Reconfigurable Linear Antenna Arrays: Tolerance Analysis Using Interval Arithmetic

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1 Fair Comparison: Tolerances over the Σ beam feeding network

In this paragraph a similar analysis to the one of Section 5 is reported. In this case we are considering:

- interval widths equal to 100% of the full normalized amplitude range (symmetric w.r.t. nominal value);
- two different scenarios (see Fig. 1 and 2) to be compared :
  1. in the first case we consider amplitude errors on all P common elements;
  2. in the second case we consider errors on the Σ beam feeding network's independent elements, considering all the possible combinations of P among Q.
- In the latter case the number of faulty elements F and the total amount of tolerance T are kept fixed (see Tab. 1).

P	2	4	6	8
F	2	4	6	8
T	1.7617	3.6347	5.5724	7.4232

Table 1. Number of faulty Σ network's elements (F) and total tolerance (T).

SLL:

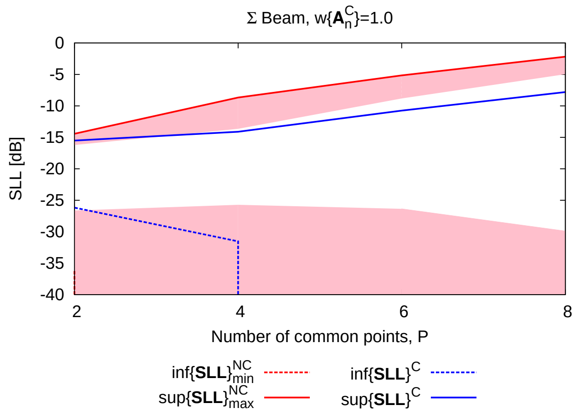

Figure 3. Sum Pattern **SLL** vs P

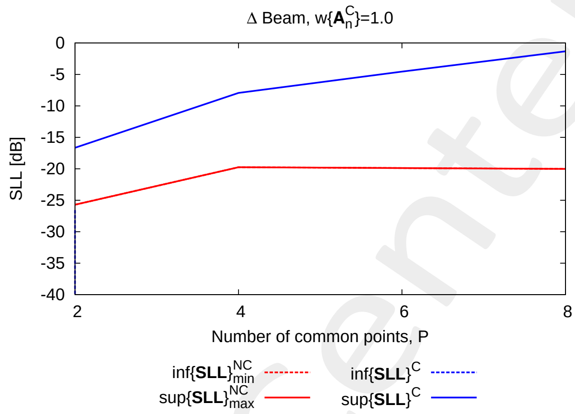

Figure 4. Difference Pattern **SLL** vs P

P	$\inf\{\mathbf{SLL}\}^C$	$\sup\{\mathbf{SLL}\}^C$	$\inf\{\mathbf{SLL}\}^{NC}_{min}$	$\inf\{\mathbf{SLL}\}^{NC}_{max}$	$\sup\{\mathbf{SLL}\}^{NC}_{min}$	$\sup\{\mathbf{SLL}\}^{NC}_{max}$
2	-26.17	-15.51	-36.24	-26.63	-16.21	-14.42
4	-31.53	-14.1	$-\infty$	-25.72	-13.65	-8.67
6	$-\infty$	-10.74	$-\infty$	-26.36	-8.81	-5.14
8	$-\infty$	-10.8	$-\infty$	-29.89	-4.96	-2.18

Table 2. Sum Pattern **SLL** values

P	$\inf\{\mathbf{SLL}\}^C$	$\sup\{\mathbf{SLL}\}^C$	$\inf\{\mathbf{SLL}\}^{NC}_{min}$	$\inf\{\mathbf{SLL}\}^{NC}_{max}$	$\sup\{\mathbf{SLL}\}^{NC}_{min}$	$\sup\{\mathbf{SLL}\}^{NC}_{max}$
2	-26.56	-16.65	-25.7	-25.7	-25.7	-25.7
4	$-\infty$	-7.93	-19.74	-19.74	-19.74	-19.74
6	$-\infty$	-4.55	-19.88	-19.88	-19.88	-19.88
8	$-\infty$	-1.32	-20.01	-20.01	-20.01	-20.01

Table 3. Difference Pattern **SLL** values

BW:

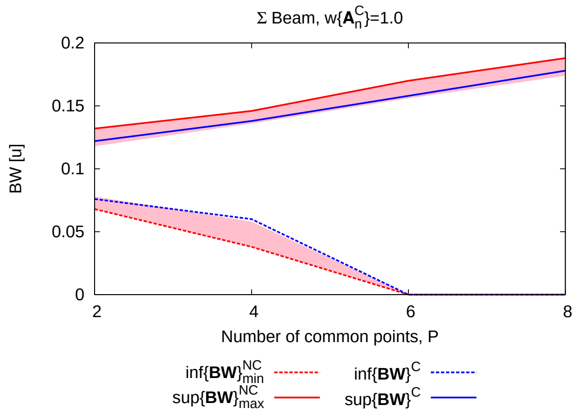

Figure 5. Sum Pattern $\mathbf{BW}$ vs P

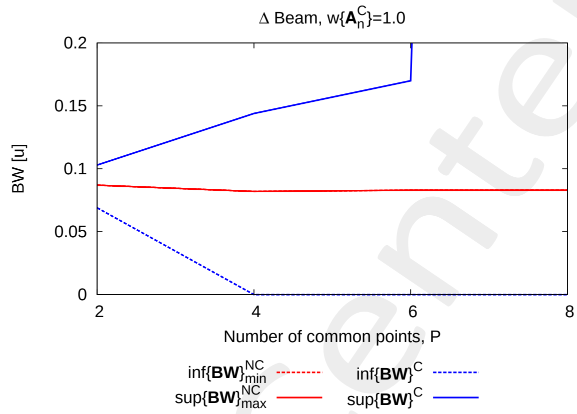

Figure 6. Difference Pattern $\mathbf{BW}$ vs P

P	$\inf\{\mathbf{BW}\}^C$	$\sup\{\mathbf{BW}\}^C$	$\inf\{\mathbf{BW}\}^{NC}_{min}$	$\inf\{\mathbf{BW}\}^{NC}_{max}$	$\sup\{\mathbf{BW}\}^{NC}_{min}$	$\sup\{\mathbf{BW}\}^{NC}_{max}$
2	0.076	0.122	0.068	0.078	0.118	0.132
4	0.06	0.138	0.038	0.058	0.136	0.146
6	0.0	0.158	0.0	0.0	0.156	0.170
8	0.0	0.178	0.0	0.0	0.174	0.188

Table 4. Sum Pattern $\mathbf{BW}$ values

P	$\inf\{\mathbf{BW}\}^C$	$\sup\{\mathbf{BW}\}^C$	$\inf\{\mathbf{BW}\}^{NC}_{min}$	$\inf\{\mathbf{BW}\}^{NC}_{max}$	$\sup\{\mathbf{BW}\}^{NC}_{min}$	$\sup\{\mathbf{BW}\}^{NC}_{max}$
2	0.069	0.103	0.087	0.087	0.087	0.087
4	0.0	0.144	0.082	0.082	0.082	0.082
6	0.0	0.17	0.083	0.083	0.083	0.083
8	0.0	4.0	0.083	0.083	0.083	0.083

Table 5. Difference Pattern $\mathbf{BW}$ values

Remembering that:

- $$\chi_{\{\cdot\}} = \left(\sup\{\cdot\}^{\Sigma}_{max} - \inf\{\cdot\}^{\Sigma}_{min}\right) + \left(\sup\{\cdot\}^{\Delta}_{max} - \inf\{\cdot\}^{\Delta}_{min}\right) \tag{1}$$

where $\sup / \inf\{\cdot\}_{max}$ is the maximum $\sup / \inf$ among all the considered combination of faulty elements; $\sup / \inf\{\cdot\}_{min}$ is the respective minimum value.

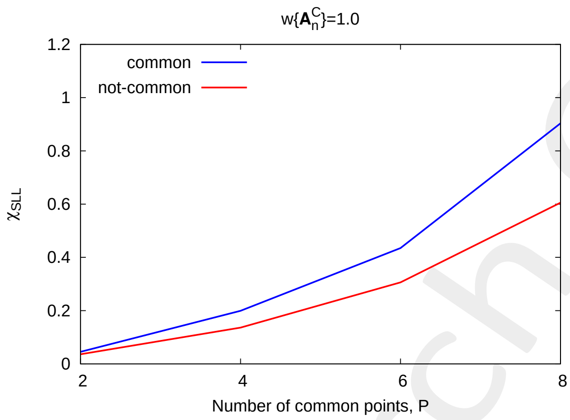

Figure 7. The χ_{SLL}relative to the SLL descriptor is plotted, for the common and not-common cases

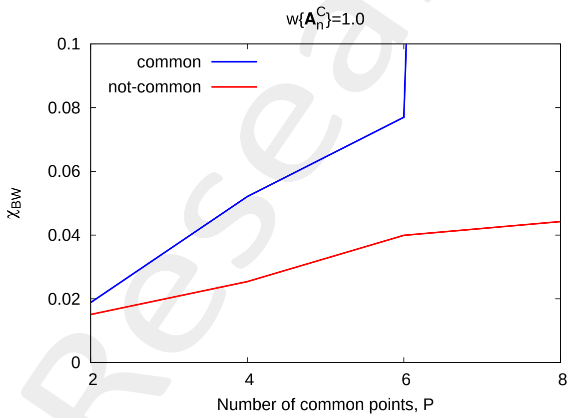

Figure 8. The χ_{BW}relative to the BW descriptor is plotted, for the common and not-common cases

Observations:

- For both the SLL and BW parameters the tolerance is higher in case of faulty common elements.

2 Fair Comparison: Tolerances over the Δ beam feeding network

In this paragraph a similar analysis to the one of Section 5 is reported. In this case we are considering:

- interval widths equal to 100% of the full normalized amplitude range (symmetric w.r.t. nominal value);
- two different scenarios (see Fig. 1 and 2) to be compared :
  1. in the first case we consider amplitude errors on all P common elements;
  2. in the second case we consider errors on the Δ beam feeding network's independent elements, considering all the possible combinations of P among Q.
- In the latter case the number of faulty elements F and the total amount of tolerance T are kept fixed (see Tab. 1).

P	2	4	6	8
F	2	4	6	8
T	1.7617	3.6347	5.5724	7.4232

Table 1. Number of faulty Δ network's elements (F) and total tolerance (T).

SLL:

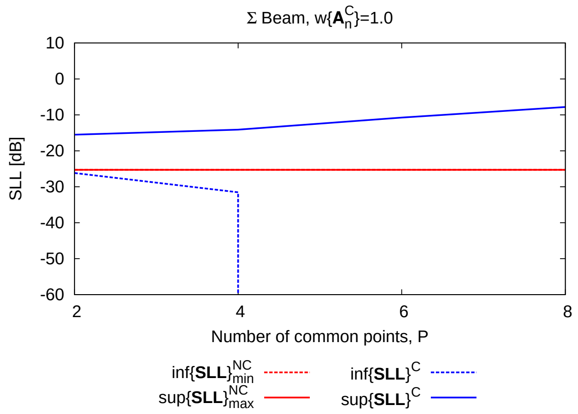

Figure 3. Sum Pattern **SLL** vs P

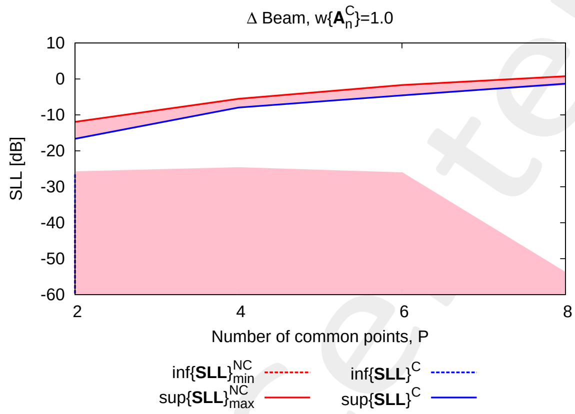

Figure 4. Difference Pattern **SLL** vs P

P	$\inf\{\mathbf{SLL}\}^C$	$\sup\{\mathbf{SLL}\}^C$	$\inf\{\mathbf{SLL}\}^{NC}_{min}$	$\inf\{\mathbf{SLL}\}^{NC}_{max}$	$\sup\{\mathbf{SLL}\}^{NC}_{min}$	$\sup\{\mathbf{SLL}\}^{NC}_{max}$
2	-26.17	-15.51	-25.28	-25.28	-25.28	-25.28
4	-31.53	-14.1	-25.28	-25.28	-25.28	-25.28
6	$-\infty$	-10.74	-25.28	-25.28	-25.28	-25.28
8	$-\infty$	-10.8	-25.28	-25.28	-25.28	-25.28

Table 2. Sum Pattern **SLL** values

P	$\inf\{\mathbf{SLL}\}^C$	$\sup\{\mathbf{SLL}\}^C$	$\inf\{\mathbf{SLL}\}^{NC}_{min}$	$\inf\{\mathbf{SLL}\}^{NC}_{max}$	$\sup\{\mathbf{SLL}\}^{NC}_{min}$	$\sup\{\mathbf{SLL}\}^{NC}_{max}$
2	-26.56	-16.65	-34.94	-25.7	-16.81	-11.94
4	$-\infty$	-7.93	$-\infty$	-24.56	-7.65	-5.52
6	$-\infty$	-4.55	$-\infty$	-25.97	-4.29	-1.69
8	$-\infty$	-1.32	$-\infty$	-53.78	-1.77	-0.75

Table 3. Difference Pattern **SLL** values

BW:

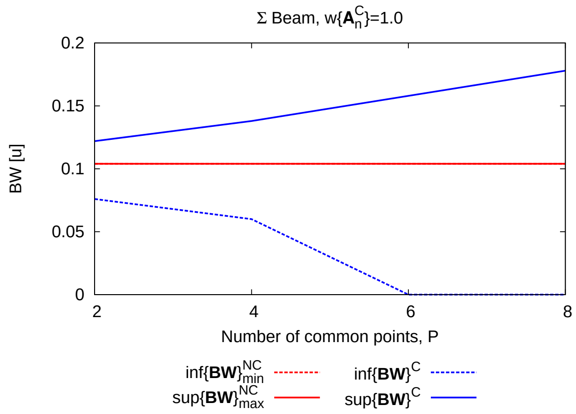

Figure 5. Sum Pattern $\mathbf{BW}$ vs P

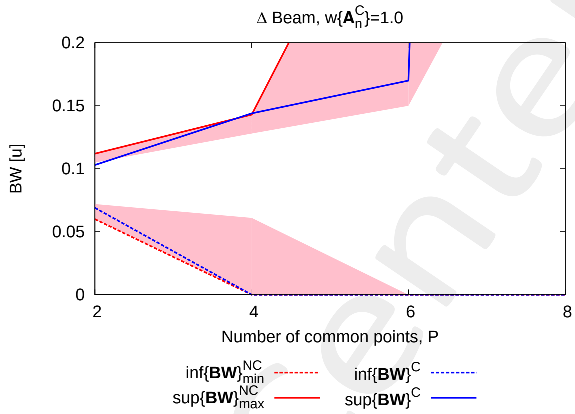

Figure 6. Difference Pattern $\mathbf{BW}$ vs P

P	$\inf\{\mathbf{BW}\}^C$	$\sup\{\mathbf{BW}\}^C$	$\inf\{\mathbf{BW}\}^{NC}_{min}$	$\inf\{\mathbf{BW}\}^{NC}_{max}$	$\sup\{\mathbf{BW}\}^{NC}_{min}$	$\sup\{\mathbf{BW}\}^{NC}_{max}$
2	0.076	0.122	0.104	0.104	0.104	0.104
4	0.06	0.138	0.104	0.104	0.104	0.104
6	0.0	0.158	0.104	0.104	0.104	0.104
8	0.0	0.178	0.104	0.104	0.104	0.104

Table 4. Sum Pattern $\mathbf{BW}$ values

P	$\inf\{\mathbf{BW}\}^C$	$\sup\{\mathbf{BW}\}^C$	$\inf\{\mathbf{BW}\}^{NC}_{min}$	$\inf\{\mathbf{BW}\}^{NC}_{max}$	$\sup\{\mathbf{BW}\}^{NC}_{min}$	$\sup\{\mathbf{BW}\}^{NC}_{max}$
2	0.069	0.103	0.06	0.072	0.105	0.112
4	0.0	0.144	0.0	0.061	0.128	0.143
6	0.0	0.17	0.0	0.0	0.15	0.384
8	0.0	4.0	0.0	0.0	0.38	4.0

Table 5. Difference Pattern $\mathbf{BW}$ values

Remembering that:

- $$\chi_{\{\cdot\}} = \left(\sup\{\cdot\}^{\Sigma}_{max} - \inf\{\cdot\}^{\Sigma}_{min}\right) + \left(\sup\{\cdot\}^{\Delta}_{max} - \inf\{\cdot\}^{\Delta}_{min}\right) \quad (2)$$

where sup / inf $\{\cdot\}_{max}$ is the maximum sup / inf among all the considered combination of faulty elements; sup / inf $\{\cdot\}_{min}$ is the respective minimum value.

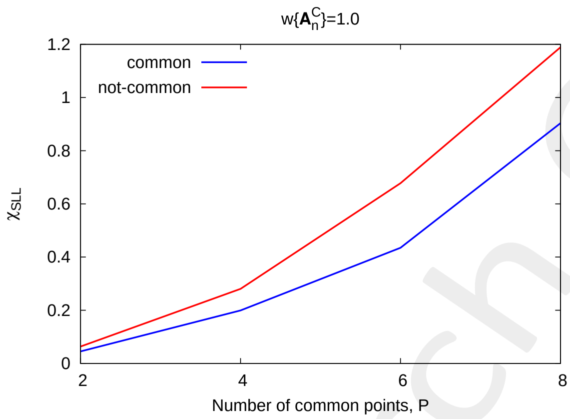

Figure 7. The χ_{SLL}relative to the SLL descriptor is plotted, for the common and not-common cases

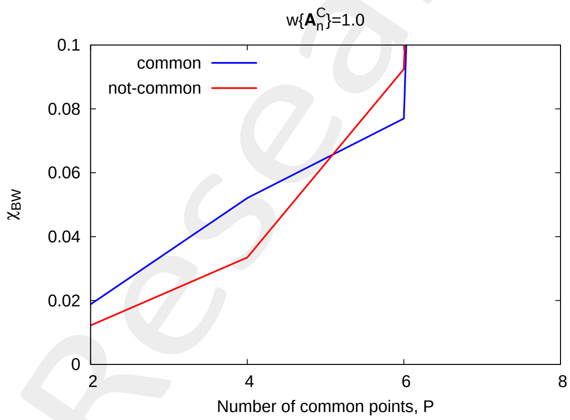

Figure 8. The χ_{BW}relative to the BW descriptor is plotted, for the common and not-common cases

Observations:

- In this case the tolerance is higher when faulty elements belong to the Δ beam forming network.
- I recall here that the Δ beam is a compromise.

3 Fair Comparison: Tolerances over the Σ and Δ beam feeding networks

In this paragraph a similar analysis to the one of Section 5 is reported. In this case we are considering:

- interval widths equal to 100% of the full normalized amplitude range (symmetric w.r.t. nominal value);
- two different scenarios (see Fig. 1 and 2) to be compared :
  1. in the first case we consider amplitude errors on all P common elements;
  2. in the second case we consider errors on the Σ and Δ beam feeding networks independent elements, considering all the possible combinations of $P/2$ among Q.
- In the latter case the number of faulty elements F and the total amount of tolerance T are kept fixed (see Tab. 1).

P	2	4	6	8
F^{Σ}	1	2	3	4
F^{Δ}	1	2	3	4
T	1.7617	3.6347	5.5724	7.4232

Table 1. Number of faulty networks elements (F^{Σ}/F^{Δ}) and total tolerance (T).

SLL:

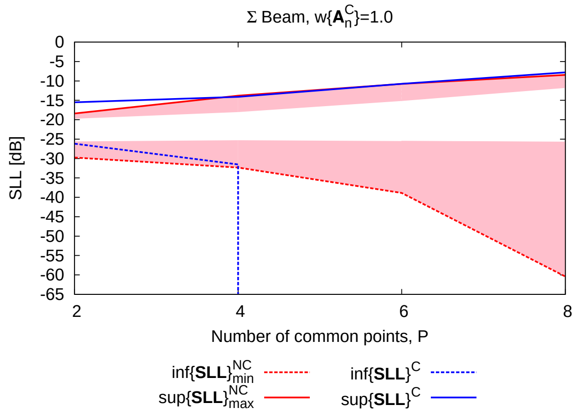

Figure 3. Sum Pattern **SLL** vs P

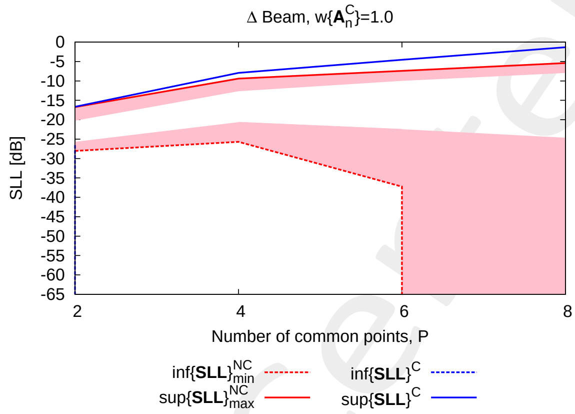

Figure 4. Difference Pattern **SLL** vs P

P	$\inf\{\mathbf{SLL}\}^C$	$\sup\{\mathbf{SLL}\}^C$	$\inf\{\mathbf{SLL}\}^{NC}_{min}$	$\inf\{\mathbf{SLL}\}^{NC}_{max}$	$\sup\{\mathbf{SLL}\}^{NC}_{min}$	$\sup\{\mathbf{SLL}\}^{NC}_{max}$
2	-26.17	-15.51	-29.74	-25.58	-19.75	-18.40
4	-31.53	-14.1	-32.33	-25.28	-18.01	-13.80
6	$-\infty$	-10.74	-38.87	-25.49	-15.16	-10.77
8	$-\infty$	-10.8	-60.41	-25.72	-11.81	-8.45

Table 2. Sum Pattern **SLL** values

P	$\inf\{\mathbf{SLL}\}^C$	$\sup\{\mathbf{SLL}\}^C$	$\inf\{\mathbf{SLL}\}^{NC}_{min}$	$\inf\{\mathbf{SLL}\}^{NC}_{max}$	$\sup\{\mathbf{SLL}\}^{NC}_{min}$	$\sup\{\mathbf{SLL}\}^{NC}_{max}$
2	-26.56	-16.65	-28.03	-25.7	-20.26	-16.74
4	$-\infty$	-7.93	-25.7	-20.64	-12.65	-9.39
6	$-\infty$	-4.55	-37.24	-22.45	-9.99	-7.41
8	$-\infty$	-1.32	$-\infty$	-24.64	-7.96	-5.4

Table 3. Difference Pattern **SLL** values

BW:

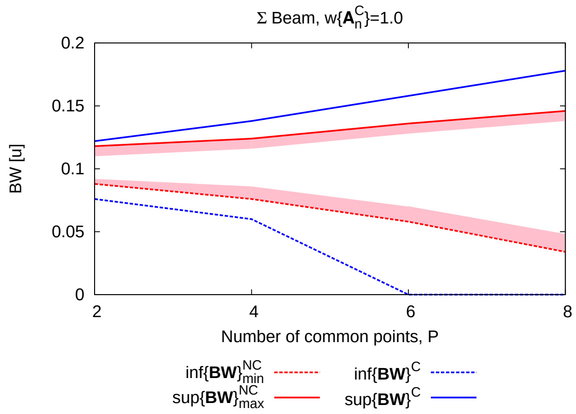

Figure 5. Sum Pattern $\mathbf{BW}$ vs P

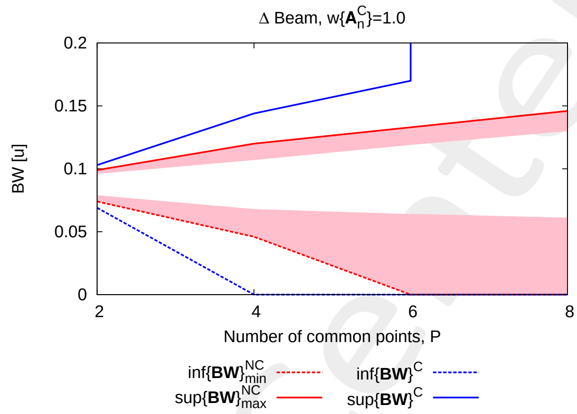

Figure 6. Difference Pattern $\mathbf{BW}$ vs P

P	$\inf\{\mathbf{BW}\}^C$	$\sup\{\mathbf{BW}\}^C$	$\inf\{\mathbf{BW}\}_{min}^{NC}$	$\inf\{\mathbf{BW}\}_{max}^{NC}$	$\sup\{\mathbf{BW}\}_{min}^{NC}$	$\sup\{\mathbf{BW}\}_{max}^{NC}$
2	0.076	0.122	0.088	0.092	0.11	0.118
4	0.06	0.138	0.076	0.086	0.116	0.124
6	0.0	0.158	0.058	0.07	0.128	0.136
8	0.0	0.178	0.034	0.048	0.138	0.146

Table 4. Sum Pattern $\mathbf{BW}$ values

P	$\inf\{\mathbf{BW}\}^C$	$\sup\{\mathbf{BW}\}^C$	$\inf\{\mathbf{BW}\}_{min}^{NC}$	$\inf\{\mathbf{BW}\}_{max}^{NC}$	$\sup\{\mathbf{BW}\}_{min}^{NC}$	$\sup\{\mathbf{BW}\}_{max}^{NC}$
2	0.069	0.103	0.074	0.079	0.096	0.099
4	0.0	0.144	0.046	0.068	0.107	0.12
6	0.0	0.17	0.0	0.064	0.119	0.133
8	0.0	4.0	0.0	0.061	0.13	0.146

Table 5. Difference Pattern $\mathbf{BW}$ values

Remembering that:

- $$\chi_{\{\cdot\}} = \left(\sup\{\cdot\}^{\Sigma}_{max} - \inf\{\cdot\}^{\Sigma}_{min}\right) + \left(\sup\{\cdot\}^{\Delta}_{max} - \inf\{\cdot\}^{\Delta}_{min}\right) \quad (3)$$

where sup / inf $\{\cdot\}_{max}$ is the maximum sup / inf among all the considered combination of faulty elements; sup / inf $\{\cdot\}_{min}$ is the respective minimum value.

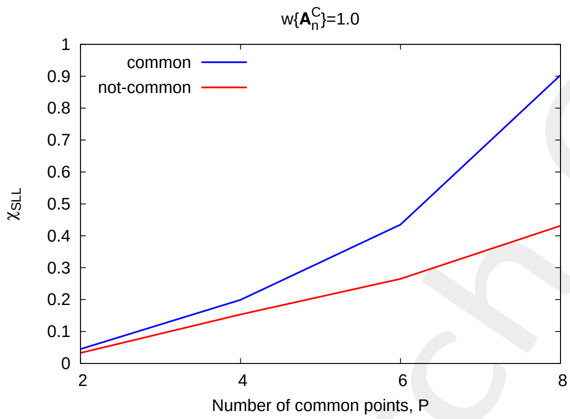

Figure 7. The χ_{SLL}relative to the SLL descriptor is plotted, for the common and not-common cases

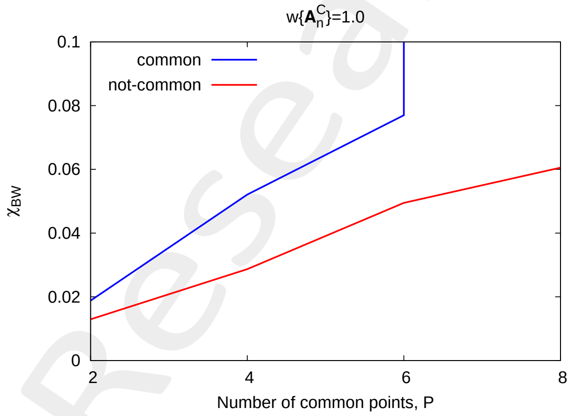

Figure 8. The χ_{BW}relative to the BW descriptor is plotted, for the common and not-common cases

Observations:

- ...

Pareto Front

Remembering that:

$$\zeta_{\{\cdot\}} = \sup\{\cdot\}_{max} - \inf\{\cdot\}_{min} \quad (4)$$

where sup / inf $\{\cdot\}_{max}$ is the maximum sup / inf among all the considered combination of faulty elements; sup / inf $\{\cdot\}_{min}$ is the respective minimum value.

SLL:

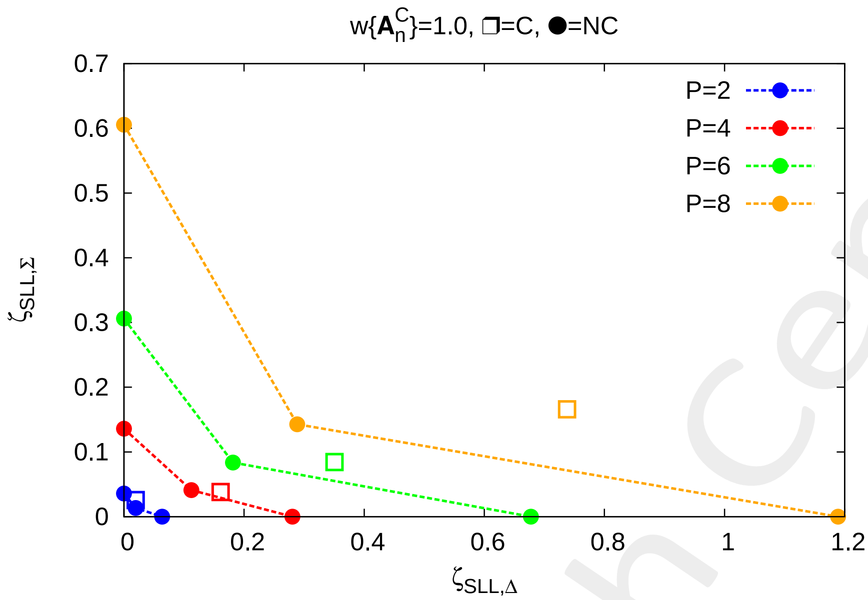

Figure 9. ζ values of the SLL, plotted in the Σ/Δ plane, when faulty elements occours on common (SQUARE) and independent (CIRCLE) elements

Observations:

- The result are very close to the "full width" intervals case.

More information on the topics of this document can be found in the following list of references.

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