Scattering Data Acquisition in Microwave Imaging using Compressive Sampling Techniques

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Contents

1 Mathematical formulation

1.1 Microwave Imaging - Mathematical Background

Let us consider an inaccessible investigation domain D irradiated by a set of incident transverse-magnetic planes $E^{v}_{inc}(\mathbf{r}^{v})$, $v = 1, ..., V$, impinging from the angular directions $\theta^{v} = \frac{2\pi}{V}(v-1)$, being V the number of views. In this working scenario, the scattered field $E^{v}_{scatt}(\mathbf{r}^{v}_{m})$, $m = 1, ..., M$, is supposed to be measured through a set of M sensors equally displaced on a circular observation domain O, external to the investigation domain ($D \cap O = 0$), having radius ρ. The exact location of the sensors are identified by the position vector $\mathbf{r}^{v}_{m} = (\rho \cos\theta^{v}_{m} \sin\theta^{v}_{m})$, being $\theta^{v}_{m} = \theta^{v} + \frac{2\pi}{M}(m-1)$. This scattered field is known to be dependent on the equivalent currents $J^{v}_{eq}(\mathbf{r}) = E^{v}_{tot}(\mathbf{r}^{v})\,\tau(\mathbf{r})$, generated in the support of the unknown scatterers placed into the domain D, according to the *data equation*

$$E^{v}_{scatt}(\mathbf{r}^{v}_{m}) = \int_{D_{inv}} G(\mathbf{r}^{v}_{m}/\mathbf{r}')\, J^{v}_{eq}(\mathbf{r}') \;\; , \;\; \mathbf{r}^{v}_{m} \in O \tag{1}$$

where $G(\mathbf{r}^{v}_{m}/\mathbf{r}')$ is the Green's function in the free space and $E^{v}_{tot}(\mathbf{r}^{v})$ is the total electric field. The material properties of the investigation domain D in terms of relative dielectric permittivity $\varepsilon_r(\mathbf{r})$ and electric conductivity $\sigma(\mathbf{r})$ are described by means of the object function $\tau(\mathbf{r}) = \varepsilon_r(\mathbf{r}) - \varepsilon_0 - \frac{\sigma(\mathbf{r})}{2\pi f \varepsilon_0}$, being f the frequency of the TM plane wave.

In matrix form (after applying the point matching method of moments) eq. (1) can be written as

$$\mathbf{E}^{v}_{scatt} = \mathbf{G}\mathbf{J}_{eq} \quad (\mathbf{y} = \mathbf{A}\mathbf{x}). \tag{2}$$

1.2 Compressive Sensing

Compressive sensing is a signal processing technique for efficiently acquiring and reconstructing signals by finding solutions to undetermined linear systems of equations.

The system of equations $\mathbf{y} = \mathbf{A}\mathbf{x}$, where the solution is found for $\mathbf{x}$, is said to be undetermined. An undetermined system has more unknowns than equations and generally has an infinite number of solution.

In compressive sensing theory there are two main conditions under which recovery is possible:

- the signal $\mathbf{x}$ is required to be sparse;
- incoherence of the matrix $\mathbf{A}$, applied through the isometric property, which is sufficient for sparse signals.

Unfortunately not all undetermined systems of equations have a sparse solution. However, if there is a sparse solution, the compressive sensing theory allows for the recovery of it.

1.3 Restricted Isometry Constant

The Restricted isometry constant (RIC) characterizes a matrix (based of its coherence), which operates on sparse vectors. In linear algebra, the coherence of a matrix is used to measure the ability of a sub-optimal algorithm to correctly identify the true representation of a sparse signal.

We define the RIC as ε. Our goal, in order to find a good solution, is to formulate the problem in such a way that, for matrix $\mathbf{A}$, $\varepsilon < 1$. Unfortunately the computation of ε is a strongly NP-hard problem. For this reason we try an upper bound estimation of ε, namely ε^*, such that $\varepsilon^* \geq \varepsilon$.

To compute ε^* for matrix $\mathbf{A}$ first we compute γ, the coherence of matrix $\mathbf{A}$. The coherence of a matrix is defined as the maximum absolute value of the cross-correlation between the columns of $\mathbf{A}$. Let $\mathbf{a}_1, \ldots, \mathbf{a}_C$ be the columns of the matrix $\mathbf{A}$, where C is the number of columns of $\mathbf{A}$

$$\gamma = \max_{1\leq i\neq j\leq C} \left\{ \frac{\mathbf{a}_i^t \mathbf{a}_j}{\|\mathbf{a}_i\| \, \|\mathbf{a}_j\|} \right\}$$

the quantities ε^{U} and ε^{L} can be computed as

$$\varepsilon^{\mathrm{U}} = \max_i \left\{ \|\mathbf{a}_i\|^2 \right\} \cdot \left[1 + \gamma\,(s-1)\right] - 1$$

$$\varepsilon^{\mathrm{L}} = 1 - \max_i \left\{ \|\mathbf{a}_i\|^2 \right\} \cdot \left[\frac{\min_i \left\{ \|\mathbf{a}_i\|^2 \right\}}{\max_i \left\{ \|\mathbf{a}_i\|^2 \right\}} - \gamma\,(s-1) \right]$$

where s is the sparsity value, finally ε^* is

$$\varepsilon^* = \max \left\{ \varepsilon^{\mathrm{U}}, \varepsilon^{\mathrm{L}} \right\}$$

1.4 Optimization Strategy

The goal of the present work is to propose a method able to compute the minimum number of antennas (number of rows of the observation matrix M) needed in an imaging system such that the investigated image is fully recoverable. Toward this aim, the positions of the antenna elements placed around the investigation domain are optimized with a Particle Swarm Optimization in order to obtain an observation matrix $\mathbf{A}$ with a low coherence, and more precisely in order to minimize the following cost function:

$$\Phi = RIO = \gamma(2S-1) \tag{3}$$

The degrees of freedom are then:

- The number of measurement points, M;
- The measurement points positions: θ_m, for $m = 1, ..., M$.

2 Numerical results, Part #1: Optimization of the Measurements Setup

2.1 Measurement Setup Optimization via PSO-based RIO Minimization

OBJECTIVE: The goal of this section is twofold: (*i*) first, the section is aimed to study the behavior of the RIO when changing the number of elements for both the non-uniform optimized measurement setup and the one where the sensors are uniformly-spaced on the circular investigation domain; (*ii*) then, the section is aimed to study the reconstruction errors as a function of the number of elements considering both the non-uniform optimized and the uniform measurement setup, also changing the scatterers' distribution within the investigation domain (and thus the sparsity factor S).

Test Case Description

Direct solver:

- Side of the investigation domain: $L = 3.0\lambda$
- Cubic domain divided in $\sqrt{D} \times \sqrt{D}$ cells
- Number of cells for the direct solver: $D = 1600$ (discretization $= \lambda/10$)

Investigation domain:

- Cubic domain divided in $\sqrt{N} \times \sqrt{N}$ cells
- Number of cells for the inversion: $N = 324$

Measurement domain:

- Total number of measurements: $M \in [2 : 38]$
- Measurement points placed on circles of radius $\rho = 3.0\lambda$

Sources:

- Plane waves
- Number of views: $V = 1$; $\theta^v_{inc} = 0°$
- Amplitude: $A = 1.0$
- Frequency: $F = 300$ MHz ($\lambda = 1$)

Background:

- $\varepsilon_r = 1.0$
- $\sigma = 0$ [S/m]
- Scatterers size/sparsity factor: $S \in [1 : 15]$

PSO parameters:

- Swarm dimension: $P = M$;
- Maximum iteration number: $I_{\max} = 1000$;
- Inertial weight: 0.4;
- $c_1 = 2.0$; $c_2 = 2.0$

Optimized Solutions: $M = 2, 3$

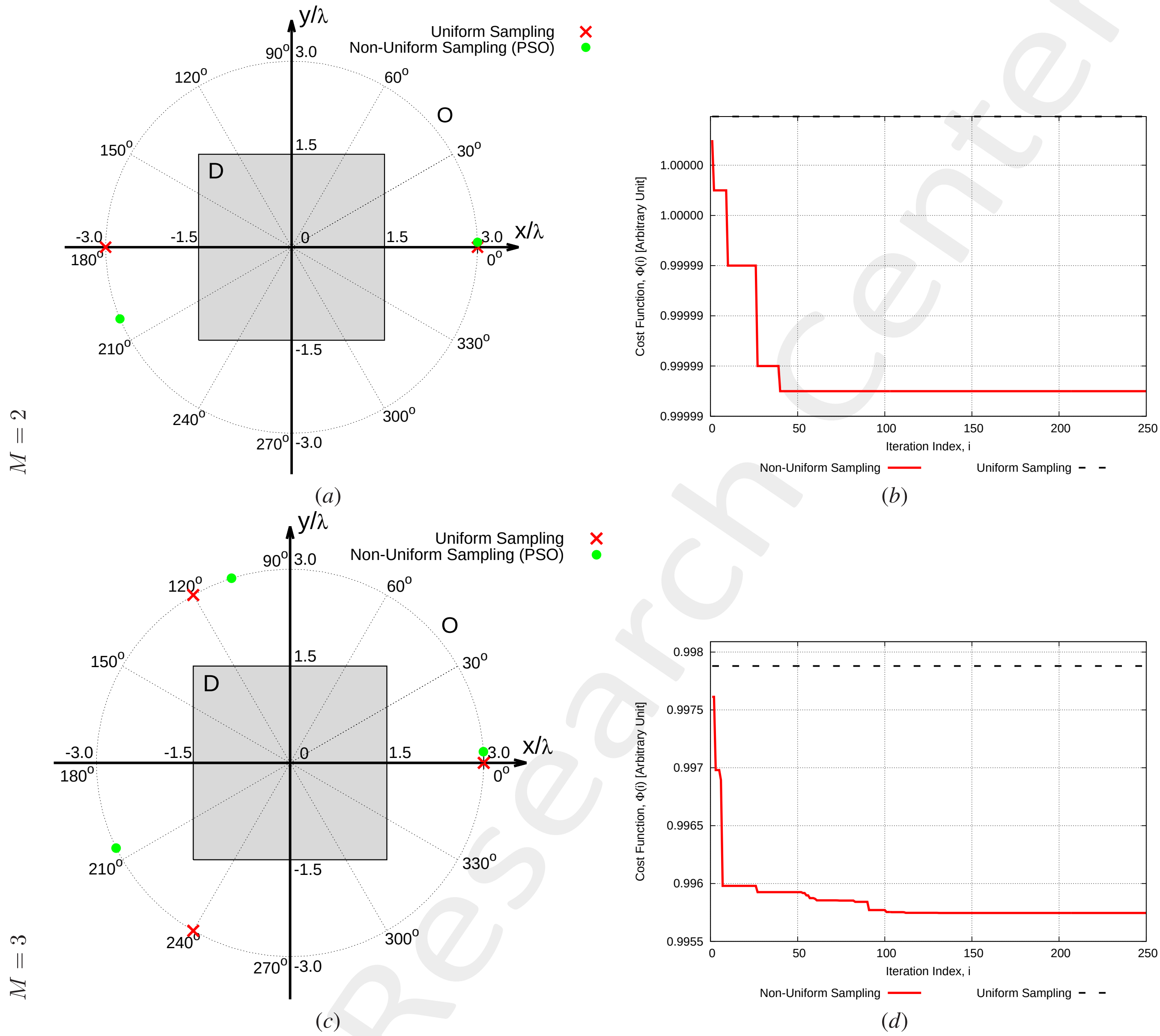

Figure 1: $(a)(c)$ Optimized vs. reference (with uniformly-spaced points) measurement setup and $(b)(d)$ fitness function (Φ) behaviour vs. iteration index for $(a)(b)$ $M = 2$, $(c)(d)$ $M = 3$.

Optimized Solutions: $M = 5, 6, 9$

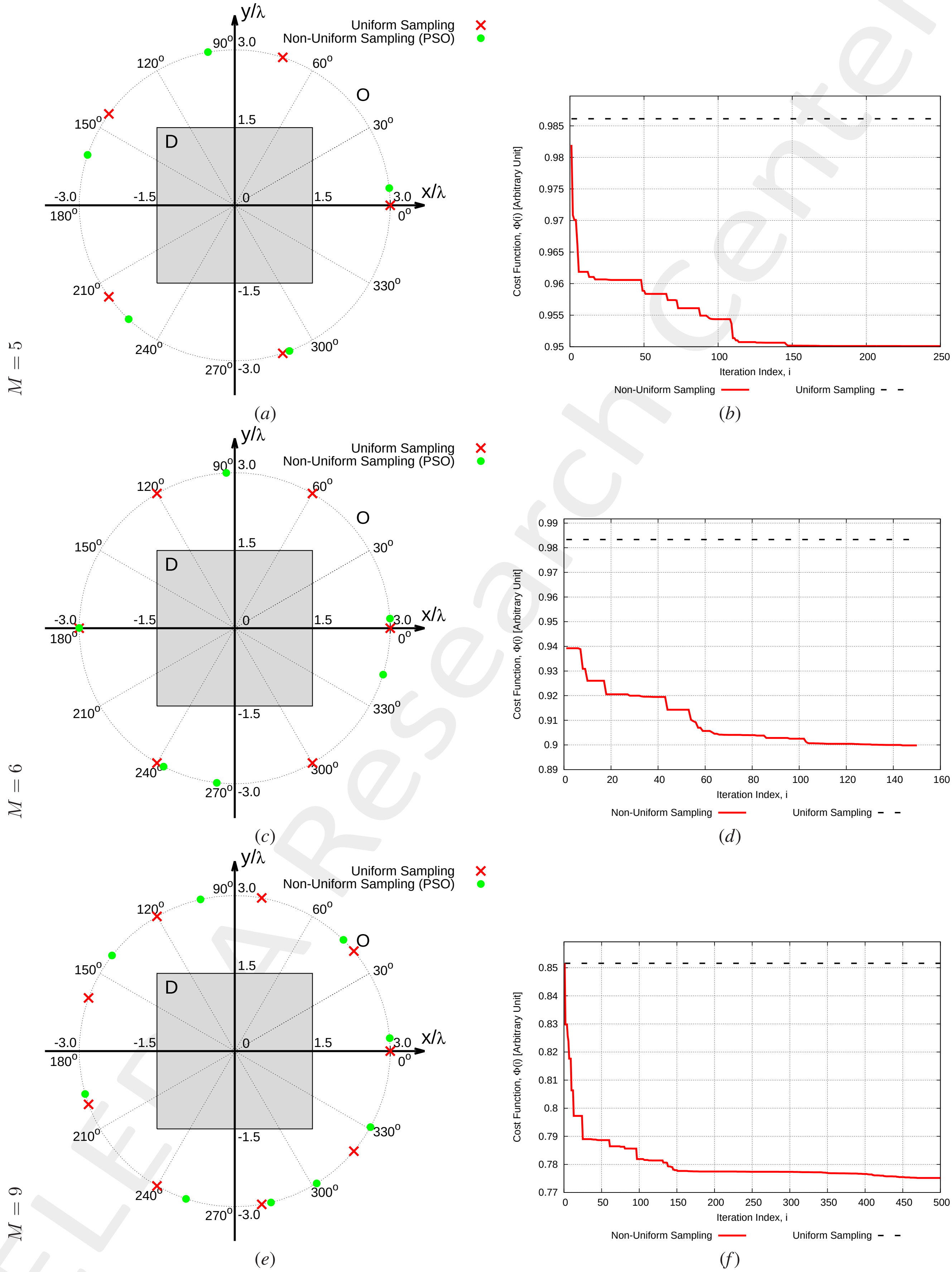

Figure 2: $(a)(c)(e)$ Optimized vs. reference (with uniformly-spaced points) measurement setup and $(b)(d)(f)$ fitness function (Φ) behaviour vs. iteration index for $(a)(b)$ $M = 5$, $(c)(d)$ $M = 6$ and $(e)(f)$ $M = 9$.

Optimized Solutions: $M = 15, 27, 38$

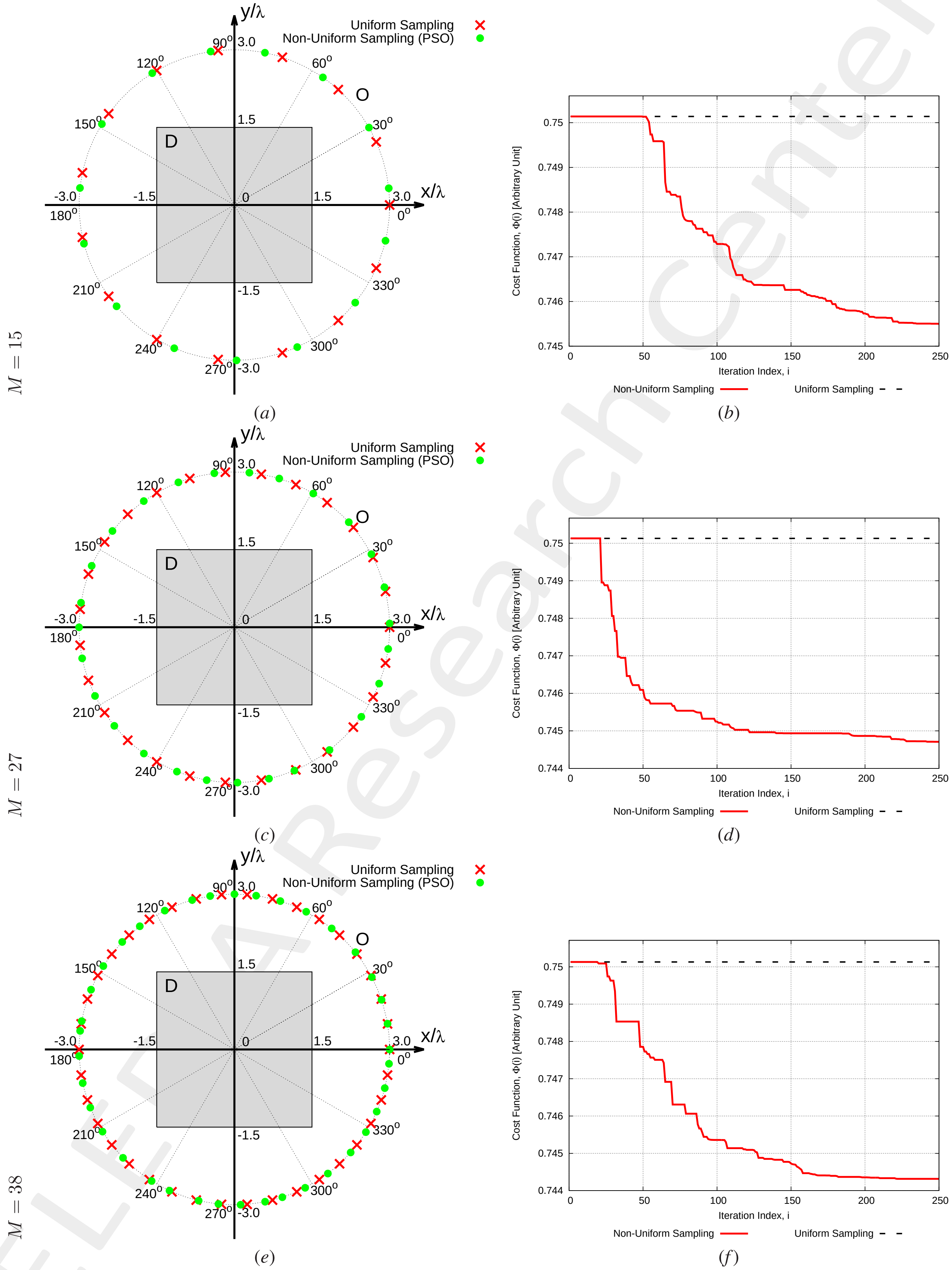

Figure 3: $(a)(c)(e)$ Optimized vs. reference (with uniformly-spaced points) measurement setup and $(b)(d)(f)$ fitness function (Φ) behaviour vs. iteration index for $(a)(b)$ $M = 15$, $(c)(d)$ $M = 27$ and $(e)(f)$ $M = 38$.

More information on the topics of this document can be found in the following list of references.

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