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# **Clustered Arrays Design via an Advanced Power Pattern Matching Method**

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# 1 Numerical Results

## 1.1 Asymmetric Flat Top, $SLL_1 = -20$ [dB] - Analysis $SLL_2$ , $N = 32$ - $Q = N/2$

### 1.1.1 $SLL_2 = -20$ [dB]

#### Antenna configuration

- Isotropic Elements
- Number of Elements:  $N = 32$
- Distance between Elements along  $x$  axis:  $d_x = \lambda/2$

#### Target excitations

- Asymmetric Flat Top,  $SLL_1 = -20$  [dB] -  $SLL_2 = -20$  [dB]

#### Sub-array generation

- number of clusters:  $Q = 16$
- excitation matching strategy: EM K-Means
- power matching strategy: PM K-means

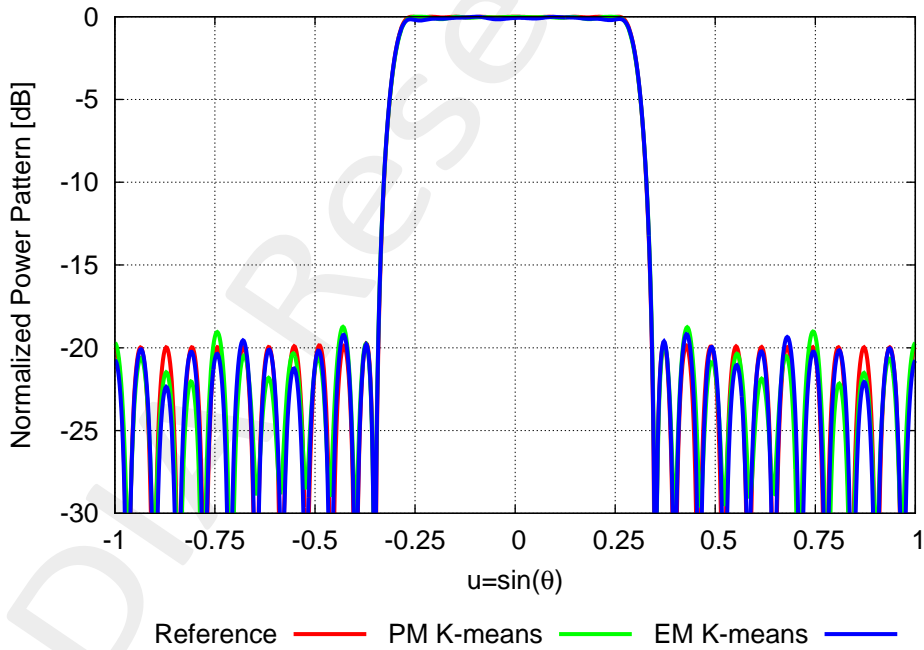


Figure 1: Power Pattern comparison PM K-means vs. EM K-means

Approach	Pattern Matching
<i>PM K-means</i>	$1.15 \times 10^{-2}$
<i>EM K-means</i>	$1.95 \times 10^{-2}$

Table I: Pattern Matching

### 1.1.2 $SLL_2 = -25$ [dB]

#### Antenna configuration

- Isotropic Elements
- Number of Elements:  $N = 32$
- Distance between Elements along  $x$  axis:  $d_x = \lambda/2$

#### Target excitations

- Asymmetric Flat Top,  $SLL_1 = -20$  [dB] -  $SLL_2 = -25$  [dB]

#### Sub-array generation

- number of clusters:  $Q = 16$
- excitation matching strategy: EM K-Means
- power matching strategy: PM K-means

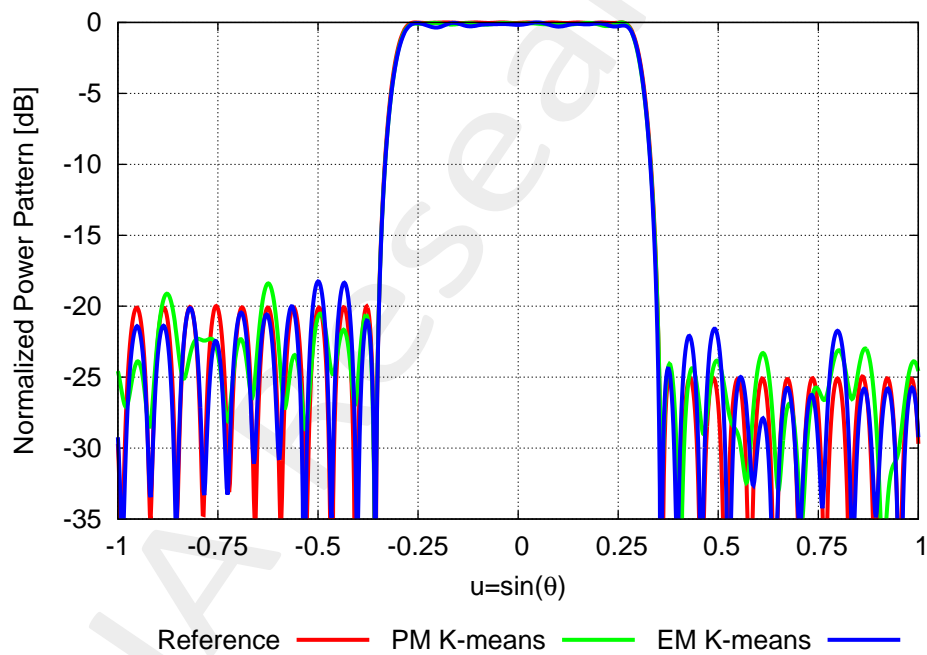


Figure 2: Power Pattern comparison PM K-means vs. EM K-means

Approach	Pattern Matching
<i>PM K-means</i>	$2.48 \times 10^{-2}$
<i>EM K-means</i>	$3.58 \times 10^{-2}$

Table II: Pattern Matching

### 1.1.3 $SLL_2 = -30$ [dB]

#### Antenna configuration

- Isotropic Elements
- Number of Elements:  $N = 32$
- Distance between Elements along  $x$  axis:  $d_x = \lambda/2$

#### Target excitations

- Asymmetric Flat Top,  $SLL_1 = -20$  [dB] -  $SLL_2 = -30$  [dB]

#### Sub-array generation

- number of clusters:  $Q = 16$
- excitation matching strategy: EM K-Means
- power matching strategy: PM K-means

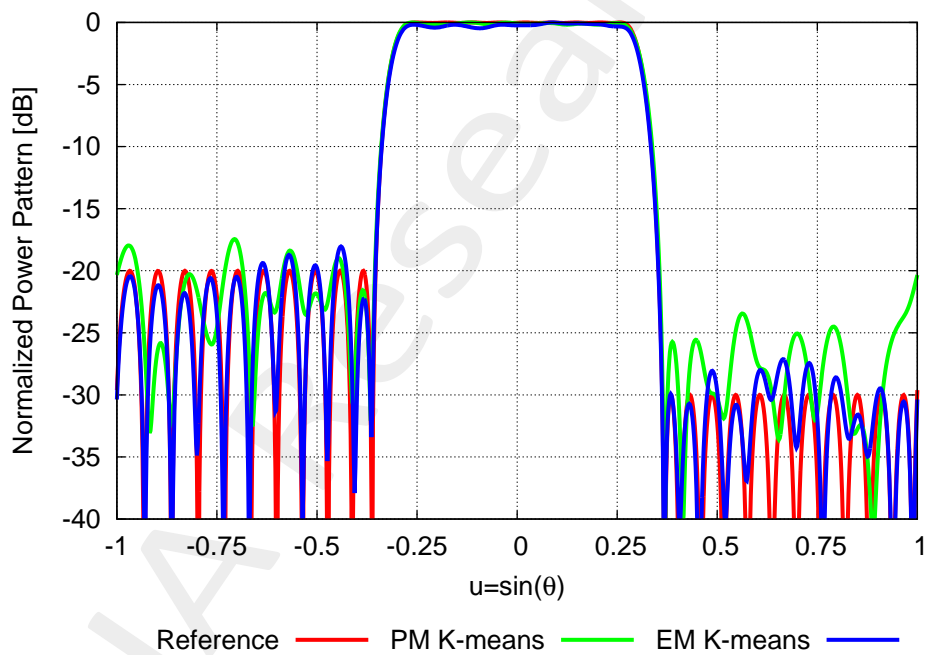


Figure 3: Power Pattern comparison PM K-means vs. EM K-means

Approach	Pattern Matching
<i>PM K-means</i>	$2.58 \times 10^{-2}$
<i>EM K-means</i>	$4.97 \times 10^{-2}$

Table III: Pattern Matching

### 1.1.4 $SLL_2 = -35$ [dB]

#### Antenna configuration

- Isotropic Elements
- Number of Elements:  $N = 32$
- Distance between Elements along  $x$  axis:  $d_x = \lambda/2$

#### Target excitations

- Asymmetric Flat Top,  $SLL_1 = -20$  [dB] -  $SLL_2 = -35$  [dB]

#### Sub-array generation

- number of clusters:  $Q = 16$
- excitation matching strategy: EM K-Means
- power matching strategy: PM K-means

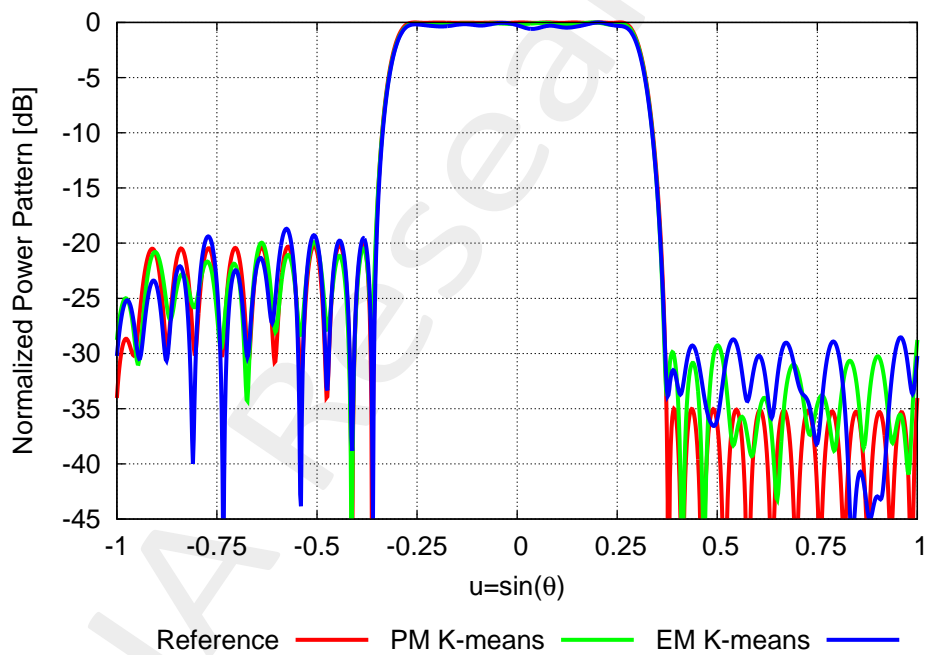


Figure 4: Power Pattern comparison PM K-means vs. EM K-means

Approach	Pattern Matching
<i>PM K-means</i>	$2.84 \times 10^{-2}$
<i>EM K-means</i>	$5.85 \times 10^{-2}$

Table IV: Pattern Matching

### 1.1.5 $SLL_2 = -40$ [dB]

#### Antenna configuration

- Isotropic Elements
- Number of Elements:  $N = 32$
- Distance between Elements along  $x$  axis:  $d_x = \lambda/2$

#### Target excitations

- Asymmetric Flat Top,  $SLL_1 = -20$  [dB] -  $SLL_2 = -40$  [dB]

#### Sub-array generation

- number of clusters:  $Q = 16$
- excitation matching strategy: EM K-Means
- power matching strategy: PM K-means

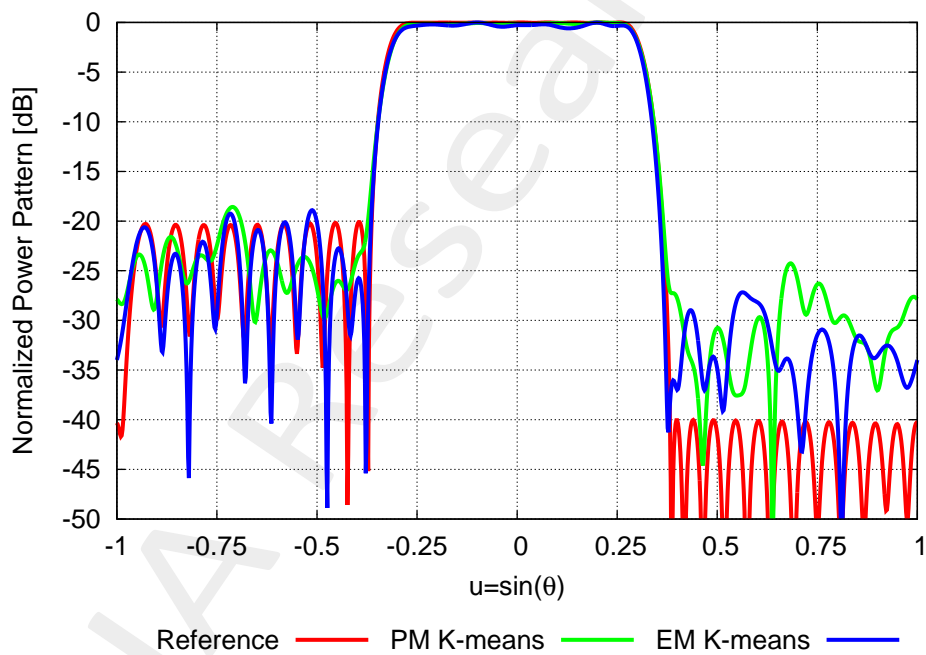


Figure 5: Power Pattern comparison PM K-means vs. EM K-means

Approach	Pattern Matching
<i>PM K-means</i>	$3.40 \times 10^{-2}$
<i>EM K-means</i>	$6.59 \times 10^{-2}$

Table V: Pattern Matching

### 1.1.6 Analysis vs. SLL

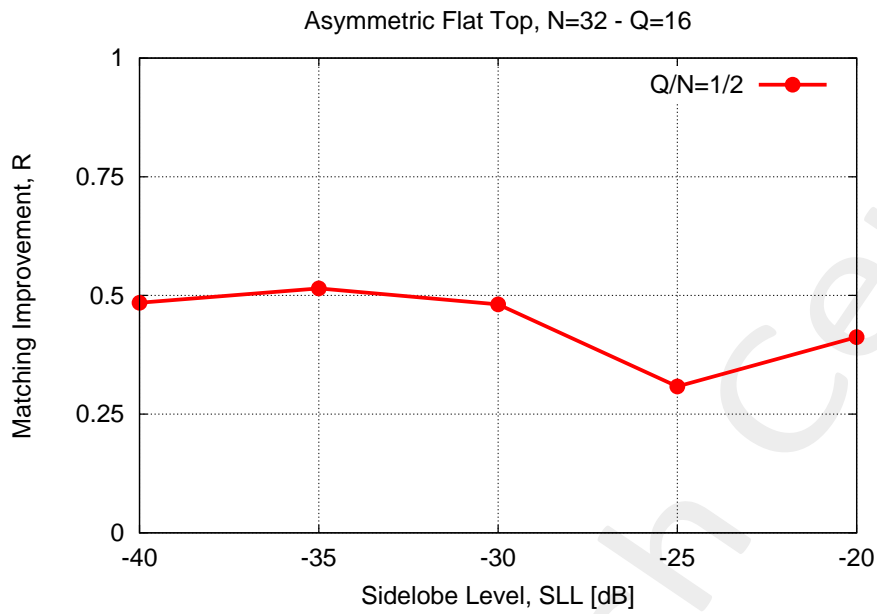


Figure 6: Analysis Matching Improvement vs. SLL

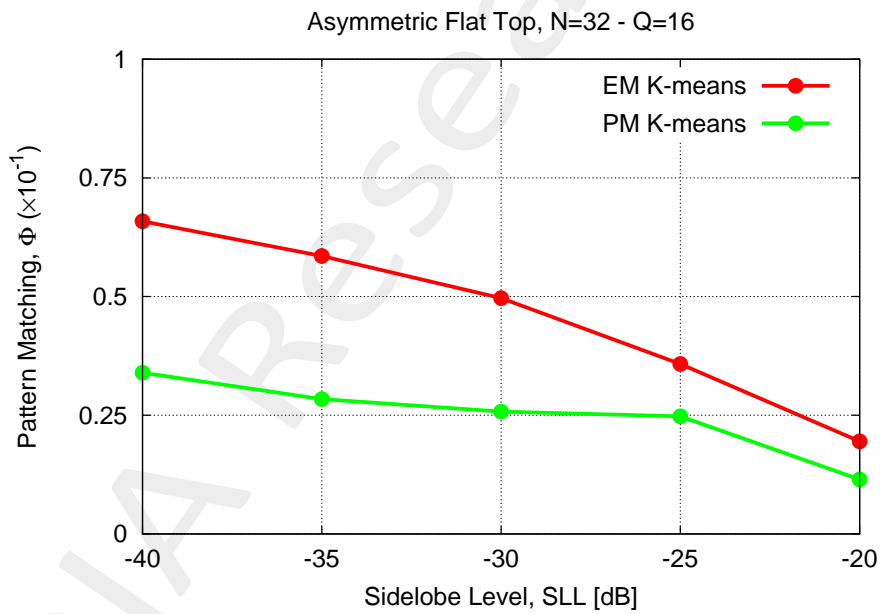


Figure 7: Pattern Matching comparison PM K-means vs. EM K-means for the different SLL for  $Q/N = 1/2$



## 1.2 Asymmetric Flat Top, $SLL_1 = -20$ [dB] - Analysis $SLL_2, N = 32 - Q = \frac{3}{4}N$

### 1.2.1 $SLL_2 = -20$ [dB]

#### Antenna configuration

- Isotropic Elements
- Number of Elements:  $N = 32$
- Distance between Elements along  $x$  axis:  $d_x = \lambda/2$

#### Target excitations

- Asymmetric Flat Top,  $SLL_1 = -20$  [dB] -  $SLL_2 = -20$  [dB]

#### Sub-array generation

- number of clusters:  $Q = 24$
- excitation matching strategy: EM K-Means
- power matching strategy: PM K-means

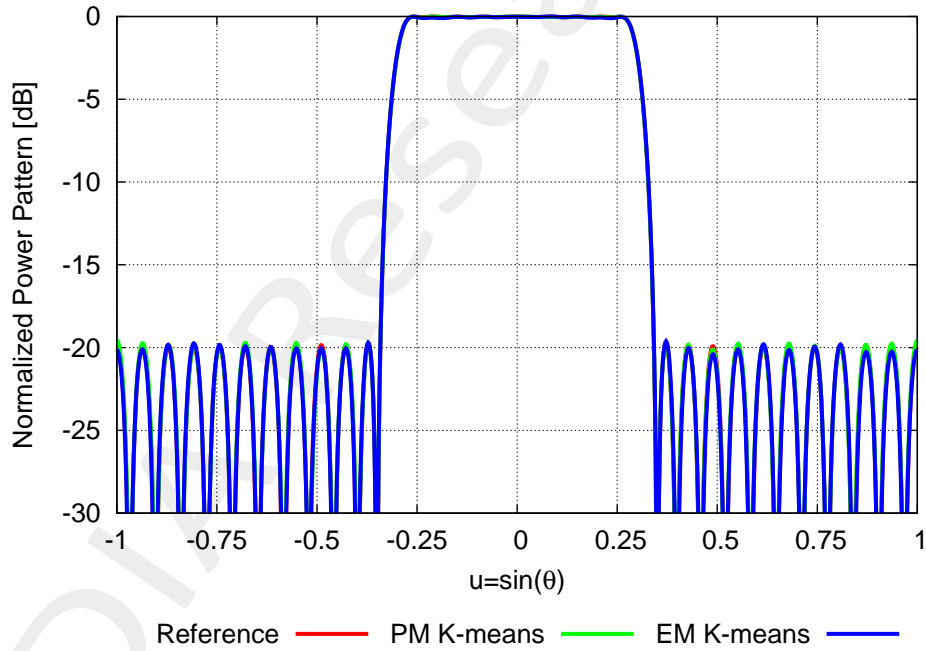


Figure 8: Power Pattern comparison PM K-means vs. EM K-means

Approach	Pattern Matching
<i>PM K-means</i>	$2.85 \times 10^{-3}$
<i>EM K-means</i>	$6.05 \times 10^{-3}$

Table VI: Pattern Matching

## 1.2.2 $SLL_2 = -25$ [dB]

### Antenna configuration

- Isotropic Elements
- Number of Elements:  $N = 32$
- Distance between Elements along  $x$  axis:  $d_x = \lambda/2$

### Target excitations

- Asymmetric Flat Top,  $SLL_1 = -20$  [dB] -  $SLL_2 = -25$  [dB]

### Sub-array generation

- number of clusters:  $Q = 24$
- excitation matching strategy: EM K-Means
- power matching strategy: PM K-means

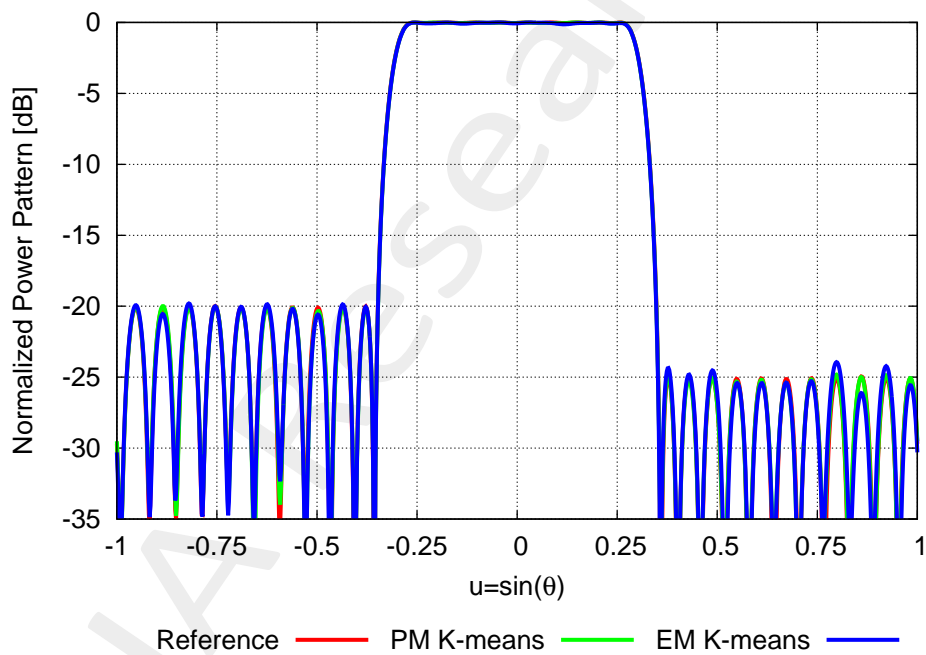


Figure 9: Power Pattern comparison PM K-means vs. EM K-means

Approach	Pattern Matching
<i>PM K-means</i>	$3.50 \times 10^{-3}$
<i>EM K-means</i>	$7.87 \times 10^{-3}$

Table VII: Pattern Matching

### 1.2.3 $SLL_2 = -30$ [dB]

#### Antenna configuration

- Isotropic Elements
- Number of Elements:  $N = 32$
- Distance between Elements along  $x$  axis:  $d_x = \lambda/2$

#### Target excitations

- Asymmetric Flat Top,  $SLL_1 = -20$  [dB] -  $SLL_2 = -30$  [dB]

#### Sub-array generation

- number of clusters:  $Q = 24$
- excitation matching strategy: EM K-Means
- power matching strategy: PM K-means

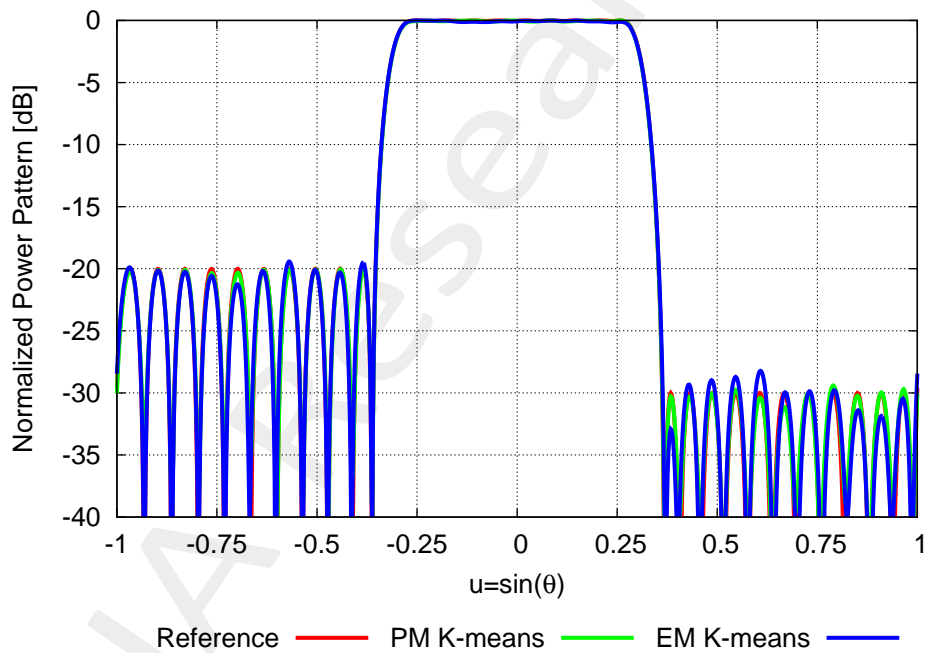


Figure 10: Power Pattern comparison PM K-means vs. EM K-means

Approach	Pattern Matching
<i>PM K-means</i>	$8.35 \times 10^{-3}$
<i>EM K-means</i>	$1.15 \times 10^{-2}$

Table VIII: Pattern Matching

## 1.2.4 $SLL_2 = -35$ [dB]

### Antenna configuration

- Isotropic Elements
- Number of Elements:  $N = 32$
- Distance between Elements along  $x$  axis:  $d_x = \lambda/2$

### Target excitations

- Asymmetric Flat Top,  $SLL_1 = -20$  [dB] -  $SLL_2 = -35$  [dB]

### Sub-array generation

- number of clusters:  $Q = 24$
- excitation matching strategy: EM K-Means
- power matching strategy: PM K-means

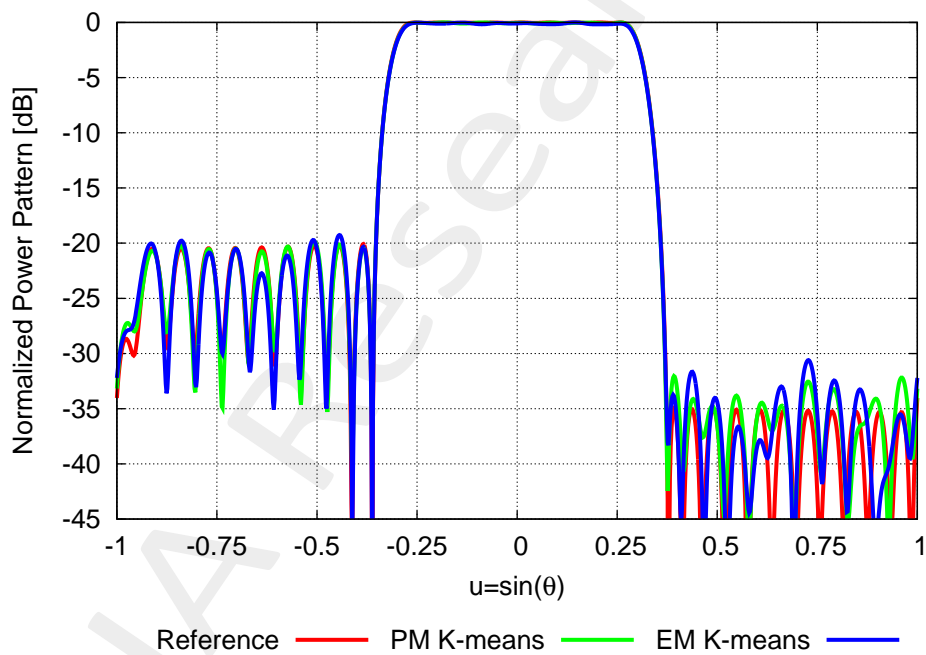


Figure 11: Power Pattern comparison PM K-means vs. EM K-means

Approach	Pattern Matching
<i>PM K-means</i>	$8.95 \times 10^{-2}$
<i>EM K-means</i>	$2.01 \times 10^{-2}$

Table IX: Pattern Matching

## 1.2.5 $SLL_2 = -40$ [dB]

### Antenna configuration

- Isotropic Elements
- Number of Elements:  $N = 32$
- Distance between Elements along  $x$  axis:  $d_x = \lambda/2$

### Target excitations

- Asymmetric Flat Top,  $SLL_1 = -20$  [dB] -  $SLL_2 = -40$  [dB]

### Sub-array generation

- number of clusters:  $Q = 16$
- excitation matching strategy: EM K-Means
- power matching strategy: PM K-means

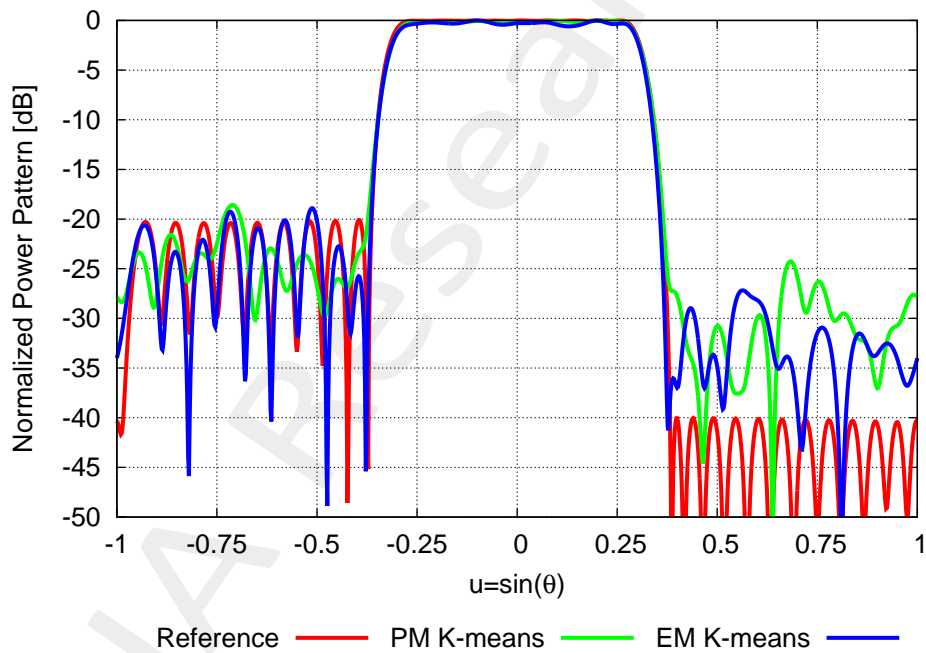


Figure 12: Power Pattern comparison PM K-means vs. EM K-means

Approach	Pattern Matching
<i>PM K-means</i>	$1.06 \times 10^{-2}$
<i>EM K-means</i>	$1.43 \times 10^{-2}$

Table X: Pattern Matching

### 1.2.6 Analysis vs. SLL

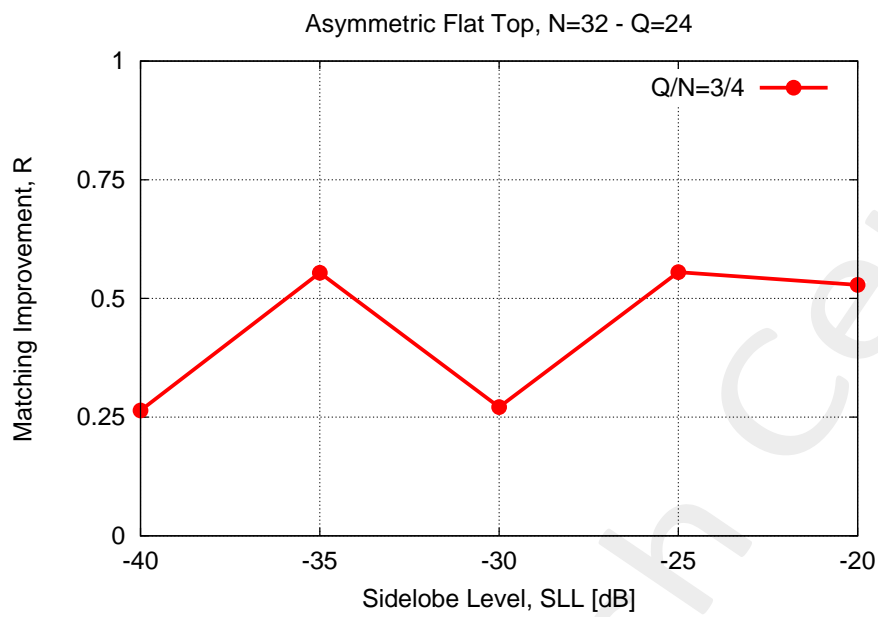


Figure 13: Analysis Matching Improvement vs. SLL

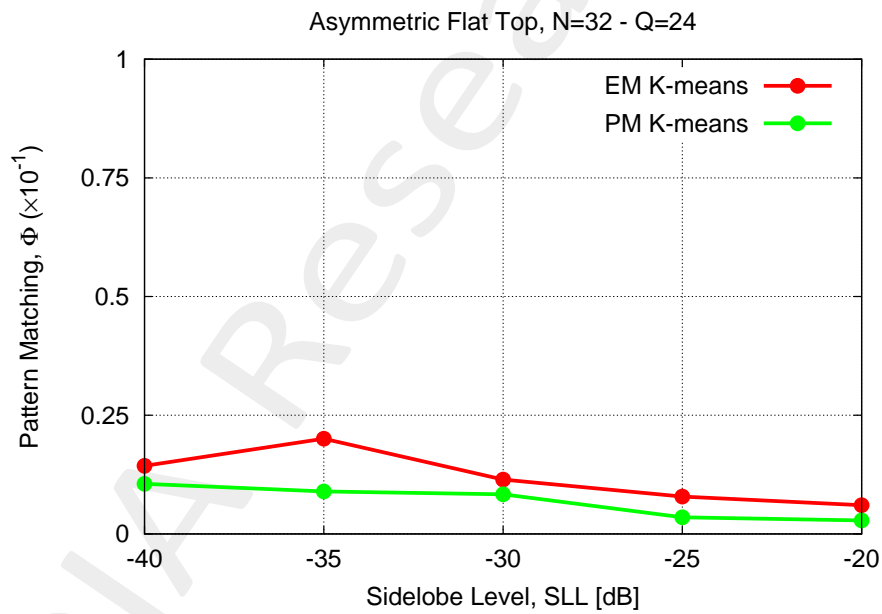


Figure 14: Pattern Matching comparison PM K-means vs. EM K-means for the different SLL for  $Q/N = 3/4$

### 1.3 Pencil Beam with Notch, SLL = -20 [dB], Analysis vs. Notch SLL

#### 1.3.1 $SLL_{notch} = -25$ [dB]

##### Antenna configuration

- Isotropic Elements
- Number of Elements:  $N = 32$
- Distance between Elements along  $x$  axis:  $d_x = \lambda/2$

##### Target excitations

- Pencil Beam Pattern,  $SLL = -20$  [dB]
- Main Lobe Steering:  $\theta_s = 10$  [deg]
- Notch Sidelobe Level:  $SLL_{notch} = -25$  [deg]

##### Sub-array generation

- number of clusters:  $Q = 16$
- excitation matching strategy: EM K-Means
- power matching strategy: PM K-means

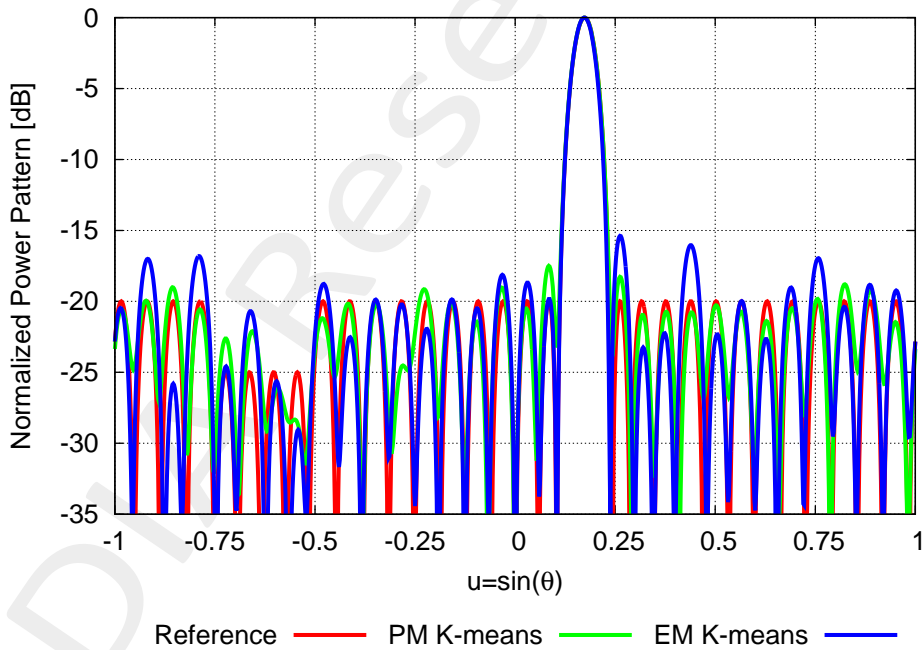


Figure 15: Power Pattern comparison PM K-means vs. EM K-means

Approach	Pattern Matching
<i>PM K-means</i>	$5.53 \times 10^{-2}$
<i>EM K-means</i>	$1.04 \times 10^{-1}$

Table XI: Pattern Matching

### 1.3.2 $SLL_{notch} = -30$ [dB]

#### Antenna configuration

- Isotropic Elements
- Number of Elements:  $N = 32$
- Distance between Elements along  $x$  axis:  $d_x = \lambda/2$

#### Target excitations

- Pencil Beam Pattern,  $SLL = -20$  [dB]
- Main Lobe Steering:  $\theta_s = 10$  [deg]
- Notch Sidelobe Level:  $SLL_{notch} = -30$  [deg]

#### Sub-array generation

- number of clusters:  $Q = 16$
- excitation matching strategy: EM K-Means
- power matching strategy: PM K-means

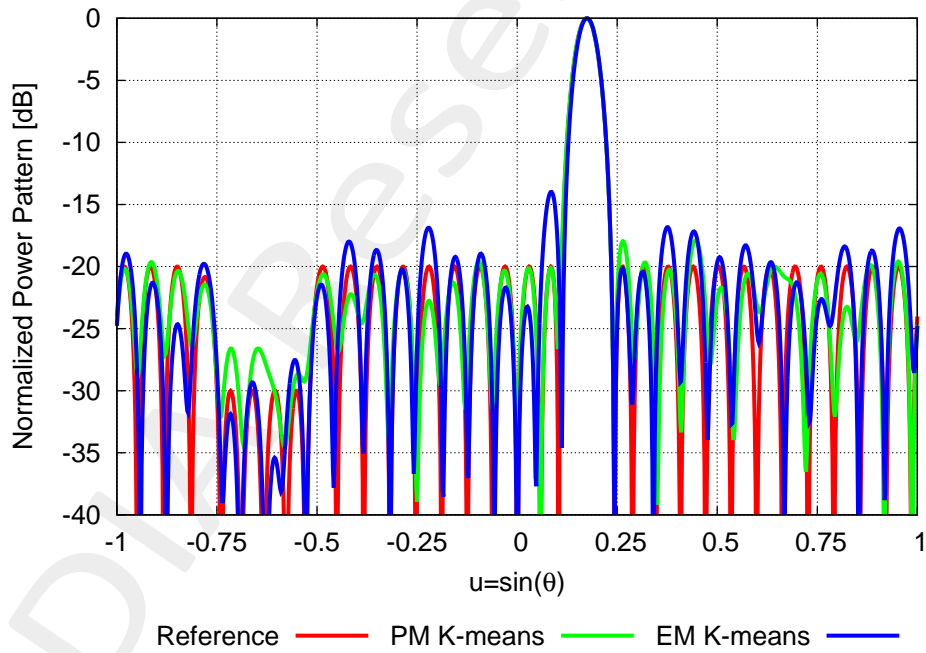


Figure 16: Power Pattern comparison PM K-means vs. EM K-means

Approach	Pattern Matching
<i>PM K-means</i>	$5.17 \times 10^{-2}$
<i>EM K-means</i>	$1.19 \times 10^{-1}$

Table XII: Pattern Matching



### 1.3.3 $SLL_{notch} = -35$ [dB]

#### Antenna configuration

- Isotropic Elements
- Number of Elements:  $N = 32$
- Distance between Elements along  $x$  axis:  $d_x = \lambda/2$

#### Target excitations

- Pencil Beam Pattern,  $SLL = -20$  [dB]
- Main Lobe Steering:  $\theta_s = 10$  [deg]
- Notch Sidelobe Level:  $SLL_{notch} = -35$  [deg]

#### Sub-array generation

- number of clusters:  $Q = 16$
- excitation matching strategy: EM K-Means
- power matching strategy: PM K-means

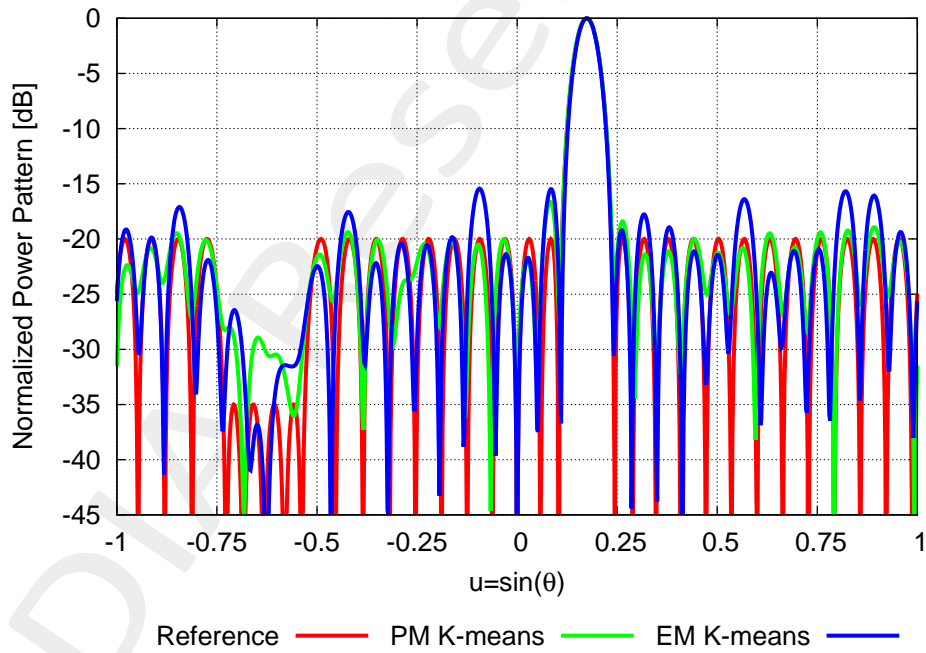


Figure 17: Power Pattern comparison PM K-means vs. EM K-means

Approach	Pattern Matching
<i>PM K-means</i>	$5.66 \times 10^{-2}$
<i>EM K-means</i>	$1.18 \times 10^{-1}$

Table XIII: Pattern Matching

### 1.3.4 $SLL_{notch} = -40$ [dB]

#### Antenna configuration

- Isotropic Elements
- Number of Elements:  $N = 32$
- Distance between Elements along  $x$  axis:  $d_x = \lambda/2$

#### Target excitations

- Pencil Beam Pattern,  $SLL = -20$  [dB]
- Main Lobe Steering:  $\theta_s = 10$  [deg]
- Notch Sidelobe Level:  $SLL_{notch} = -40$  [deg]

#### Sub-array generation

- number of clusters:  $Q = 16$
- excitation matching strategy: EM K-Means
- power matching strategy: PM K-means

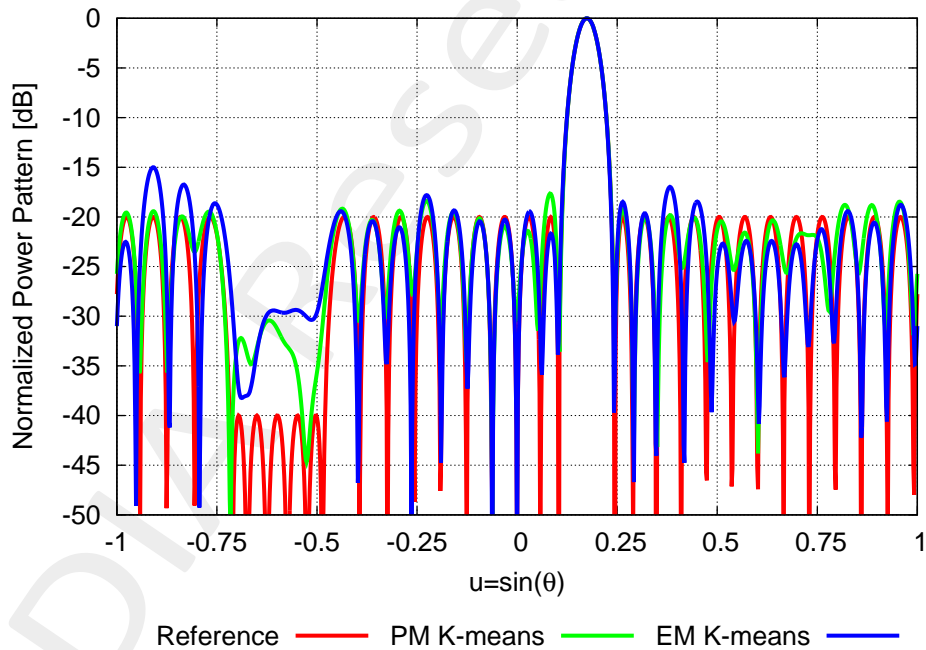


Figure 18: Power Pattern comparison PM K-means vs. EM K-means

Approach	Pattern Matching
<i>PM K-means</i>	$4.81 \times 10^{-2}$
<i>EM K-means</i>	$7.61 \times 10^{-2}$

Table XIV: Pattern Matching

### 1.3.5 $SLL_{notch} = -45$ [dB]

#### Antenna configuration

- Isotropic Elements
- Number of Elements:  $N = 32$
- Distance between Elements along  $x$  axis:  $d_x = \lambda/2$

#### Target excitations

- Pencil Beam Pattern,  $SLL = -20$  [dB]
- Main Lobe Steering:  $\theta_s = 10$  [deg]
- Notch Sidelobe Level:  $SLL_{notch} = -45$  [deg]

#### Sub-array generation

- number of clusters:  $Q = 16$
- excitation matching strategy: EM K-Means
- power matching strategy: PM K-means

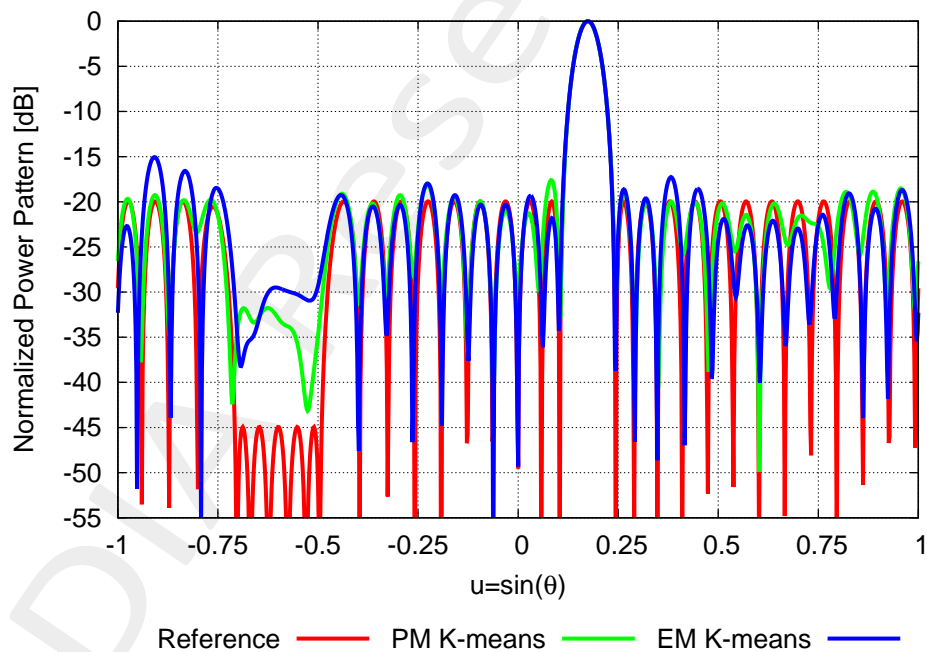


Figure 19: Power Pattern comparison PM K-means vs. EM K-means

Approach	Pattern Matching
<i>PM K-means</i>	$4.76 \times 10^{-2}$
<i>EM K-means</i>	$7.50 \times 10^{-2}$

Table XV: Pattern Matching

### 1.3.6 Analysis vs. Notch SLL

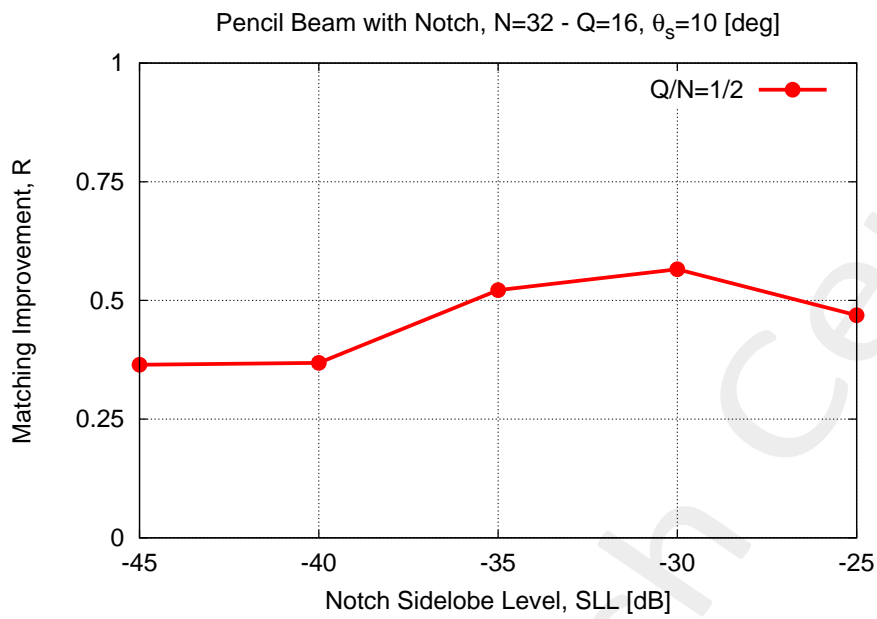


Figure 20: Analysis Matching Improvement vs. Notch Sidelobe Level for  $Q/N = 1/2$

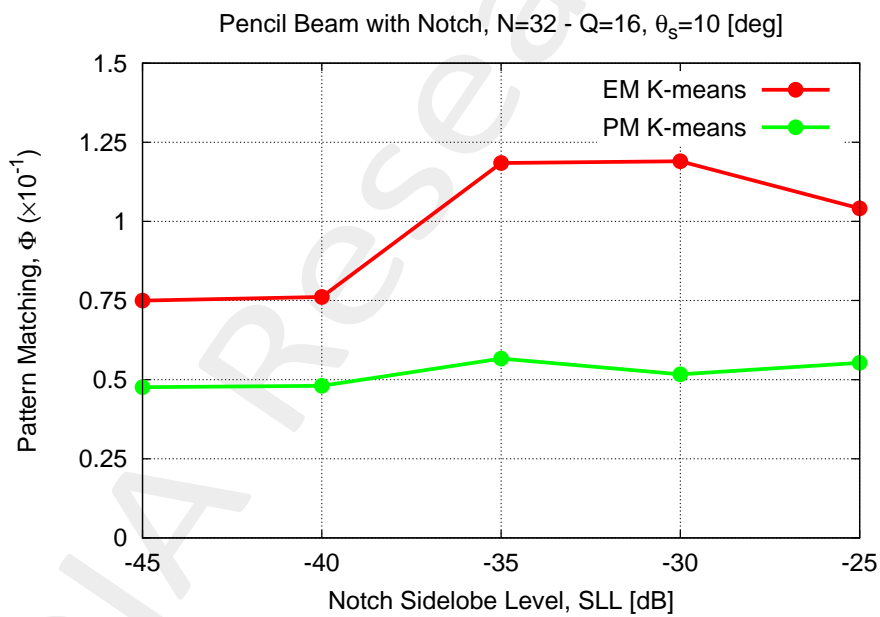


Figure 21: Pattern Matching comparison PM K-means vs. EM K-means varying the Notch SLL for  $Q/N = 1/2$

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## **Observations**

Let us consider the matching error improvement, defined as:

$$R = \frac{\Phi_{EM}(\underline{I}) - \Phi_{PM}(\underline{I})}{\Phi_{EM}(\underline{I})} \quad (1)$$

being  $\Phi_{EM}(\underline{I})$  and  $\Phi_{PM}(\underline{I})$  the power pattern matching of the *EM* K-means and the *PM* K-means and the pattern matching of the *PM* K-means.

From the analysis, it emerges that a good matching improvement can be reach when:

- $R > 0.5$ ;
- $\Phi_{PM}(\underline{I}) \rightarrow 10^{-2}$ .

Moreover, from the analysis it is possible to see a greater improvement when we are dealing with shaped patterns or pencil been patterns with the presence of a notch.

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More information on the topics of this document can be found in the following list of references.

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