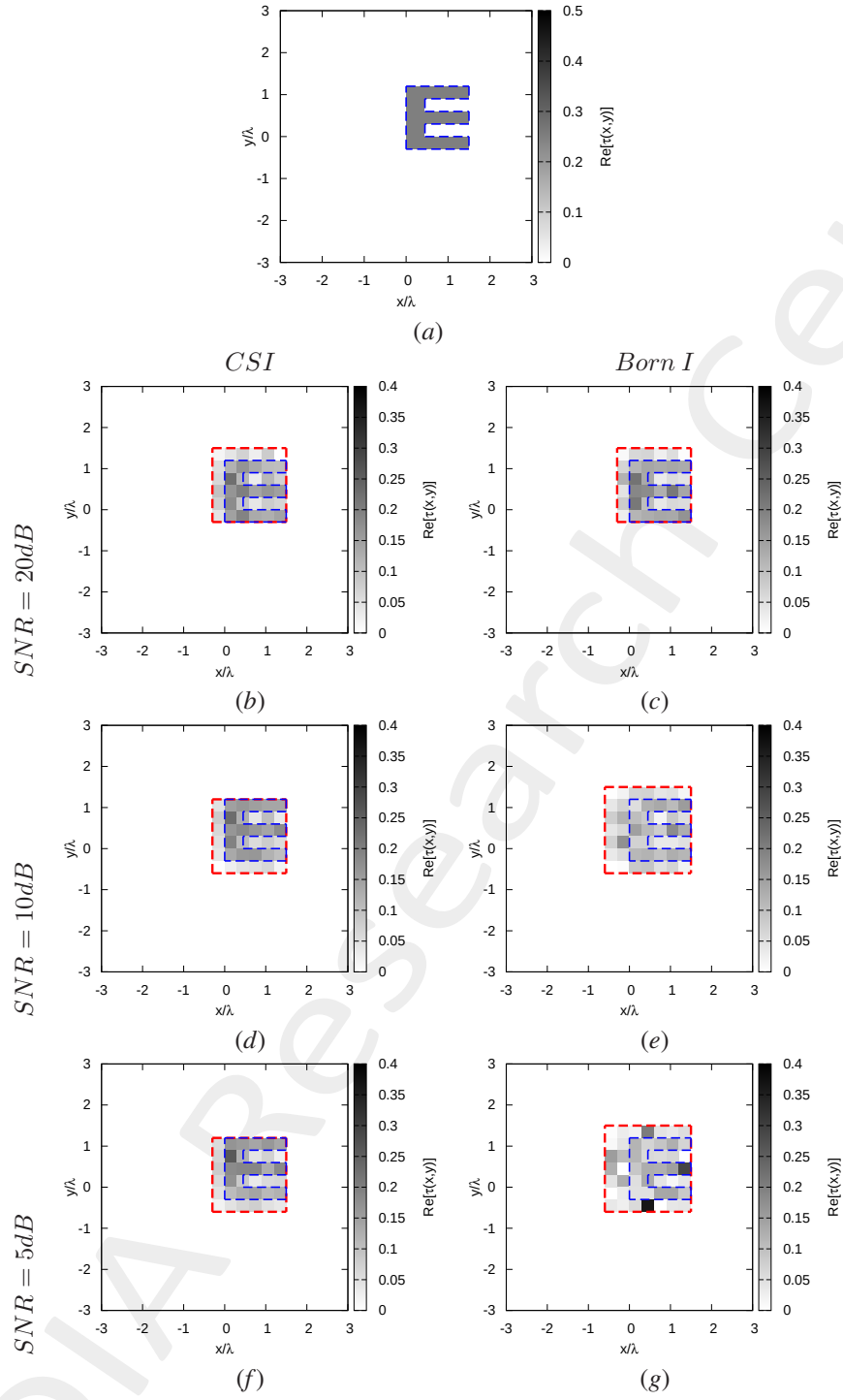


Figure 2: *E-shaped Object*, $\ell = 1.5\lambda$, $\tau = 0.20$ - *IMSA-BCS CSI vs. Born I* - (a) Actual profile, (b)(d)(f) *IMSA-BCS CSI* and (c)(e)(g) *IMSA-BCS Born I* reconstructed profiles for (b)(c) $SNR = 20$ [dB], (d)(e) $SNR = 10$ [dB] and (f)(g) $SNR = 5$ [dB].

	$SNR = 50dB$	
	CSI	$BORN$
ξ_{tot}	4.23×10^{-3}	5.14×10^{-3}
ξ_{int}	3.86×10^{-2}	4.33×10^{-2}
ξ_{ext}	2.40×10^{-3}	2.23×10^{-3}
	$SNR = 20dB$	
	CSI	$BORN$
ξ_{tot}	4.82×10^{-3}	5.42×10^{-3}
ξ_{int}	4.38×10^{-2}	4.39×10^{-2}
ξ_{ext}	2.81×10^{-3}	2.52×10^{-3}
	$SNR = 10dB$	
	CSI	$BORN$
ξ_{tot}	4.45×10^{-3}	8.31×10^{-3}
ξ_{int}	4.03×10^{-2}	7.27×10^{-2}
ξ_{ext}	2.55×10^{-3}	4.44×10^{-3}
	$SNR = 5dB$	
	CSI	$BORN$
ξ_{tot}	4.62×10^{-3}	1.15×10^{-2}
ξ_{int}	4.36×10^{-2}	8.38×10^{-2}
ξ_{ext}	2.61×10^{-3}	5.40×10^{-3}

Table II: *E-shaped Object*, $\ell = 1.5\lambda$, $\tau = 0.20$ - *CSI* vs. *BORN*- Reconstruction errors: total (ξ_{tot}), internal (ξ_{int}) and external (ξ_{ext}) errors.

1.1.3 E-shaped Object, $\ell = 1.5\lambda$, $\tau = 0.50$ - (IMSA-BCS) CSI vs. BORN reconstructed profiles

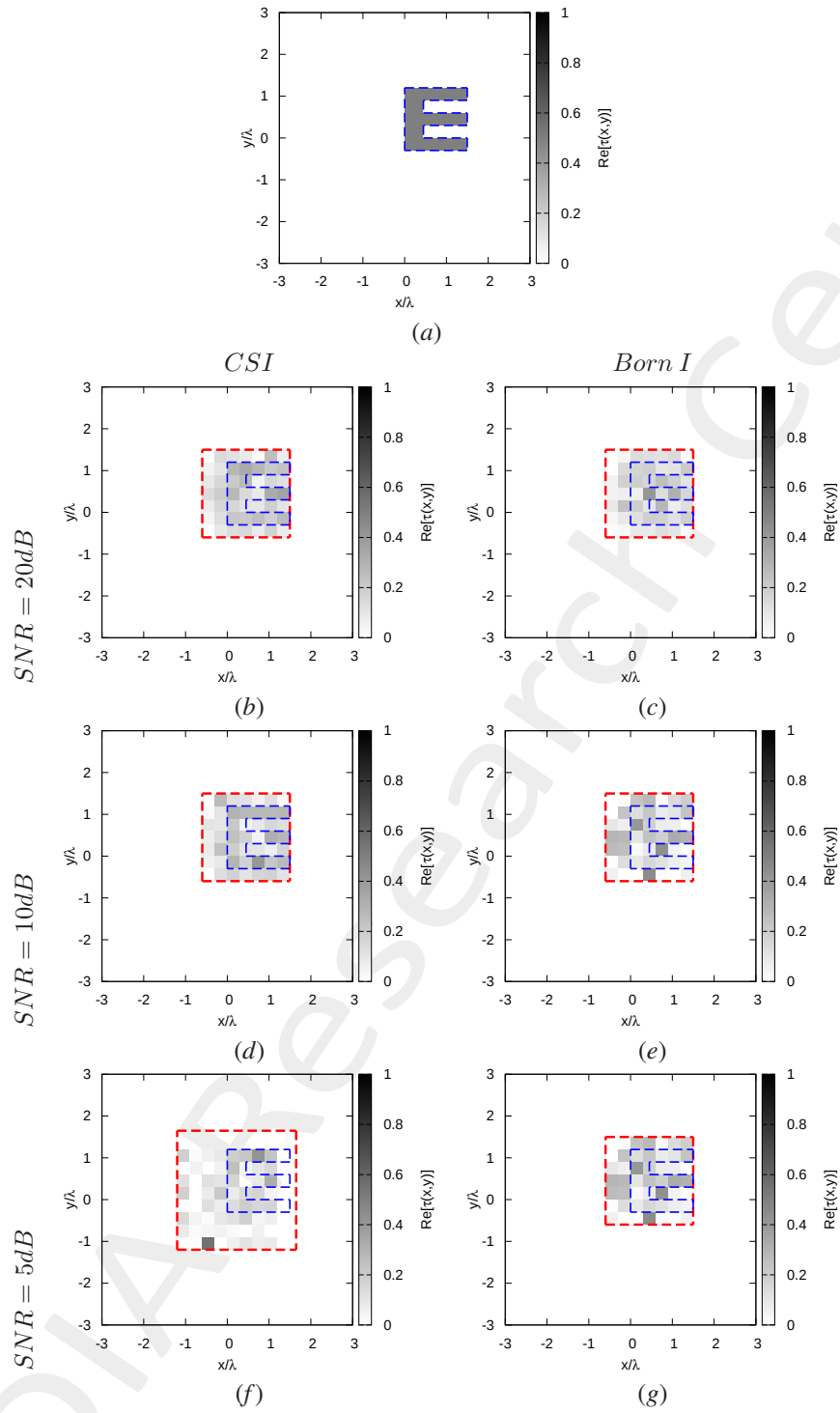


Figure 3: *E-shaped Object*, $\ell = 1.5\lambda$, $\tau = 0.50$ - *IMSA-BCS CSI vs. Born I* - (a) Actual profile, (b)(d)(f) *IMSA-BCS CSI* and (c)(e)(g) *IMSA-BCS Born* reconstructed profiles for (b)(c) $SNR = 20$ [dB], (d)(e) $SNR = 10$ [dB] and (f)(g) $SNR = 5$ [dB].

	$SNR = 50dB$	
	CSI	$BORN$
ξ_{tot}	1.65×10^{-2}	2.26×10^{-2}
ξ_{int}	1.64×10^{-1}	2.18×10^{-1}
ξ_{ext}	8.76×10^{-3}	8.38×10^{-3}
	$SNR = 20dB$	
	CSI	$BORN$
ξ_{tot}	1.76×10^{-2}	2.22×10^{-2}
ξ_{int}	1.76×10^{-1}	2.16×10^{-1}
ξ_{ext}	9.07×10^{-3}	8.28×10^{-3}
	$SNR = 10dB$	
	CSI	$BORN$
ξ_{tot}	1.72×10^{-2}	2.78×10^{-2}
ξ_{int}	1.72×10^{-1}	2.23×10^{-1}
ξ_{ext}	8.78×10^{-3}	1.12×10^{-2}
	$SNR = 5dB$	
	CSI	$BORN$
ξ_{tot}	2.75×10^{-2}	1.50×10^{-2}
ξ_{int}	3.33×10^{-1}	3.33×10^{-1}
ξ_{ext}	1.12×10^{-2}	0.00×10^{-1}

Table III: *E-shaped Object*, $\ell = 1.5\lambda$, $\tau = 0.50$ - *CSI* vs. *BORN* - Reconstruction errors: total (ξ_{tot}), internal (ξ_{int}) and external (ξ_{ext}) errors.

1.1.4 E-shaped Object, $\ell = 1.5\lambda$ - (IMSA-BCS) CSI vs. BORN errors resume vs. τ

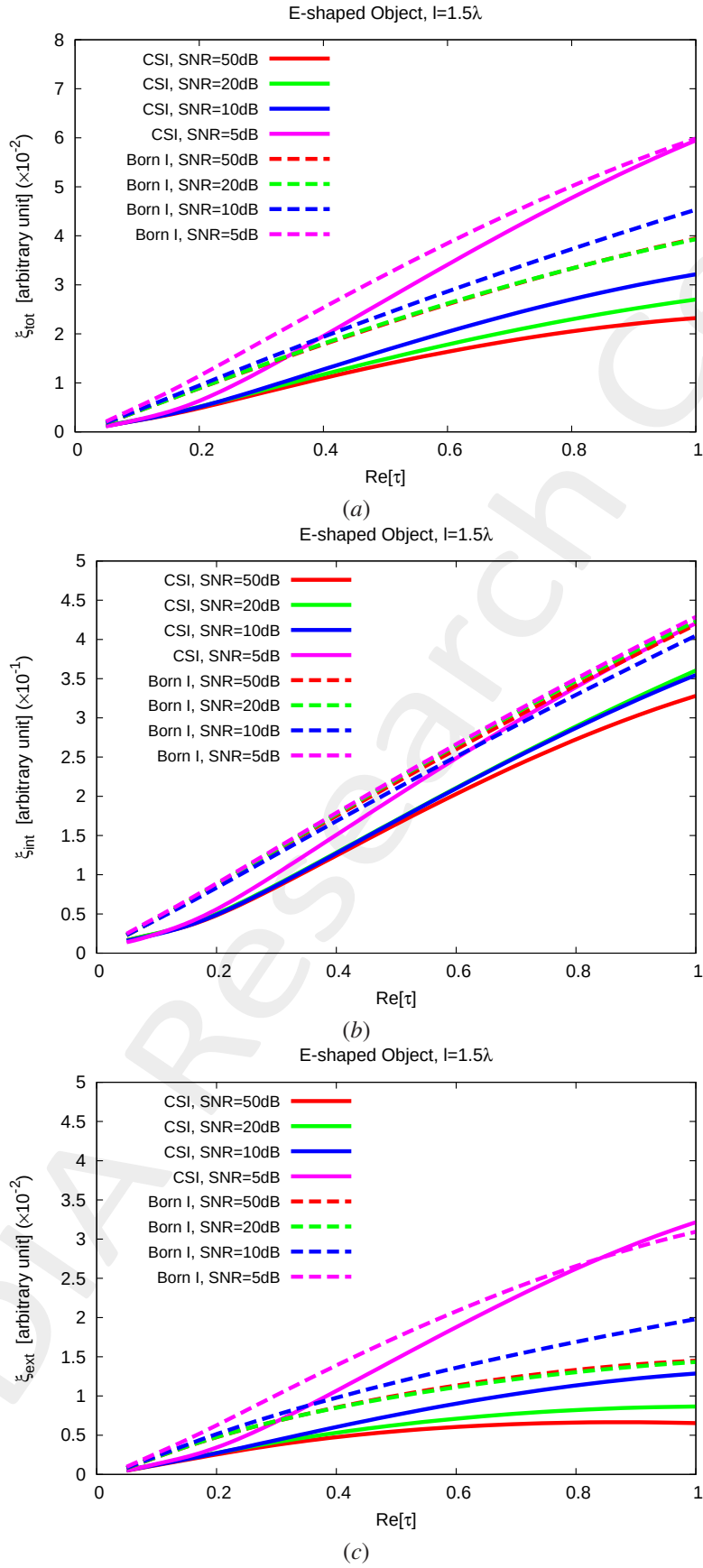


Figure 4: *E-shaped Object*, $\ell = 1.5\lambda$ - Reconstruction errors vs. τ : (a) total error, (b) internal error and (c) external error.

1.2 Hollow Square, $\ell = 1.5\lambda$

Test Case Description

Direct solver:

- Side of the investigation domain: $L = 6.0\lambda$
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- Number of cells for the direct solver: $D = 1600$ (discretization = $\lambda/10$)

Investigation domain:

- Cubic domain divided in $\sqrt{N} \times \sqrt{N}$ cells
- Number of cells for the inversion:
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Measurement domain:

- Total number of measurements: $M = 60$
- Measurement points placed on circles of radius $\rho = 4.5\lambda$

Sources:

- Plane waves
- Number of views: $V = 60$; $\theta_{inc}^v = 0 + (v - 1) \times (360/V)$
- Amplitude: $A = 1.0$
- Frequency: $F = 300$ MHz ($\lambda = 1$)

Background:

- $\varepsilon_r = 1.0$
- $\sigma = 0$ [S/m]

Scatterer

- Hollow square object, $\ell = 1.5\lambda$
- $\varepsilon_r \in \{1.10, 1.20, 1.25\}$
- $\sigma = 0$ [S/m]

1.2.1 Hollow Square, $\ell = 1.5\lambda$, $\tau = 0.10$ - (IMSA-BCS) CSI vs. BORN reconstructed profiles

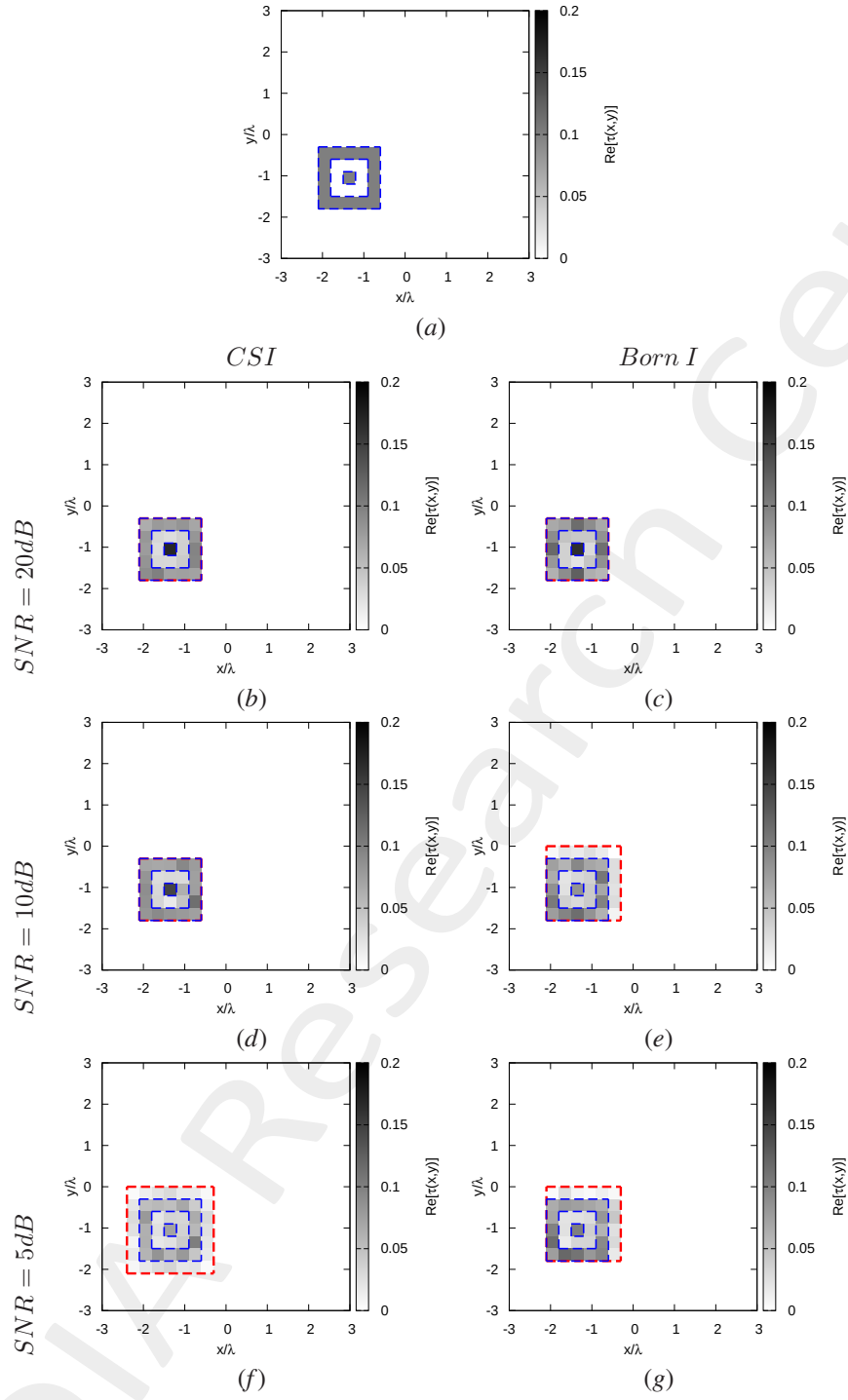


Figure 5: *Hollow Square*, $\ell = 1.5\lambda$, $\tau = 0.10$ - *IMSA-BCS* vs. *BARE-BCS* - (a) Actual profile, (b)(d)(f) *IMSA-BCS* and *BARE-BCS* reconstructed profiles for (b)(c) $SNR = 20$ [dB], (d)(e) $SNR = 10$ [dB] and (f)(g) $SNR = 5$ [dB].

	$SNR = 50dB$	
	CSI	$BORN$
ξ_{tot}	1.58×10^{-3}	2.05×10^{-3}
ξ_{int}	2.17×10^{-2}	1.99×10^{-2}
ξ_{ext}	6.33×10^{-4}	1.06×10^{-3}
	$SNR = 20dB$	
	CSI	$BORN$
ξ_{tot}	1.56×10^{-3}	1.86×10^{-3}
ξ_{int}	2.00×10^{-2}	2.26×10^{-2}
ξ_{ext}	6.82×10^{-4}	7.88×10^{-4}
	$SNR = 10dB$	
	CSI	$BORN$
ξ_{tot}	1.56×10^{-3}	2.13×10^{-3}
ξ_{int}	1.94×10^{-2}	1.84×10^{-2}
ξ_{ext}	7.20×10^{-4}	1.19×10^{-3}
	$SNR = 5dB$	
	CSI	$BORN$
ξ_{tot}	3.15×10^{-3}	2.17×10^{-3}
ξ_{int}	3.28×10^{-2}	1.82×10^{-2}
ξ_{ext}	1.75×10^{-3}	1.11×10^{-3}

Table IV: *Hollow Square*, $\ell = 1.5\lambda$, $\tau = 0.10$ - CSI vs. $BORN$ - Reconstruction errors: total (ξ_{tot}), internal (ξ_{int}) and external (ξ_{ext}) errors.

1.2.2 Hollow Square, $\ell = 1.5\lambda$, $\tau = 0.20$ - (IMSA-BCS) CSI vs. BORN reconstructed profiles

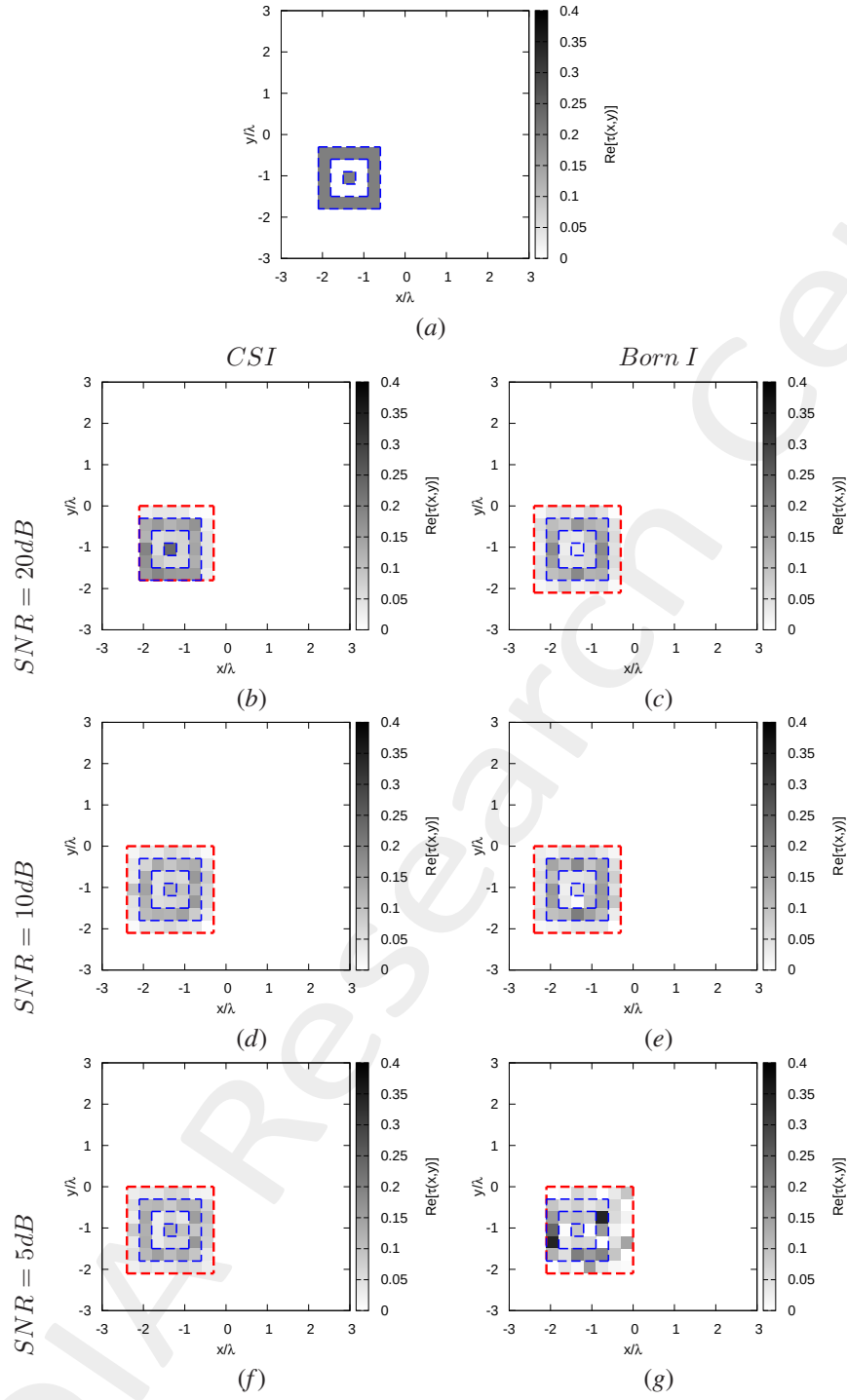


Figure 6: *Hollow Square*, $\ell = 1.5\lambda$, $\tau = 0.20$ - *IMSA-BCS CSI* vs. *Born I* - (a) Actual profile, (b)(d)(f) *IMSA-BCS CSI* and (c)(e)(g) *IMSA-BCS Born I* reconstructed profiles for (b)(c) $\text{SNR} = 20$ [dB], (d)(e) $\text{SNR} = 10$ [dB] and (f)(g) $\text{SNR} = 5$ [dB].

	$SNR = 50dB$	
	CSI	$BORN$
ξ_{tot}	4.48×10^{-3}	6.54×10^{-3}
ξ_{int}	4.44×10^{-2}	5.72×10^{-2}
ξ_{ext}	2.54×10^{-3}	3.40×10^{-3}
	$SNR = 20dB$	
	CSI	$BORN$
ξ_{tot}	3.98×10^{-3}	6.68×10^{-3}
ξ_{int}	4.05×10^{-2}	5.84×10^{-2}
ξ_{ext}	2.23×10^{-3}	3.50×10^{-3}
	$SNR = 10dB$	
	CSI	$BORN$
ξ_{tot}	6.72×10^{-3}	6.62×10^{-3}
ξ_{int}	6.62×10^{-2}	5.78×10^{-2}
ξ_{ext}	3.99×10^{-3}	3.52×10^{-3}
	$SNR = 5dB$	
	CSI	$BORN$
ξ_{tot}	6.82×10^{-3}	1.04×10^{-2}
ξ_{int}	6.62×10^{-2}	8.28×10^{-2}
ξ_{ext}	4.02×10^{-3}	4.05×10^{-3}

Table V: *Hollow Square*, $\ell = 1.5\lambda$, $\tau = 0.20$ - CSI vs. $BORN$ - Reconstruction errors: total (ξ_{tot}), internal (ξ_{int}) and external (ξ_{ext}) errors.

1.2.3 Hollow Square, $\ell = 1.5\lambda$, $\tau = 0.25$ - (IMSA-BCS) CSI vs. BORN reconstructed profiles

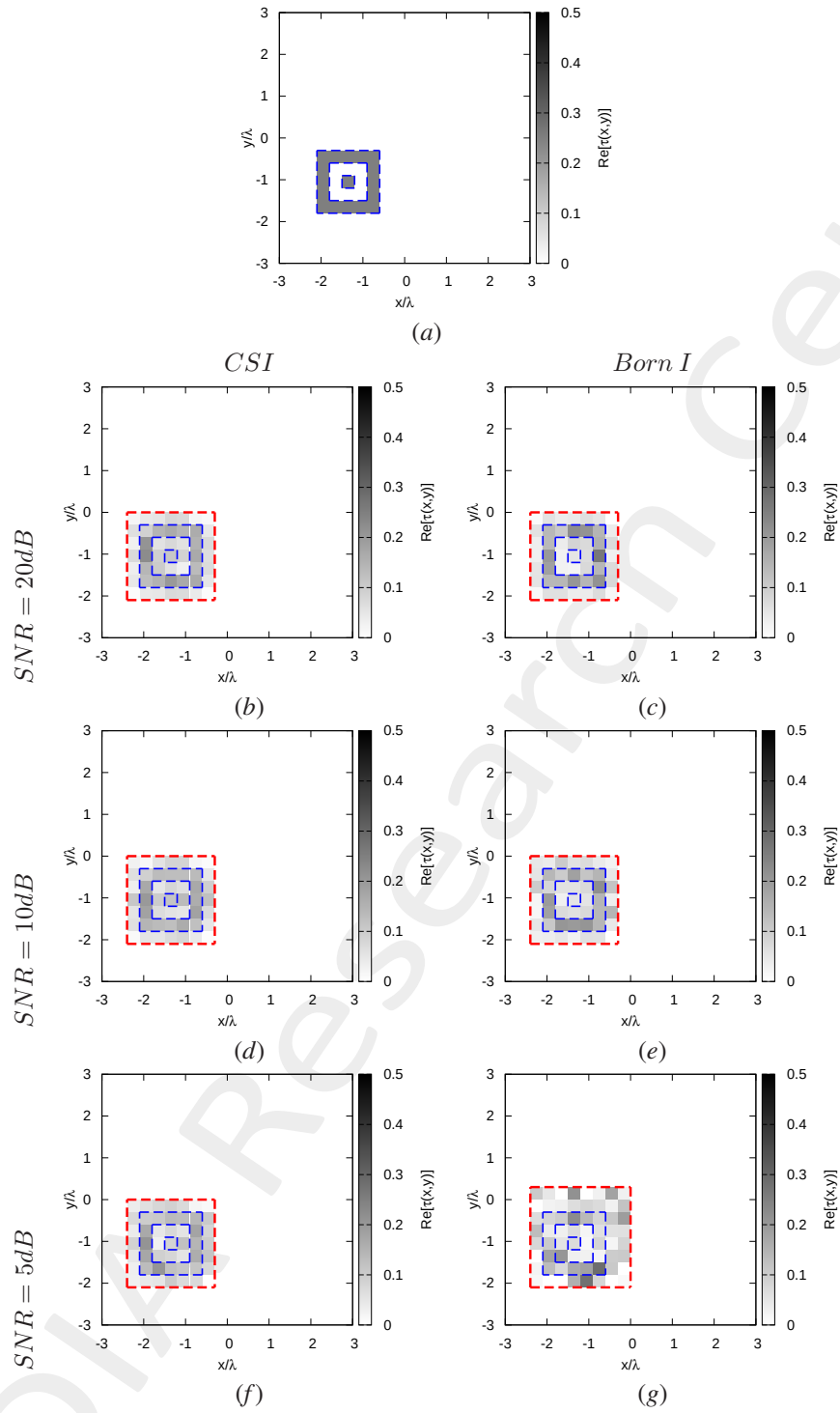


Figure 7: Hollow Square, $\ell = 1.5\lambda$, $\tau = 0.25$ - IMSA-BCS CSI vs. Born I - (a) Actual profile, (b)(d)(f) IMSA - BCS CSI and (c)(e)(g) IMSA - BCS Born I reconstructed profiles for (b)(c) SNR = 20 [dB], (d)(e) SNR = 10 [dB] and (f)(g) SNR = 5 [dB].

	$SNR = 50dB$	
	CSI	$BORN$
ξ_{tot}	7.40×10^{-3}	7.73×10^{-3}
ξ_{int}	7.43×10^{-2}	6.61×10^{-2}
ξ_{ext}	4.22×10^{-3}	3.64×10^{-3}
	$SNR = 20dB$	
	CSI	$BORN$
ξ_{tot}	7.66×10^{-3}	8.00×10^{-3}
ξ_{int}	7.49×10^{-2}	6.96×10^{-2}
ξ_{ext}	4.45×10^{-3}	3.87×10^{-3}
	$SNR = 10dB$	
	CSI	$BORN$
ξ_{tot}	8.52×10^{-3}	9.22×10^{-3}
ξ_{int}	8.31×10^{-2}	7.93×10^{-2}
ξ_{ext}	5.00×10^{-3}	4.60×10^{-3}
	$SNR = 5dB$	
	CSI	$BORN$
ξ_{tot}	9.14×10^{-3}	2.20×10^{-2}
ξ_{int}	8.62×10^{-2}	1.08×10^{-1}
ξ_{ext}	5.42×10^{-3}	1.04×10^{-2}

Table VI: *Hollow Square*, $\ell = 1.5\lambda$, $\tau = 0.25$ - CSI vs. $BORN$ - Reconstruction errors: total (ξ_{tot}), internal (ξ_{int}) and external (ξ_{ext}) errors.

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