Analyzing Tolerances in Monopulse Linear Antenna Arrays Using Interval Arithmetic

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2024/12/13

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1 Common / Not-common Faulty Elements Robustness Analysis

1.1 Common Elements - $P = 4$

Array Parameters:

- Number Elements: $N = 20$
- Services: Sum / Difference Beams
- Number of Common Elements: $P = 4$
- Element Spacing: $\lambda/2$

Constraints:

- Main Sum Lobe Width: $BW^{\Sigma} = 0.24u$
- Main Difference Lobe Width: $BW^{\Delta} = 0.38u$

Simulation Parameters:

- Sample Points: 2001
- Max Function Evaluations: 6000
- Max Iterations Number: 1000
- Function Tolerance: 1.0×10^{-8}
- Constraint Tolerance: 1×10^{-8}

Algorithm Behaviour:

- Simulation Time Pattern: 46 sec.

In the following figures are reported, for each iteration, the max values evaluated by the objective function and by the constraint function for the sum and difference pattern synthesis.

Sum Beam:

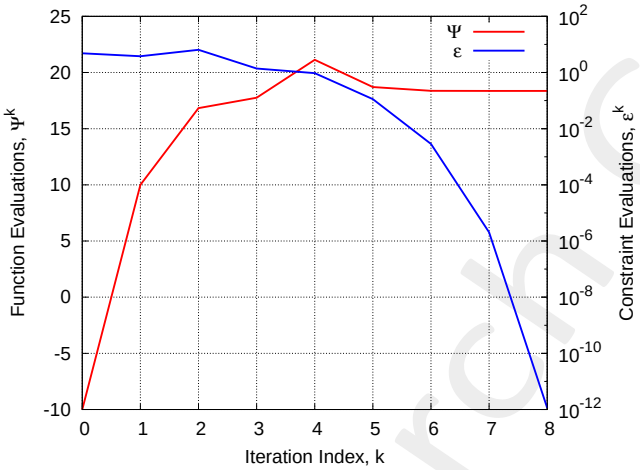

Figure 1. Sum Beam optimization's fitness

$max\{\psi(k)\}$	$min\{\varepsilon(k)\}$
18.36	1.1×10^{-12}

Table 1. Max. value evaluated by ψ; min value evaluated by ε; simulation time.

Difference Beam:

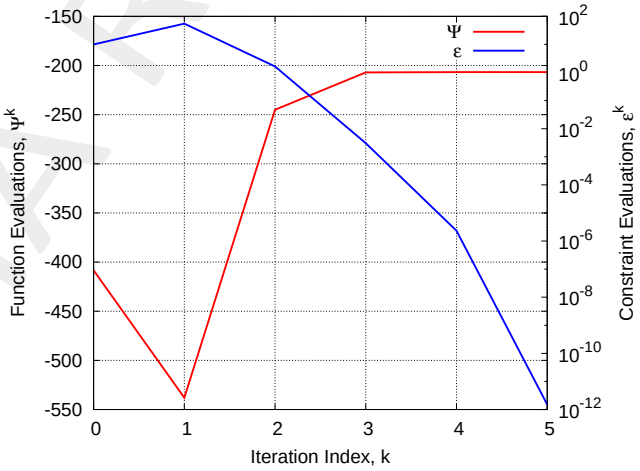

Figure 2. Difference beam optimization's fitness

$max\{\psi(k)\}$	$min\{\varepsilon(k)\}$
-206.7	1.5×10^{-12}

Table 2. Max. value evaluated by ψ; min value evaluated by ε; simulation time.

Excitations:

- Number of Common Elements: $P = 4$

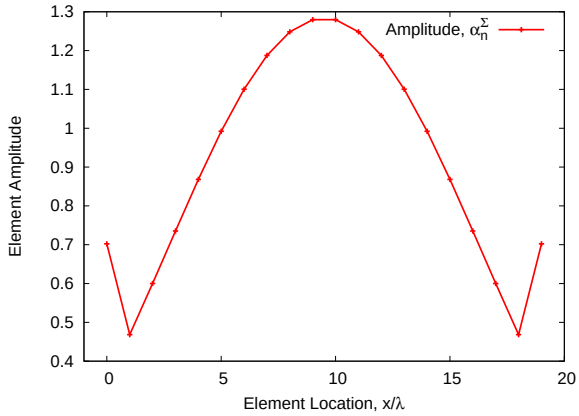

Figure 3. Sum beam's excitations amplitudes

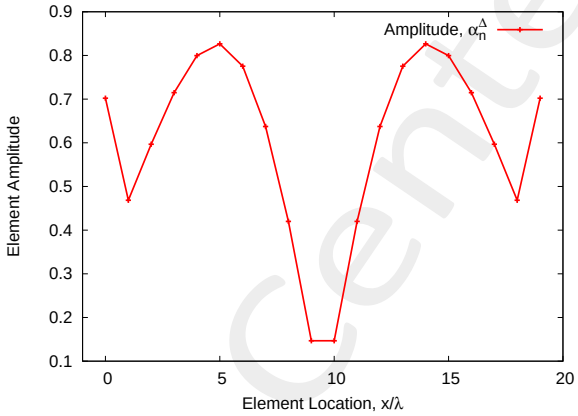

Figure 4. Difference beam's excitations amplitudes

Normalized Excitations:

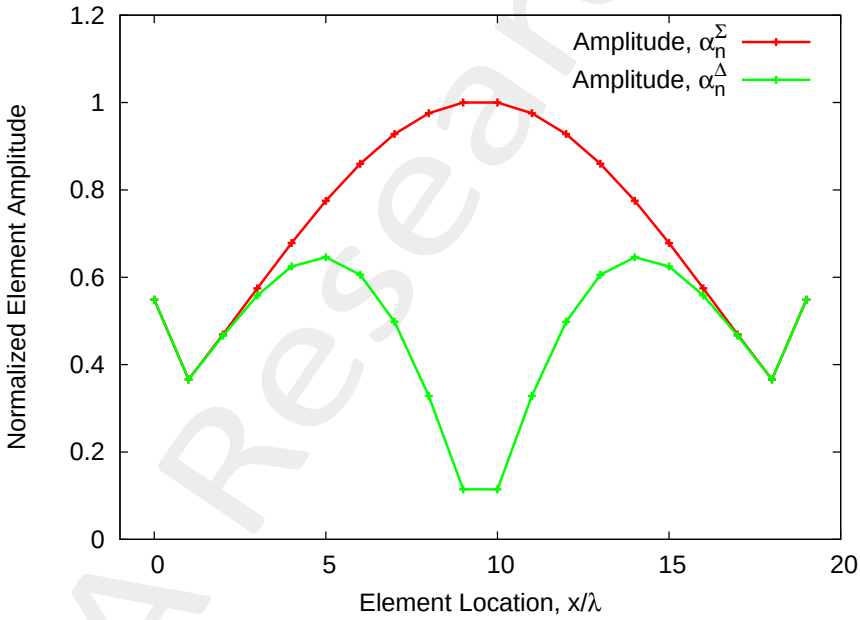

Figure 5. Sum and Difference beam's normalized excitations amplitudes

n	1	2	3	4	5	6	7	8	9	10
α_n^{Σ}	0.5487	0.366	0.4689	0.5746	0.6785	0.7752	0.8599	0.9279	0.9755	1.0
α_n^{Δ}	0.5487	0.366	0.4663	0.5585	0.6249	0.6459	0.6059	0.4981	0.3281	0.1146

Table 3. Sum and Difference beam's nominal amplitudes values (symmetric excitations)

Sum Pattern:

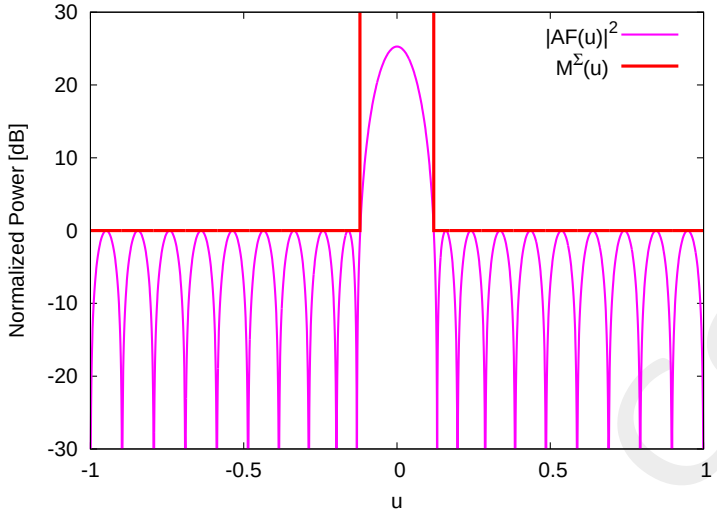

Figure 6. Synthesized Sum Pattern

SLL^{nom} [dB]	$D^{nom}_{\max}$ [dB]	BW^{nom} [u]	ψ_1 [u]
-25.28	12.65	0.104	0.131

Table 4. Sum beam's features values

Difference Pattern:

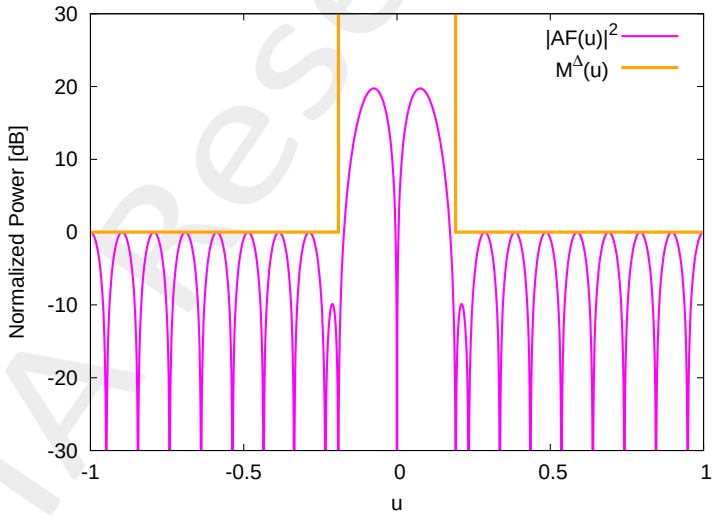

Figure 7. Synthesized Difference Pattern

SLL^{nom} [dB]	K^{nom} [1/rad]	BW^{nom} [u]	ψ_1 [u]
-19.74	1.2084	0.082	0.192

Table 5. Difference beam's features values

Tolerance Over Common Elements - $F = 4$

- Total tolerance: $T = 4$

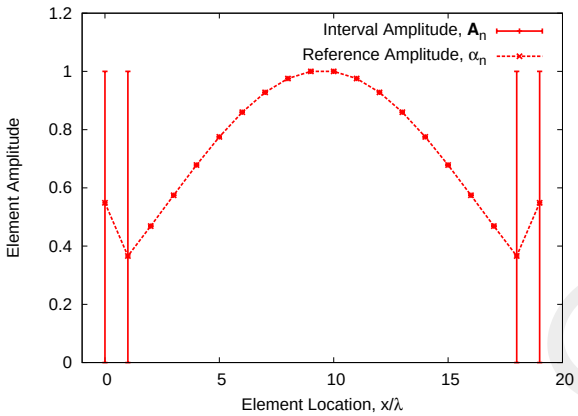

Figure 8. Sum beam's excitations amplitudes

n	1	2	3	4	5	6	7	8	9	10
nominal	0.5487	0.366	0.4689	0.5746	0.6785	0.7752	0.8599	0.9279	0.9755	1.0
$\inf\{\mathbf{A}_n\}$	0.0	0.0	0.4689	0.5746	0.6785	0.7752	0.8599	0.9279	0.9755	1.0
$\sup\{\mathbf{A}_n\}$	1.0	1.0	0.4689	0.5746	0.6785	0.7752	0.8599	0.9279	0.9755	1.0
$w\{\mathbf{A}_n\}$	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 6. Sum beam's nominal and interval amplitudes values (symmetric excitations)

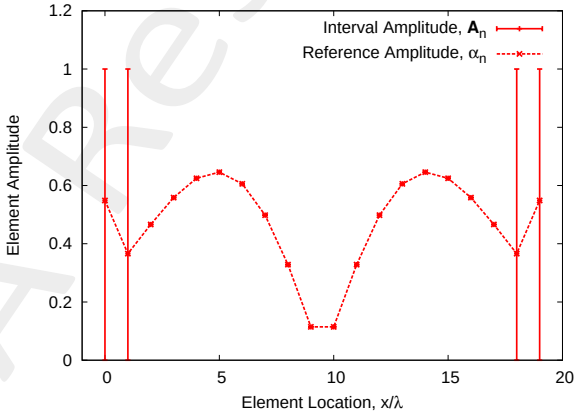

Figure 9. Difference beam's excitations amplitudes

n	1	2	3	4	5	6	7	8	9	10
nominal	0.5487	0.366	0.4663	0.5585	0.6249	0.6459	0.6059	0.4981	0.3281	0.1146
$\inf\{\mathbf{A}_n\}$	0.0	0.0	0.4663	0.5585	0.6249	0.6459	0.6059	0.4981	0.3281	0.1146
$\sup\{\mathbf{A}_n\}$	1.0	1.0	0.4663	0.5585	0.6249	0.6459	0.6059	0.4981	0.3281	0.1146
$w\{\mathbf{A}_n\}$	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 7. Difference beam's nominal and interval amplitudes values (symmetric excitations)

Sum / Difference Interval Patterns:

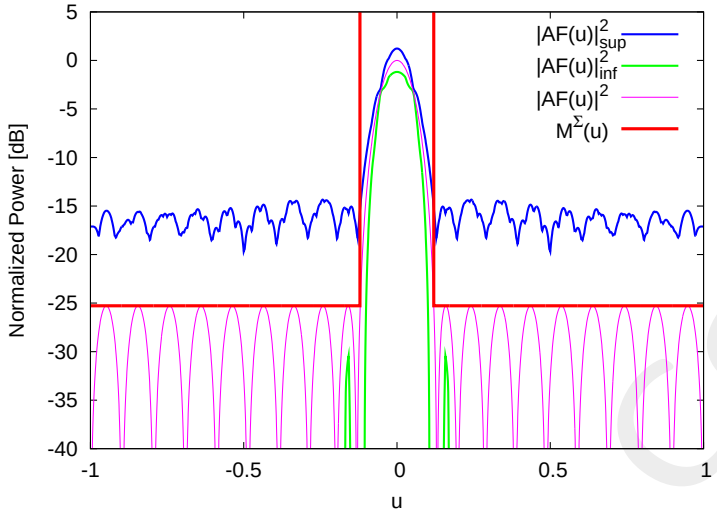

Figure 10. Synthesized Sum Pattern

	SLL [dB]	**BW** [u]	$\mathbf{D}_{\max}$ [dB]	$\mathbf{P}_{\max}$ [dB]	Δ	Δ_n
nominal	-25.28	0.104	12.65	0.0	0.01	0.4951
inf	-31.68	0.056	10.5	-1.18		
sup	-13.16	0.142	14.16	1.22		

Table 8. Sum Pattern Features

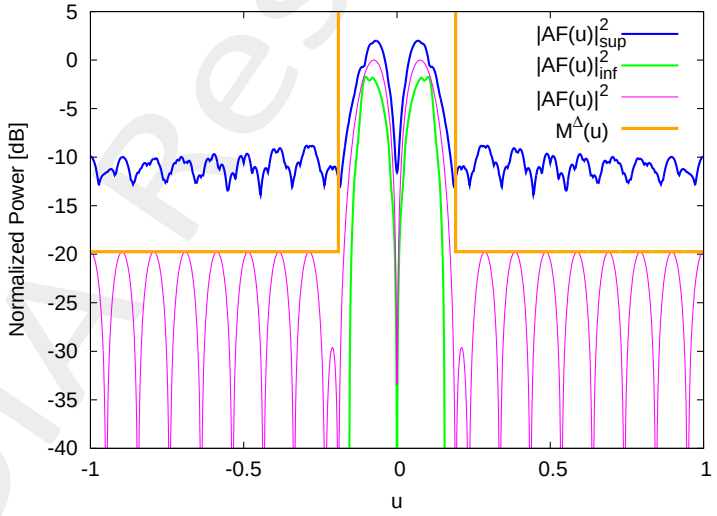

Figure 11. Synthesized Difference Pattern

	SLL [dB]	**BW** [u]	**K** [1/rad]	$\mathbf{P}_{\max}$ [dB]	Δ	Δ_n
nominal	-19.74	0.082	1.2084	0.0	0.318	0.9936
inf	$-\infty$	0.0	0.59	-1.7		
sup	-7.11	0.147	2.08	2.0		

Table 9. Difference Pattern Features

Tolerance Over Not-Common Elements - $F = 16$

- Total tolerance: $T = 4$

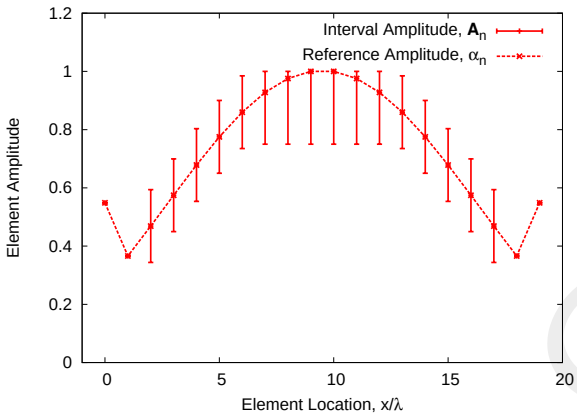

Figure 8. Sum beam's excitations amplitudes

n	1	2	3	4	5	6	7	8	9	10
nominal	0.5487	0.366	0.4689	0.5746	0.6785	0.7752	0.8599	0.9279	0.9755	1.0
$\inf\{\mathbf{A}_n\}$	0.5487	0.366	0.3439	0.4496	0.5535	0.6502	0.7349	0.75	0.75	0.75
$\sup\{\mathbf{A}_n\}$	0.5487	0.366	0.5939	0.6996	0.8035	0.9002	0.9849	1.0	1.0	1.0
$w\{\mathbf{A}_n\}$	0.0	0.0	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25

Table 6. Sum beam's nominal and interval amplitudes values (symmetric excitations)

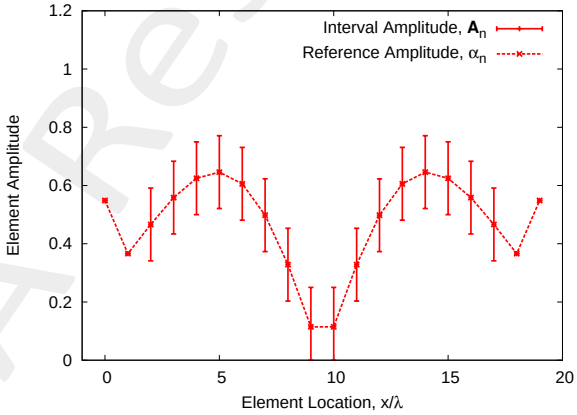

Figure 9. Difference beam's excitations amplitudes

n	1	2	3	4	5	6	7	8	9	10
nominal	0.5487	0.366	0.4663	0.5585	0.6249	0.6459	0.6059	0.4981	0.3281	0.1146
$\inf\{\mathbf{A}_n\}$	0.5487	0.366	0.3413	0.4335	0.4999	0.5209	0.4809	0.3731	0.2031	0.0
$\sup\{\mathbf{A}_n\}$	0.5487	0.366	0.5913	0.6835	0.7499	0.7709	0.7309	0.6231	0.4531	0.25
$w\{\mathbf{A}_n\}$	0.0	0.0	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25

Table 7. Difference beam's nominal and interval amplitudes values (symmetric excitations)

Sum / Difference Interval Patterns:

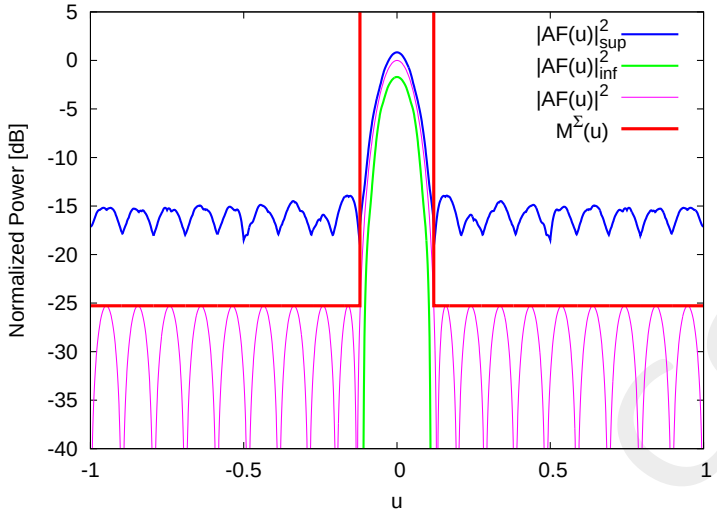

Figure 10. Synthesized Sum Pattern

	SLL [dB]	**BW** [u]	$\mathbf{D}_{\max}$ [dB]	$\mathbf{P}_{\max}$ [dB]	Δ	Δ_n
nominal	-25.28	0.104	12.65	0.0	0.1115	0.5524
inf	$-\infty$	0.1	10.34	-1.83		
sup	-12.22	0.104	15.16	1.7		

Table 8. Sum Pattern Features

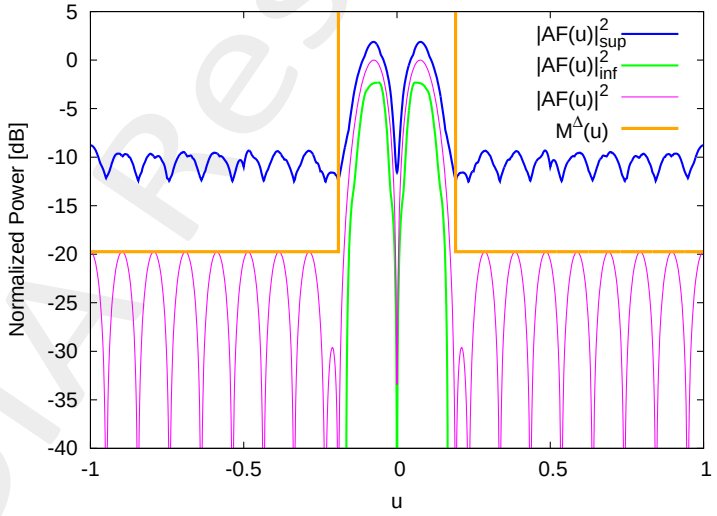

Figure 11. Synthesized Difference Pattern

	SLL [dB]	**BW** [u]	**K** [1/rad]	$\mathbf{P}_{\max}$ [dB]	Δ	Δ_n
nominal	-19.74	0.082	1.2084	0.0	0.3343	1.0445
inf	$-\infty$	0.0	0.85	-2.32		
sup	-6.46	0.14	1.7	1.88		

Table 9. Difference Pattern Features

1.2 Common Elements - $P = 6$

Array Parameters:

- Number Elements: $N = 20$
- Services: Sum / Difference Beams
- Number of Common Elements: $P = 6$
- Element Spacing: $\lambda/2$

Constraints:

- Main Sum Lobe Width: $BW^{\Sigma} = 0.24u$
- Main Difference Lobe Width: $BW^{\Delta} = 0.38u$

Simulation Parameters:

- Sample Points: 2001
- Max Function Evaluations: 6000
- Max Iterations Number: 1000
- Function Tolerance: 1.0×10^{-8}
- Constraint Tolerance: 1×10^{-8}

Algorithm Behaviour:

- Simulation Time Pattern: 30 sec.

In the following figures are reported, for each iteration, the max values evaluated by the objective function and by the constraint function for the sum and difference pattern synthesis.

Sum Beam:

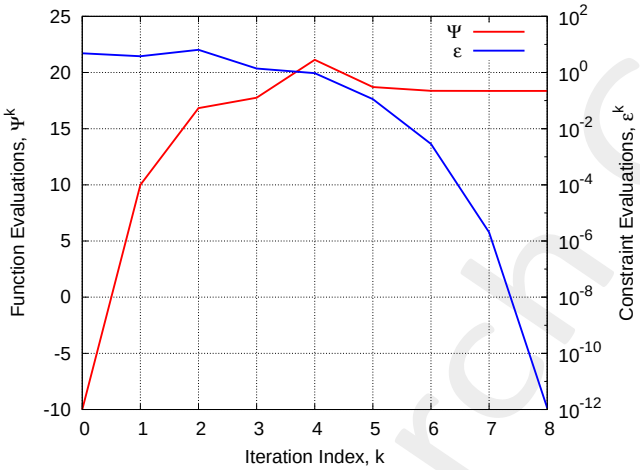

Figure 1. Sum Beam optimization's fitness

$max\{\psi(k)\}$	$min\{\varepsilon(k)\}$
-18.36	1.1×10^{-12}

Table 1. Max. value evaluated by ψ; min value evaluated by ε; simulation time.

Difference Beam:

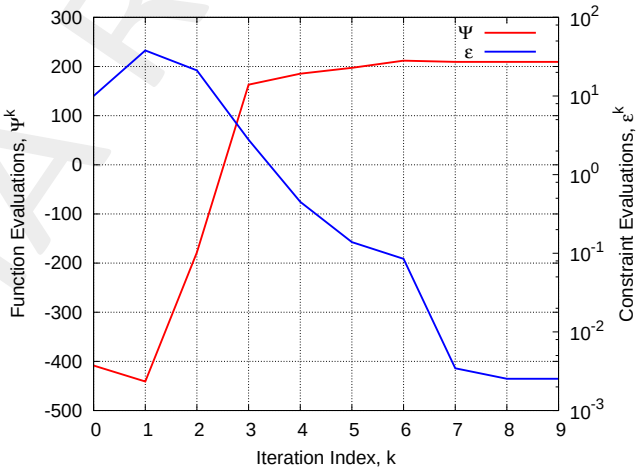

Figure 2. Difference beam optimization's fitness

$max\{\psi(k)\}$	$min\{\varepsilon(k)\}$
209.5	2.5×10^{-3}

Table 2. Max. value evaluated by ψ; min value evaluated by ε; simulation time.

Excitations:

- Number of Common Elements: $P = 6$

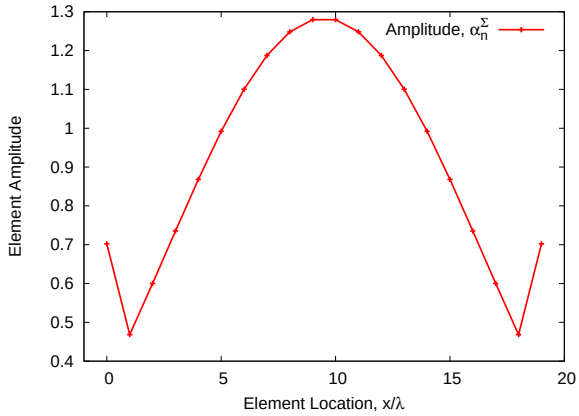

Figure 3. Sum beam's excitations amplitudes

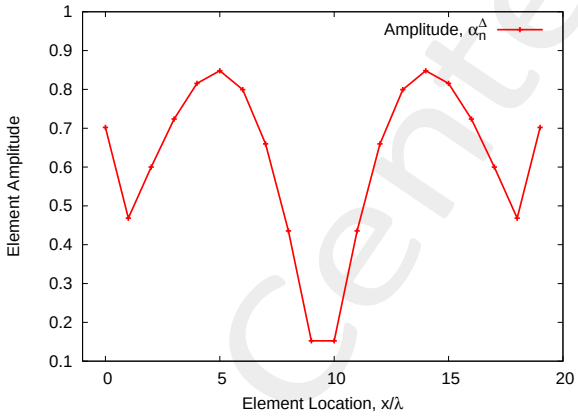

Figure 4. Difference beam's excitations amplitudes

Normalized Excitations:

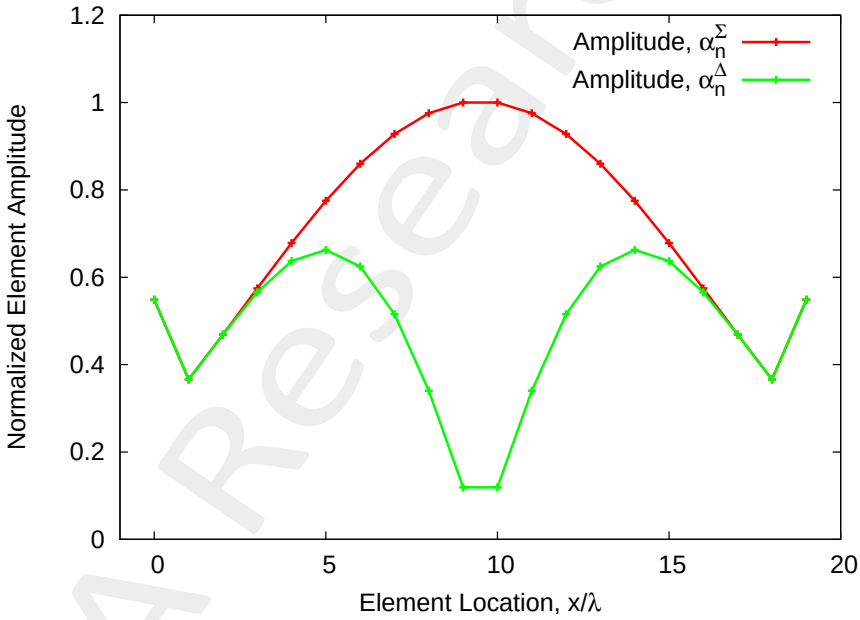

Figure 5. Sum and Difference beam's normalized excitations amplitudes

n	1	2	3	4	5	6	7	8	9	10
α_n^Σ	0.5487	0.366	0.4689	0.5746	0.6785	0.7752	0.8599	0.9279	0.9755	1.0
α_n^Δ	0.5487	0.366	0.4689	0.5655	0.6372	0.6627	0.6247	0.5154	0.3404	0.119

Table 3. Sum and Difference beam's nominal amplitudes values (symmetric excitations)

Sum Pattern:

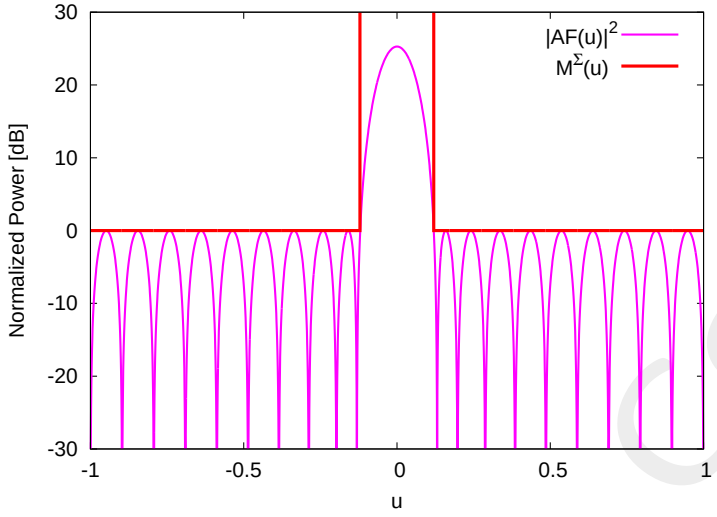

Figure 6. Synthesized Sum Pattern

SLL^{nom} [dB]	$D_{\max}^{nom}$ [dB]	BW^{nom} [u]	ψ_1 [u]
-25.28	12.65	0.104	0.131

Table 4. Sum beam's features values

Difference Pattern:

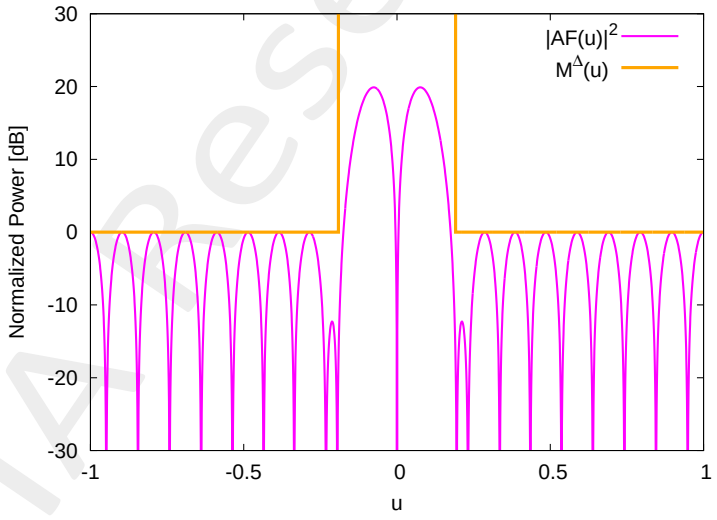

Figure 7. Synthesized Difference Pattern

SLL^{nom} [dB]	K^{nom} [1/rad]	BW^{nom} [u]	ψ_1 [u]
-19.8	1.2011	0.083	0.195

Table 5. Difference beam's features values

Tolerance Over Common Elements - $F = 6$

- Total tolerance: $T = 6$

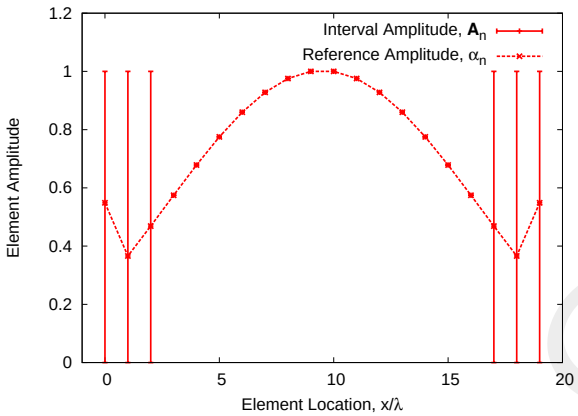

Figure 8. Sum beam's excitations amplitudes

n	1	2	3	4	5	6	7	8	9	10
nominal	0.5487	0.366	0.4689	0.5746	0.6785	0.7752	0.8599	0.9279	0.9755	1.0
$\inf\{\mathbf{A}_n\}$	0.0	0.0	0.0	0.5746	0.6785	0.7752	0.8599	0.9279	0.9755	1.0
$\sup\{\mathbf{A}_n\}$	1.0	1.0	1.0	0.5746	0.6785	0.7752	0.8599	0.9279	0.9755	1.0
$w\{\mathbf{A}_n\}$	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 6. Sum beam's nominal and interval amplitudes values (symmetric excitations)

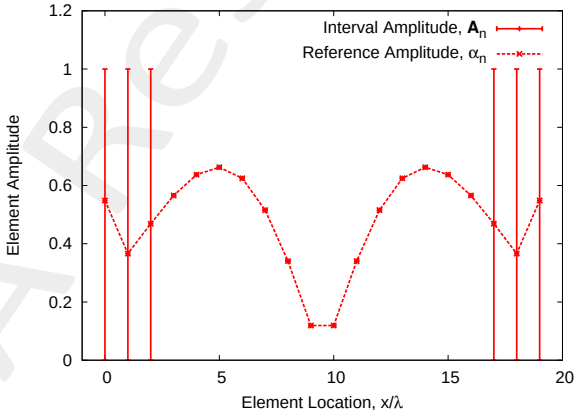

Figure 9. Difference beam's excitations amplitudes

n	1	2	3	4	5	6	7	8	9	10
nominal	0.5487	0.366	0.4689	0.5655	0.6372	0.6627	0.6247	0.5154	0.3404	0.119
$\inf\{\mathbf{A}_n\}$	0.0	0.0	0.0	0.5655	0.6372	0.6627	0.6247	0.5154	0.3404	0.119
$\sup\{\mathbf{A}_n\}$	1.0	1.0	1.0	0.5655	0.6372	0.6627	0.6247	0.5154	0.3404	0.119
$w\{\mathbf{A}_n\}$	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 7. Difference beam's nominal and interval amplitudes values (symmetric excitations)

Sum / Difference Interval Patterns:

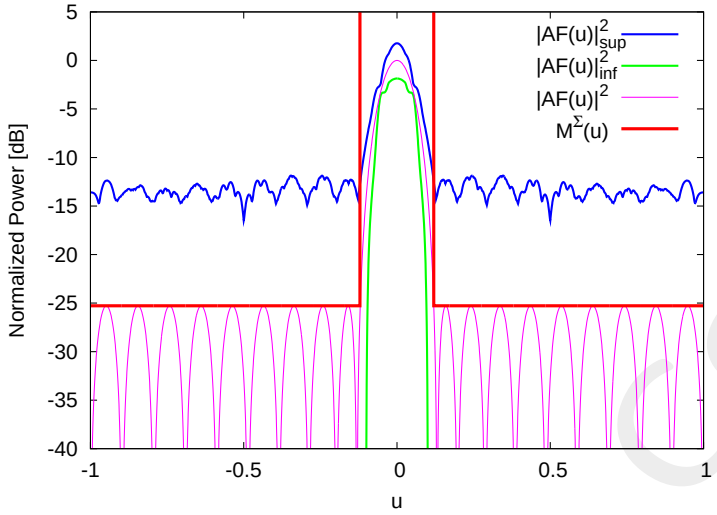

Figure 10. Synthesized Sum Pattern

	SLL [dB]	**BW** [u]	$\mathbf{D}_{\max}$ [dB]	$\mathbf{P}_{\max}$ [dB]	Δ	Δ_n
nominal	-25.28	0.104	12.65	0.0	0.1722	0.8529
inf	$-\infty$	0.104	9.46	-1.86		
sup	-10.00	0.160	14.87	1.76		

Table 8. Sum Pattern Features

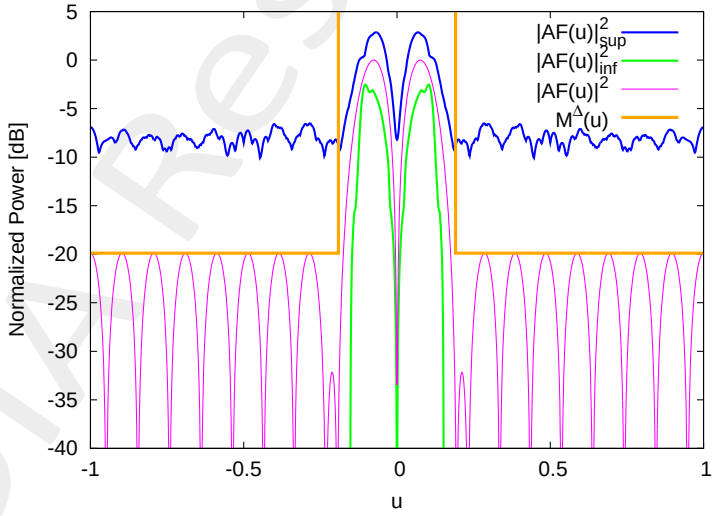

Figure 11. Synthesized Difference Pattern

	SLL [dB]	**BW** [u]	**K** [1/rad]	$\mathbf{P}_{\max}$ [dB]	Δ	Δ_n
nominal	-19.8	0.083	1.2011	0.0	0.5431	1.6989
inf	$-\infty$	0.0	0.4753	-2.5		
sup	-3.99	0.176	2.1159	2.9		

Table 9. Difference Pattern Features

Tolerance Over Not-Common Elements - $F = 14$

- Total tolerance: $T = 6$

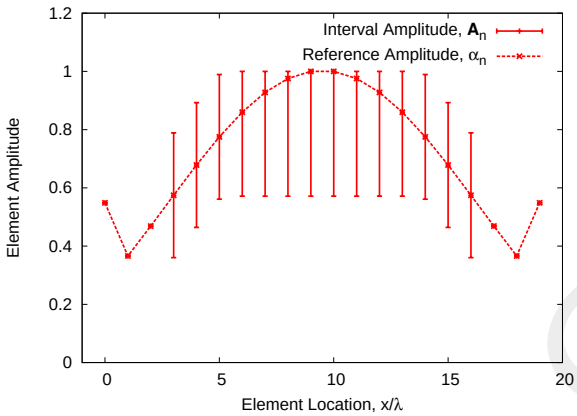

Figure 8. Sum beam's excitations amplitudes

n	1	2	3	4	5	6	7	8	9	10
nominal	0.5487	0.366	0.4689	0.5746	0.6785	0.7752	0.8599	0.9279	0.9755	1.0
$\inf\{\mathbf{A}_n\}$	0.5487	0.366	0.4689	0.3603	0.4642	0.5609	0.5714	0.5714	0.5714	0.5714
$\sup\{\mathbf{A}_n\}$	0.5487	0.366	0.4689	0.7889	0.8928	0.9895	1,0	1.0	1.0	1.0
$w\{\mathbf{A}_n\}$	0.0	0.0	0.0	0.4286	0.4286	0.4286	0.4286	0.4286	0.4286	0.4286

Table 6. Sum beam's nominal and interval amplitudes values (symmetric excitations)

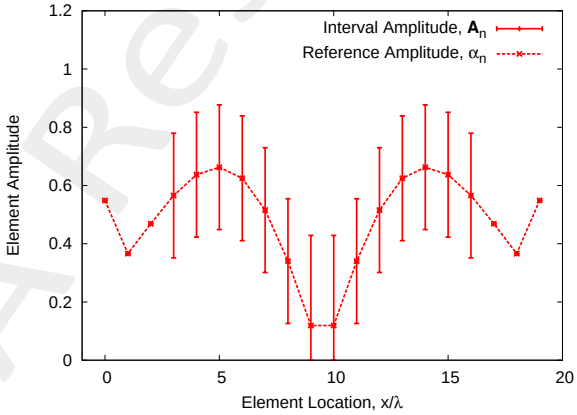

Figure 9. Difference beam's excitations amplitudes

n	1	2	3	4	5	6	7	8	9	10
nominal	0.5487	0.366	0.4689	0.5655	0.6372	0.6627	0.6247	0.5154	0.3404	0.119
$\inf\{\mathbf{A}_n\}$	0.5487	0.366	0.4689	0.3512	0.4229	0.4484	0.4105	0.3011	0.1261	0.0
$\sup\{\mathbf{A}_n\}$	0.5487	0.366	0.4689	0.7798	0.8515	0.877	0.839	0.7297	0.5547	0.4286
$w\{\mathbf{A}_n\}$	0.0	0.0	0.0	0.4286	0.4286	0.4286	0.4286	0.4286	0.4286	0.4286

Table 7. Difference beam's nominal and interval amplitudes values (symmetric excitations)

Sum / Difference Interval Patterns:

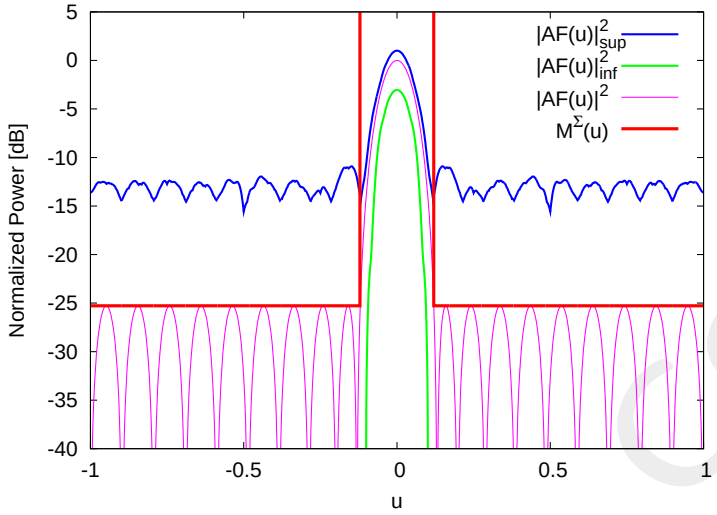

Figure 10. Synthesized Sum Pattern

	SLL [dB]	**BW** [u]	$\mathbf{D}_{\max}$ [dB]	$\mathbf{P}_{\max}$ [dB]	Δ	Δ_n
nominal	-25.28	0.104	12.65	0.0	0.1833	0.9079
inf	$-\infty$	0.0	8.96	-3.04		
sup	-7.87	0.164	16.8	1.0		

Table 8. Sum Pattern Features

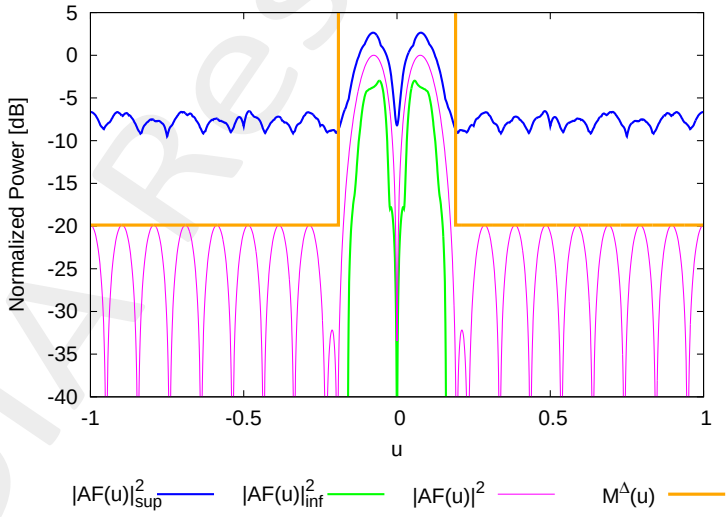

Figure 11. Synthesized Difference Pattern

	SLL [dB]	**BW** [u]	**K** [1/rad]	$\mathbf{P}_{\max}$ [dB]	Δ	Δ_n
nominal	-19.8	0.083	1.2011	0.0	0.5606	1.7536
inf	$-\infty$	0.0	0.73	-2.98		
sup	-3.56	0.171	1.95	2.64		

Table 9. Difference Pattern Features

1.3 Common Elements - $P = 8$

Array Parameters:

- Number Elements: $N = 20$
- Services: Sum / Difference Beams
- Number of Common Elements: $P = 8$
- Element Spacing: $\lambda/2$

Constraints:

- Main Sum Lobe Width: $BW^{\Sigma} = 0.24u$
- Main Difference Lobe Width: $BW^{\Delta} = 0.38u$

Simulation Parameters:

- Sample Points: 2001
- Max Function Evaluations: 6000
- Max Iterations Number: 1000
- Function Tolerance: 1.0×10^{-8}
- Constraint Tolerance: 1×10^{-8}

Algorithm Behaviour:

- Simulation Time Pattern: 30 sec.

In the following figures are reported, for each iteration, the max values evaluated by the objective function and by the constraint function for the sum and difference pattern synthesis.

Sum Beam:

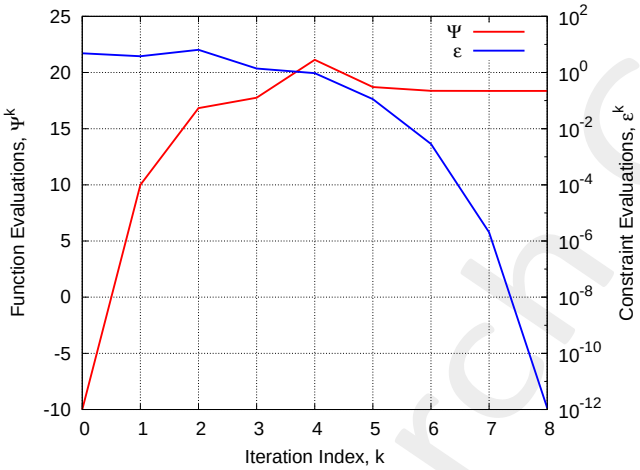

Figure 1. Sum Beam optimization's fitness

$max\{\psi(k)\}$	$min\{\varepsilon(k)\}$
-18.36	1.1×10^{-12}

Table 1. Max. value evaluated by ψ; min value evaluated by ε; simulation time.

Difference Beam:

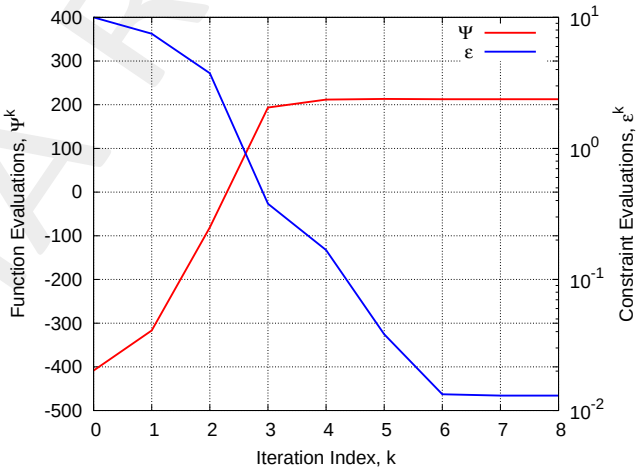

Figure 2. Difference beam optimization's fitness

$max\{\psi(k)\}$	$min\{\varepsilon(k)\}$
212.7	1.3×10^{-2}

Table 2. Max. value evaluated by ψ; min value evaluated by ε; simulation time.

Excitations:

- Number of Common Elements: $P = 8$

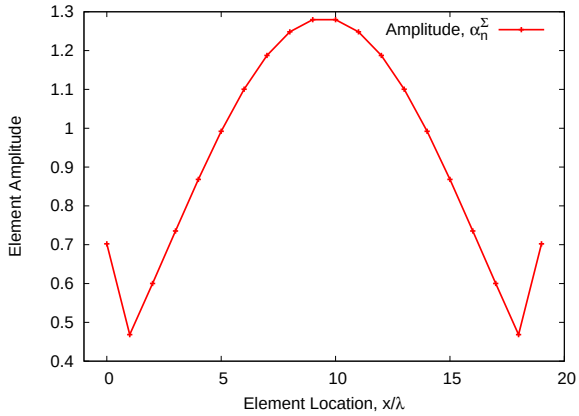

Figure 3. Sum beam's excitations amplitudes

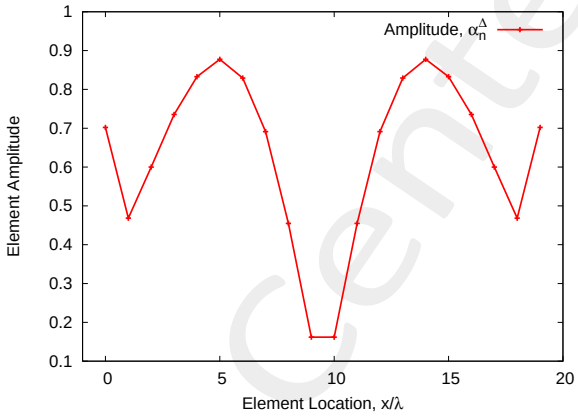

Figure 4. Difference beam's excitations amplitudes

Normalized Excitations:

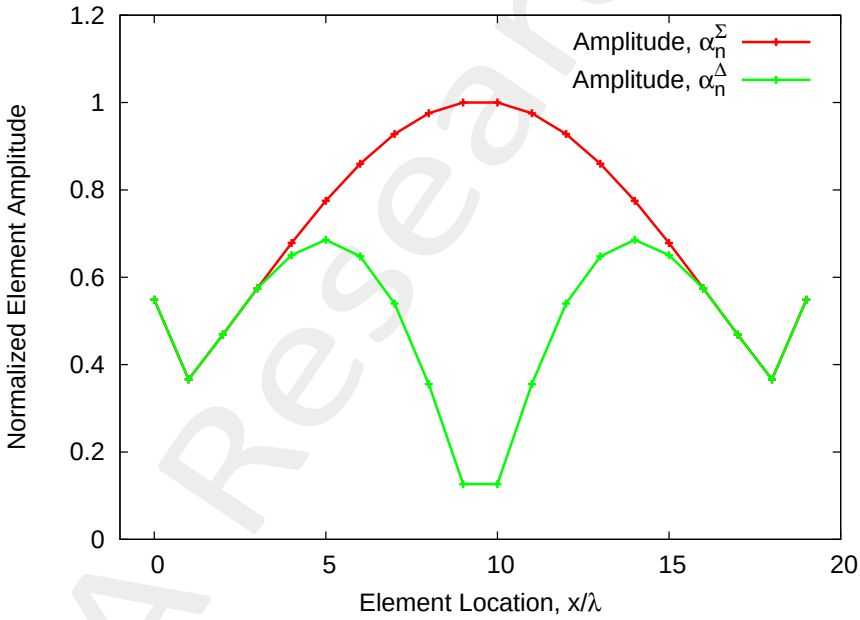

Figure 5. Sum and Difference beam's normalized excitations amplitudes

n	1	2	3	4	5	6	7	8	9	10
α_n^Σ	0.5487	0.366	0.4689	0.5746	0.6785	0.7752	0.8599	0.9279	0.9755	1.0
α_n^Δ	0.5487	0.366	0.4689	0.5746	0.6507	0.6858	0.648	0.5401	0.3554	0.1265

Table 3. Sum and Difference beam's nominal amplitudes values (symmetric excitations)

Sum Pattern:

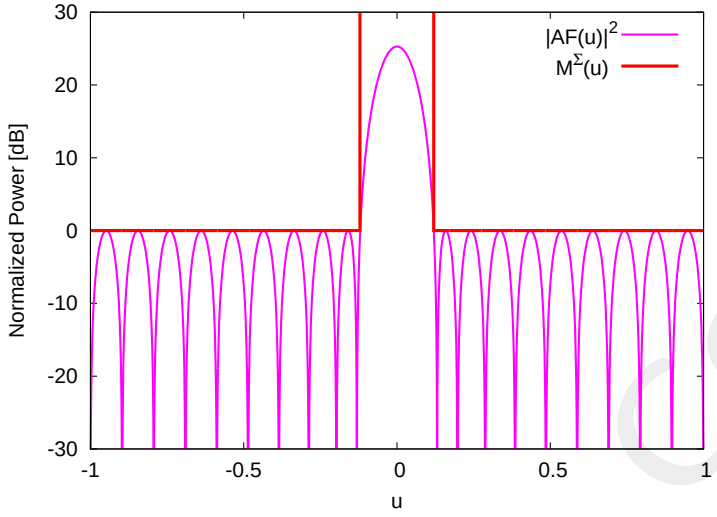

Figure 6. Synthesized Sum Pattern

SLL^{nom} [dB]	$D^{nom}_{\max}$ [dB]	BW^{nom} [u]	ψ_1 [u]
-25.28	12.65	0.104	0.131

Table 4. Sum beam's features values

Difference Pattern:

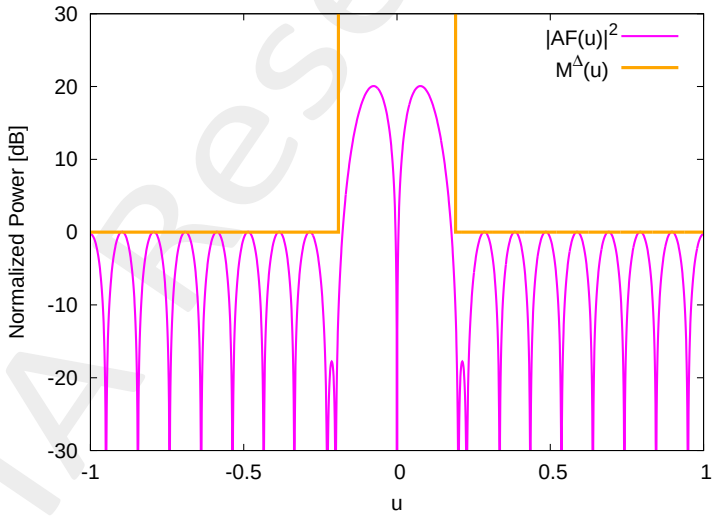

Figure 7. Synthesized Difference Pattern

SLL^{nom} [dB]	K^{nom} [1/rad]	BW^{nom} [u]	ψ_1 [u]
-20.01	1.1912	0.83	0.2

Table 5. Difference beam's features values

Tolerance Over Common Elements - $F = 8$

- Total tolerance: $T = 8$

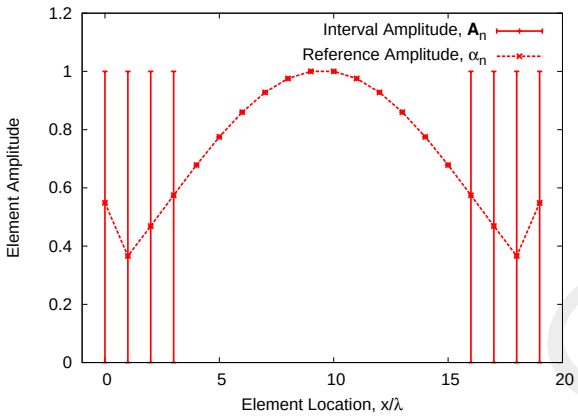

Figure 8. Sum beam's excitations amplitudes

n	1	2	3	4	5	6	7	8	9	10
nominal	0.5487	0.366	0.4689	0.5746	0.6785	0.7752	0.8599	0.9279	0.9755	1.0
$\inf\{\mathbf{A}_n\}$	0.0	0.0	0.0	0.0	0.6785	0.7752	0.8599	0.9279	0.9755	1.0
$\sup\{\mathbf{A}_n\}$	1.0	1.0	1.0	0.0	0.6785	0.7752	0.8599	0.9279	0.9755	1.0
$w\{\mathbf{A}_n\}$	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 6. Sum beam's nominal and interval amplitudes values (symmetric excitations)

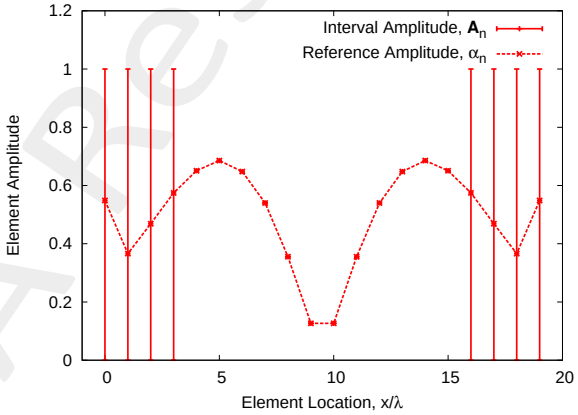

Figure 9. Difference beam's excitations amplitudes

n	1	2	3	4	5	6	7	8	9	10
nominal	0.5487	0.366	0.4689	0.5746	0.6507	0.6858	0.648	0.5401	0.3554	0.1265
$\inf\{\mathbf{A}_n\}$	0.0	0.0	0.0	0.0	0.6507	0.6858	0.648	0.5401	0.3554	0.1265
$\sup\{\mathbf{A}_n\}$	1.0	1.0	1.0	1.0	0.6507	0.6858	0.648	0.5401	0.3554	0.1265
$w\{\mathbf{A}_n\}$	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 7. Difference beam's nominal and interval amplitudes values (symmetric excitations)

Sum / Difference Interval Patterns:

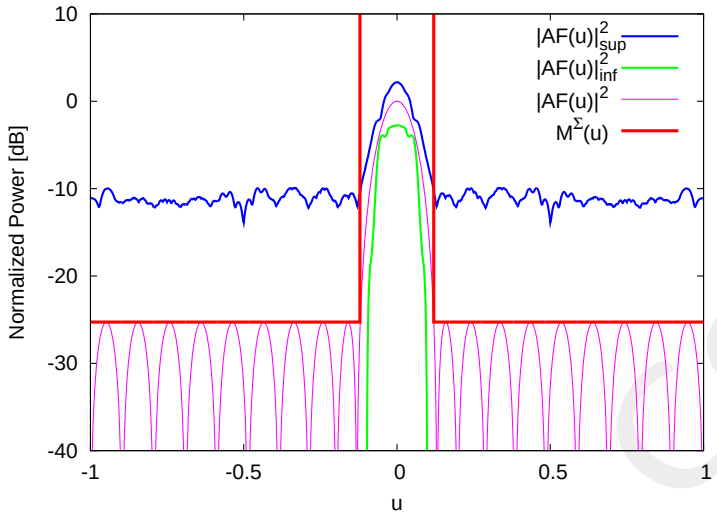

Figure 10. Synthesized Sum Pattern

	SLL [dB]	**BW** [u]	$\mathbf{D}_{\max}$ [dB]	$\mathbf{P}_{\max}$ [dB]	Δ	Δ_n
nominal	-25.28	0.104	12.65	0.0	0.26	1.288
inf	$-\infty$	0.0	8.32	-2.77		
sup	-7.15	0.184	15.56	2.18		

Table 8. Sum Pattern Features

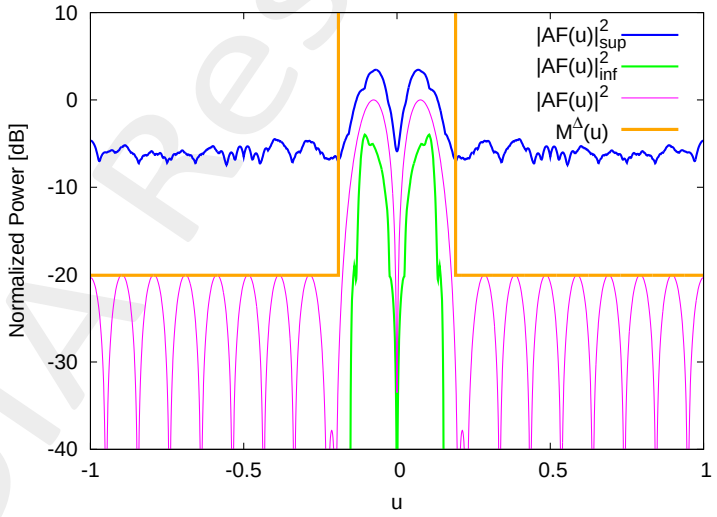

Figure 11. Synthesized Difference Pattern

	SLL [dB]	**BW** [u]	**K** [1/rad]	$\mathbf{P}_{\max}$ [dB]	Δ	Δ_n
nominal	-20.01	0.083	1.1912	0.0	0.7999	5.5009
inf	$-\infty$	0.0	0.3378	-3.97		
sup	-0.52	4.0	2.4143	3.45		

Table 9. Difference Pattern Features

Tolerance Over Not-Common Elements - $F = 12$

- Total tolerance: $T = 8$

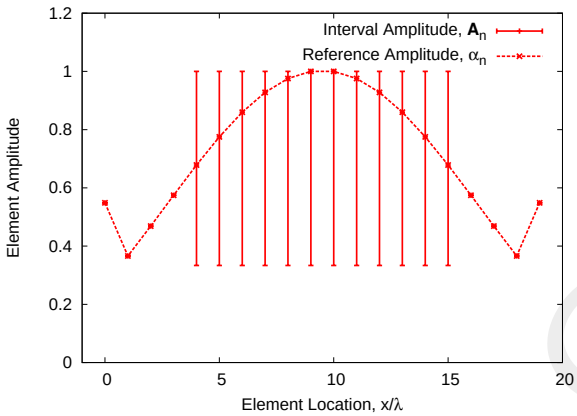

Figure 8. Sum beam's excitations amplitudes

n	1	2	3	4	5	6	7	8	9	10
nominal	0.5487	0.366	0.4689	0.5746	0.6785	0.7752	0.8599	0.9279	0.9755	1.0
$\inf\{\mathbf{A}_n\}$	0.5487	0.366	0.4689	0.5746	0.3333	0.7752	0.3333	0.3333	0.3333	0.3333
$\sup\{\mathbf{A}_n\}$	0.5487	0.366	0.4689	0.5746	1.0	1.0	1.0	1.0	1.0	1.0
$w\{\mathbf{A}_n\}$	0.0	0.0	0.0	0.0	0.6667	0.6667	0.6667	0.6667	0.6667	0.6667

Table 6. Sum beam's nominal and interval amplitudes values (symmetric excitations)

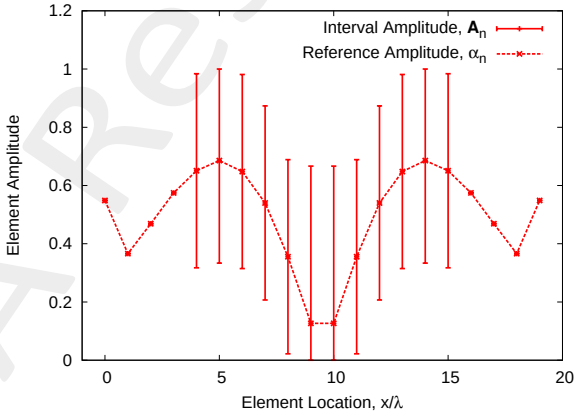

Figure 9. Difference beam's excitations amplitudes

n	1	2	3	4	5	6	7	8	9	10
nominal	0.5487	0.366	0.4689	0.5746	0.6507	0.6858	0.648	0.5401	0.3554	0.1265
$\inf\{\mathbf{A}_n\}$	0.5487	0.366	0.4689	0.5746	0.3173	0.3333	0.3147	0.2068	0.0221	0.0
$\sup\{\mathbf{A}_n\}$	0.5487	0.366	0.4689	0.5746	0.984	1.0	0.9813	0.8735	0.6887	0.6667
$w\{\mathbf{A}_n\}$	1.0	1.0	1.0	1.0	0.6667	0.6667	0.6667	0.6667	0.6667	0.6667

Table 7. Difference beam's nominal and interval amplitudes values (symmetric excitations)

Sum / Difference Interval Patterns:

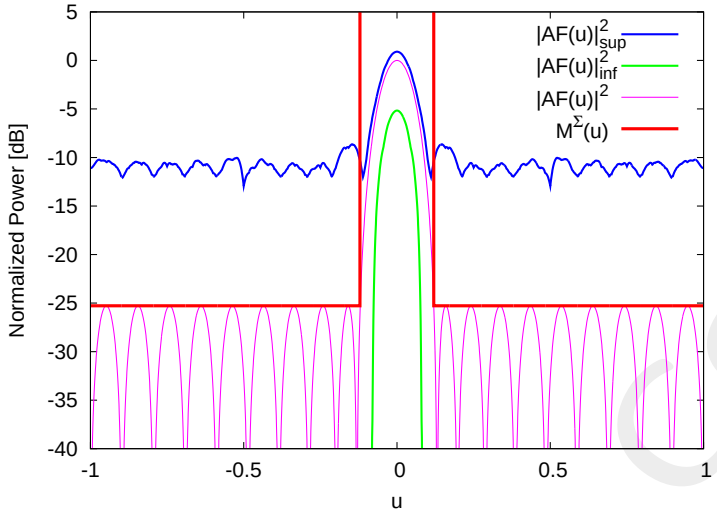

Figure 10. Synthesized Sum Pattern

	SLL [dB]	**BW** [u]	$\mathbf{D}_{\max}$ [dB]	$\mathbf{P}_{\max}$ [dB]	Δ	Δ_n
nominal	-25.28	0.104	12.65	0.0	0.261	1.293
inf	$-\infty$	0.0	7.2	-5.17		
sup	-3.47	0.186	18.7	0.9		

Table 8. Sum Pattern Features

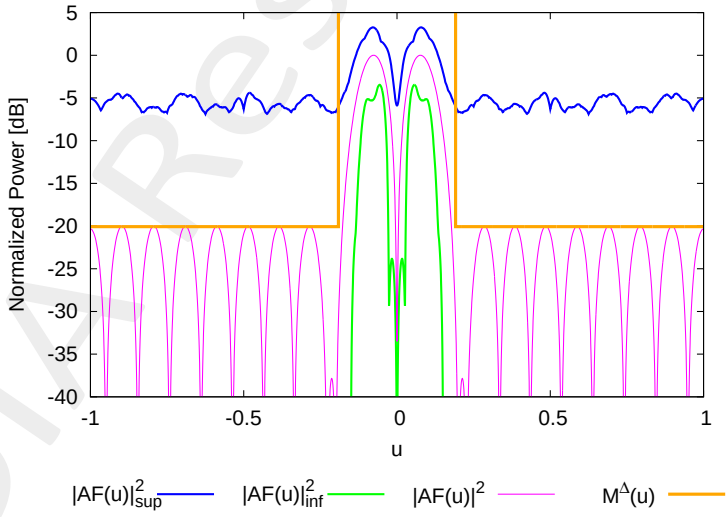

Figure 11. Synthesized Difference Pattern

	SLL [dB]	**BW** [u]	**K** [1/rad]	$\mathbf{P}_{\max}$ [dB]	Δ	Δ_n
nominal	-20.01	0.083	1.1912	0.0	0.8244	2.5772
inf	$-\infty$	0.00	0.644	-3.46		
sup	-0.9	4.0	2.089	3.26		

Table 9. Difference Pattern Features

More information on the topics of this document can be found in the following list of references.

References

[1] G. Ding, N. Anselmi, W. Xu, P. Li, and P. Rocca, "Interval-bounded optimal power pattern synthesis of array antenna excitations robust to mutual coupling," *IEEE Antennas Wireless Propag. Lett.*, vol. 22, no. 11, pp. 2725-2729, Nov. 2023.

[2] N. Anselmi, P. Rocca, and A. Massa, "Tolerance analysis of reconfigurable monopulse linear antenna arrays through interval arithmetic," *J. Electromagn. Waves Appl. J*, pp. 1066-1081, 2023.

[3] P. Rocca, N. Anselmi, A. Benoni, and A. Massa, "Probabilistic interval analysis for the analytic prediction of the pattern tolerance distribution in linear phased arrays with random excitation errors," *IEEE Trans. Antennas Propag.*, vol. 68, no. 2, pp. 7866-7878, Dec. 2020.

[4] L. Tenuti, N. Anselmi, P. Rocca, M. Salucci, and A. Massa, "Minkowski sum method for planar arrays sensitivity analysis with uncertain-but-bounded excitation tolerances" *IEEE Trans. Antennas Propag.*, vol. 65, no. 1, pp. 167-177, Jan. 2017.

[5] N. Anselmi, P. Rocca, M. Salucci, and A. Massa, "Optimization of excitation tolerances for robust beamforming in linear arrays," *IET Microwaves, Antennas & Propagation*, vol. 10, no. 2, pp. 208-214, 2016.

[6] N. Anselmi, P. Rocca, M. Salucci, and A. Massa, "Power pattern sensitivity to calibration errors and mutual coupling in linear arrays through circular interval arithmetics," *Sensors*, vol. 16, no. 6 (791), pp. 1-14, 2016.

[7] L. Poli, P. Rocca, N. Anselmi, and A. Massa, "Dealing with uncertainties on phase weighting of linear antenna arrays by means of interval-based tolerance analysis," *IEEE Trans. Antennas Propag.*, vol. 63, no. 7, pp. 3299-3234, Jul. 2015.

[8] P. Rocca, N. Anselmi, and A. Massa, "Optimal synthesis of robust array configurations exploiting interval analysis and convex optimization," *IEEE Trans. Antennas Propag.*, vol. 62, no. 7, pp. 3603-3612, July 2014.

[9] T. Moriyama, L. Poli, N. Anselmi, M. Salucci, and P. Rocca, "Real array pattern tolerances from amplitude excitation errors," *IEICE Electronics Express*, vol. 11, no. 17, pp. 1-8, Sep. 2014.

[10] N. Anselmi, L. Manica, P. Rocca, and A. Massa, "Tolerance analysis of antenna arrays through interval arithmetic," *IEEE Trans. Antennas Propag.*, vol. 61, no. 11, pp. 5496-5507, Nov. 2013.

[11] L. Manica, N. Anselmi, P. Rocca, and A. Massa, "Robust mask-constrained linear array synthesis through an interval-based particle swarm optimisation," *IET Microwaves, Antennas and Propagation*, vol. 7, no. 12, pp. 976-984, Sep. 2013.

[12] P. Rocca, L. Manica, N. Anselmi, and A. Massa, "Analysis of the pattern tolerances in linear arrays with arbitrary amplitude errors," *IEEE Antennas Wireless Propag. Lett.*, vol. 12, pp. 639-642, 2013.

[13] L. Manica, P. Rocca, N. Anselmi, and A. Massa, "On the synthesis of reliable linear arrays through interval arithmetic," *IEEE International Symposium on Antennas Propag. (APS/URSI 2013)*, *Orlando, Florida*, *USA*, Jul. 7-12, 2013.

[14] L. Manica, P. Rocca, G. Oliveri, and A. Massa, "Designing radiating systems through interval analysis tools," *IEEE International Symposium on Antennas Propag. (APS/URSI 2013), Orlando, Florida, USA*, Jul. 7-12, 2013.

[15] M. Carlin, N. Anselmi, L. Manica, P. Rocca, and A. Massa, "Exploiting interval arithmetic for predicting real arrays performances - The linear case," *IEEE International Symposium on Antennas Propag. (APS/URSI 2013), Orlando, Florida, USA*, Jul. 7-12, 2013.

[16] N. Anselmi, M. Salucci, P. Rocca, and A. Massa, "Generalised interval-based analysis tool for pattern distortions in reflector antennas with bump-like surface deformations," *IET Microwaves, Antennas & Propagation*, vol. 10, no. 9, p. 909-916, June 2016.

[17] P. Rocca, L. Poli, N. Anselmi, M. Salucci, and A. Massa, "Predicting antenna pattern degradations in microstrip reflectarrays through interval arithmetic," *IET Microwaves, Antennas & Propagation*, vol. 10, no. 8, pp. 817-826, May 2016.

[18] P. Rocca, L. Manica, and A. Massa, "Interval-based analysis of pattern distortions in reflector antennas with bump-like surface deformations," *IET Microwaves, Antennas and Propagation*, vol. 8, no. 15, pp. 1277-1285, Dec. 2014.

[19] P. Rocca, N. Anselmi, and A. Massa, "Interval Arithmetic for pattern tolerance analysis of parabolic reflectors," *IEEE Trans. Antennas Propag.*, vol. 62, no. 10, pp. 4952-4960, Oct. 2014.