Hilbert-Based Clustering Approach for Linear Arrays

A. Benoni, P. Rocca, N. Anselmi, and A. Massa

2024/06/07

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1 Numerical Results

1.1 Perfomance Analysis

Let us evaluate the performance of the three different approaches, varying the number of elements and clusters. An exhaustive search has been performed to find the optimum achievable configuration given a sorted listed. More in details,

- *Hilbert/Moore Sorting*:
  - number curve order evaluated: $H = [1,\ 5]$
  - number of starting points computed: 4 (*BOTTOM/TOP/LEFT/RIGHT)*

  Thus, the number of configuration taken into account is given by this formula:

$$\#\,\text{Simulation} = \sum_{n=10}^{25}\sum_{q=5}^{10}\begin{pmatrix} n-1 \\ q-1 \end{pmatrix} \times 20 = 142079680$$

- *Spectral LPM Sorting:*

  There is no a curve order or a possible different starting point, since the sorted list is given by an analytical computation. Therefore, the total amount of simulation computed is given by:

$$\#\,\text{Simulation} = \sum_{n=10}^{25}\sum_{q=5}^{10}\begin{pmatrix} n-1 \\ q-1 \end{pmatrix} = 7103984$$

To compare the proposed methodologies and the K-means approach, two differential quantities have been calculated:

- *difference excitation mathing*: $\Delta\Psi = \Psi_i - \Psi_{K-means},\ i \in \{Hilbert,\ Moore,\ Spectral\ LPM\}$
- *difference field matching*: $\Delta\Phi = \Phi_i - \Phi_{K-means}\ i \in \{Hilbert,\ Moore,\ Spectral\ LPM\}$
- *difference pattern matching*: $\Delta\Gamma = \Gamma_i - \Gamma_{K-means}\ i \in \{Hilbert,\ Moore,\ Spectral\ LPM\}$

In the following, different array excitations have been considered and 3 colors maps have been computed for the difference excitation matching and the difference field matching. In the 3 colors map the following rules have been applied:

- ***green***: the proposed approach (Hilbert, Moore or Spectral LPM) outperforms the K-means method,
- ***yellow***: the proposed approach (Hilbert, Moore or Spectral LPM) gives the same results of the K-means method,
- ***red***: the proposed approach fails compared to the K-means method.

1.1.1 Steered Pencil Beam Pattern, $\mathbf{N = [10,\ 25]}$, $\mathbf{Q = [5,\ 10]}$, $\boldsymbol{\theta_0 = -10}$[deg]

Test Case Description

Antenna configuration

- isotropic elements
- number of elements: $N = [10,\ 25]$
- distance between elements along x axis: $d_x = \lambda/2$

Target excitations

- $w_n = \alpha_n e^{j\varphi_n}$ with $n = 1, .., N$
- main lobe steering: $\theta_0 = -10$ [deg]
- Taylor pattern
  - nominal sidelobe level: $SLL_{ref} = -30$ [dB]
  - polinomial order: $\bar{n} = 7$

Sub-array generation

- number of clusters: $Q = [5,\ 10]$
- excitation matching strategies:
  - K-Means
  - Hilbert + Exhaustive Search
  - Moore + Exhaustive Search
  - Spectral LPM + Exhaustive Search

Hilbert Curve Sorting + Exhaustive Search vs. K-means

Analysis varying Hilbert order:

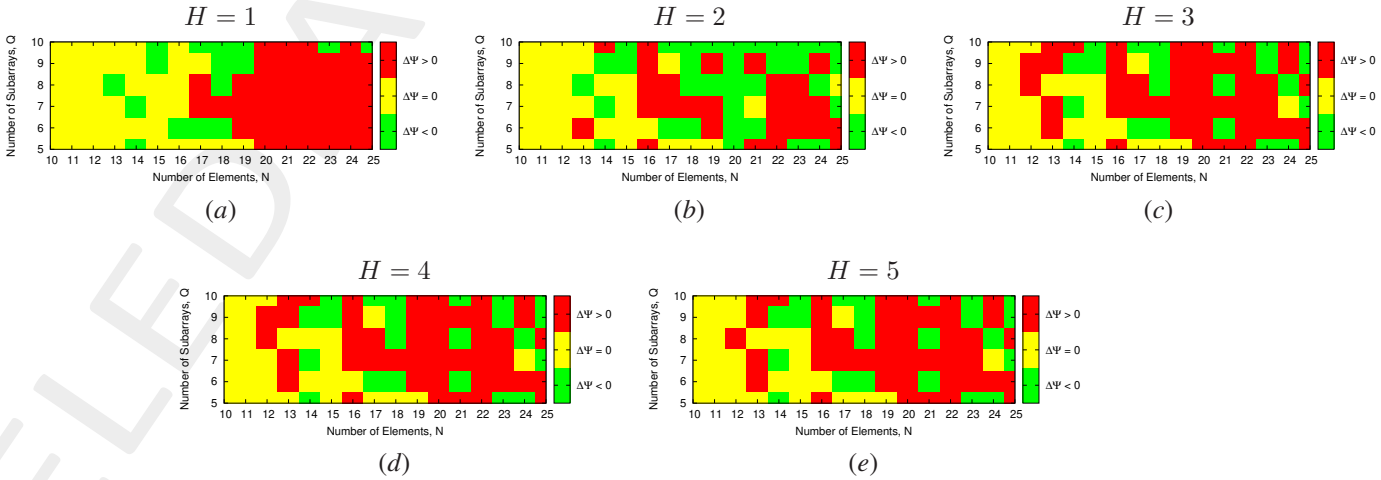

Figure 1: *Steered Pencil Beam Pattern, $\theta_0 = -10$ [deg], Performance Evaluation:* difference excitation matching evaluation, for (*a*) $H = 1$, (*b*) $H = 2$, (*c*) $H = 3$, (*d*) $H = 4$ and (*e*) $H = 5$, taking into account 4 starting points (*BOTTOM/TOP/RIGHT/LEFT*).

Resume:

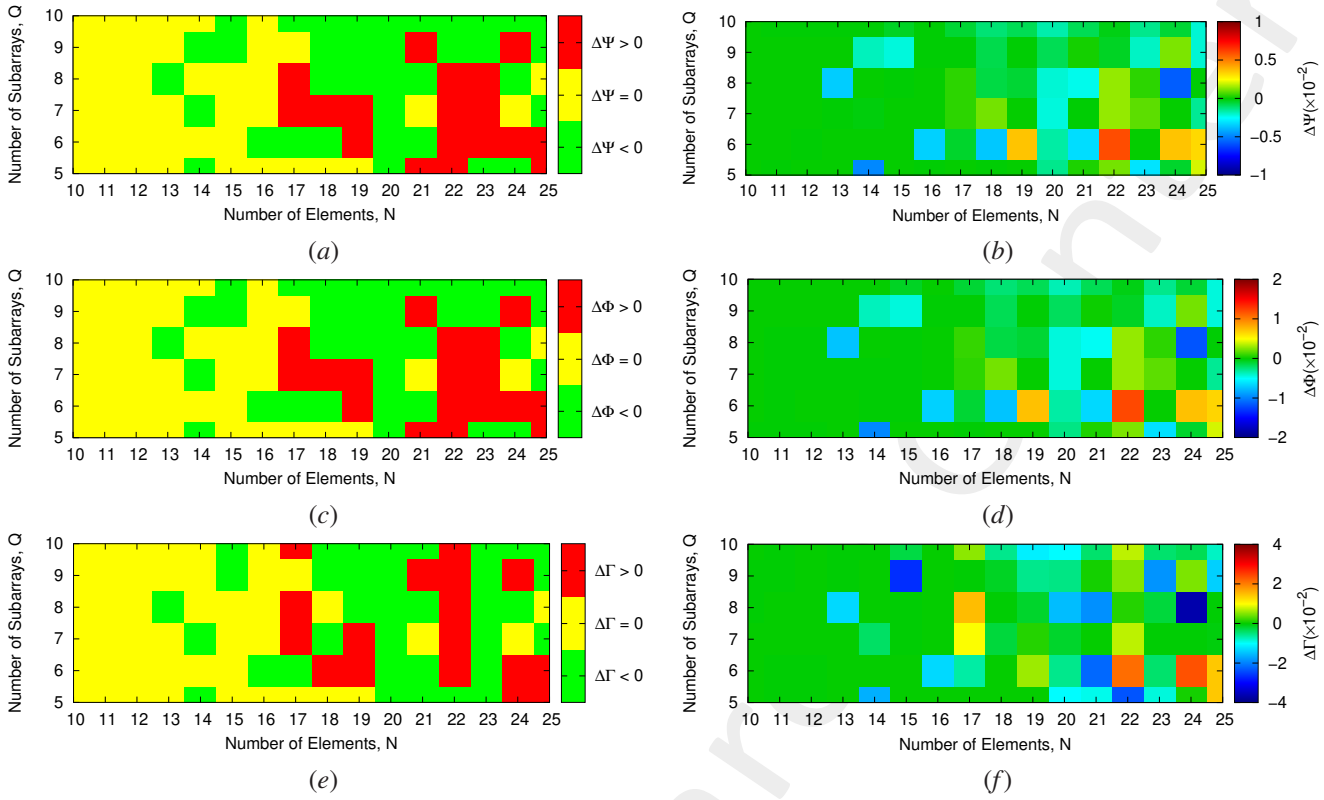

Figure 2: *Steered Pencil Beam Pattern, $\theta_0 = -10$ [deg], Performance Evaluation:* $(a)(c)(e)$ 3 colors map and $(b)(d)(f)$ quantitative color map for the $(a)(b)$ difference excitation matching, $(c)(d)$ difference field matching and $(e)(f)$ difference power pattern matching.

N/Q	$N=10$	$N=11$	$N=12$	$N=13$	$N=14$	$N=15$	$N=16$	$N=17$
$Q=5$	N/A	N/A	N/A	N/A	[1 : 5]	N/A	N/A	N/A
$Q=6$	N/A	N/A	N/A	N/A	N/A	N/A	1	[1 : 5]
$Q=7$	N/A	N/A	N/A	N/A	[1 : 5]	N/A	N/A	N/A
$Q=8$	N/A	N/A	N/A	[1 : 2]	N/A	N/A	N/A	N/A
$Q=9$	N/A	N/A	N/A	N/A	[2 : 5]	[1 : 5]	N/A	N/A
$Q=10$	N/A	N/A	N/A	N/A	N/A	[1 : 5]	N/A	[1 : 5] [3 : 4]

N/Q	$N=18$	$N=19$	$N=20$	$N=21$	$N=22$	$N=23$	$N=24$	$N=25$
$Q=5$	N/A	N/A	2	N/A	N/A	[2 : 5]	[2 : 5]	N/A
$Q=6$	[1 : 5] [2 : 5]	N/A	2	[2 : 5] 2	N/A	N/A	N/A	N/A
$Q=7$	N/A	N/A	2	N/A	N/A	N/A	N/A	[2 : 5] [3 : 5]
$Q=8$	[1 : 5] 1	2	2	[2 : 5]	N/A	N/A	[2 : 5] 2	N/A
$Q=9$	[1 : 5] 1	1	2	N/A	2	[2 : 5]	N/A	[2 : 5] 2
$Q=10$	[1 : 5] [2 : 5]	[1 : 2] 1	2	[2 : 5] 2	2	[1 : 5] [3 : 5]	2	[2 : 5] 2

Table I: *Steered Pencil Beam Pattern, $\theta_0 = -10$ [deg], Performance Evaluation:* Hilbert orders for which the proposed approach works better than the K-means algorithm in terms of excitations and field matching, in ***black*** the curve orders where the Hilbert Curve outoperforms the K-means and in ***green*** the cases where the best excitation matching is obtained.

Moore Curve Sorting + Exhaustive Search vs. K-means

Analysis varying Hilbert order:

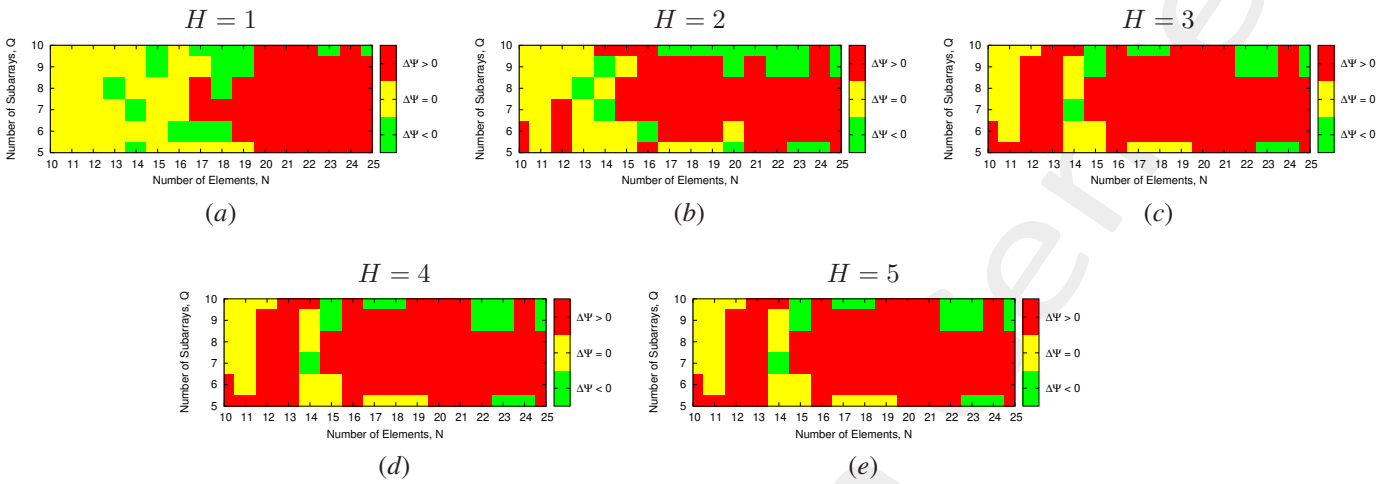

Figure 3: *Steered Pencil Beam Pattern, $\theta_0 = -10$ [deg], Performance Evaluation:* difference excitation matching evaluation, for (*a*) $H = 1$, (*b*) $H = 2$, (*c*) $H = 3$, (*d*) $H = 4$ and (*e*) $H = 5$, taking into account 4 starting points (*BOTTOM/TOP/RIGHT/LEFT*).

Resume:

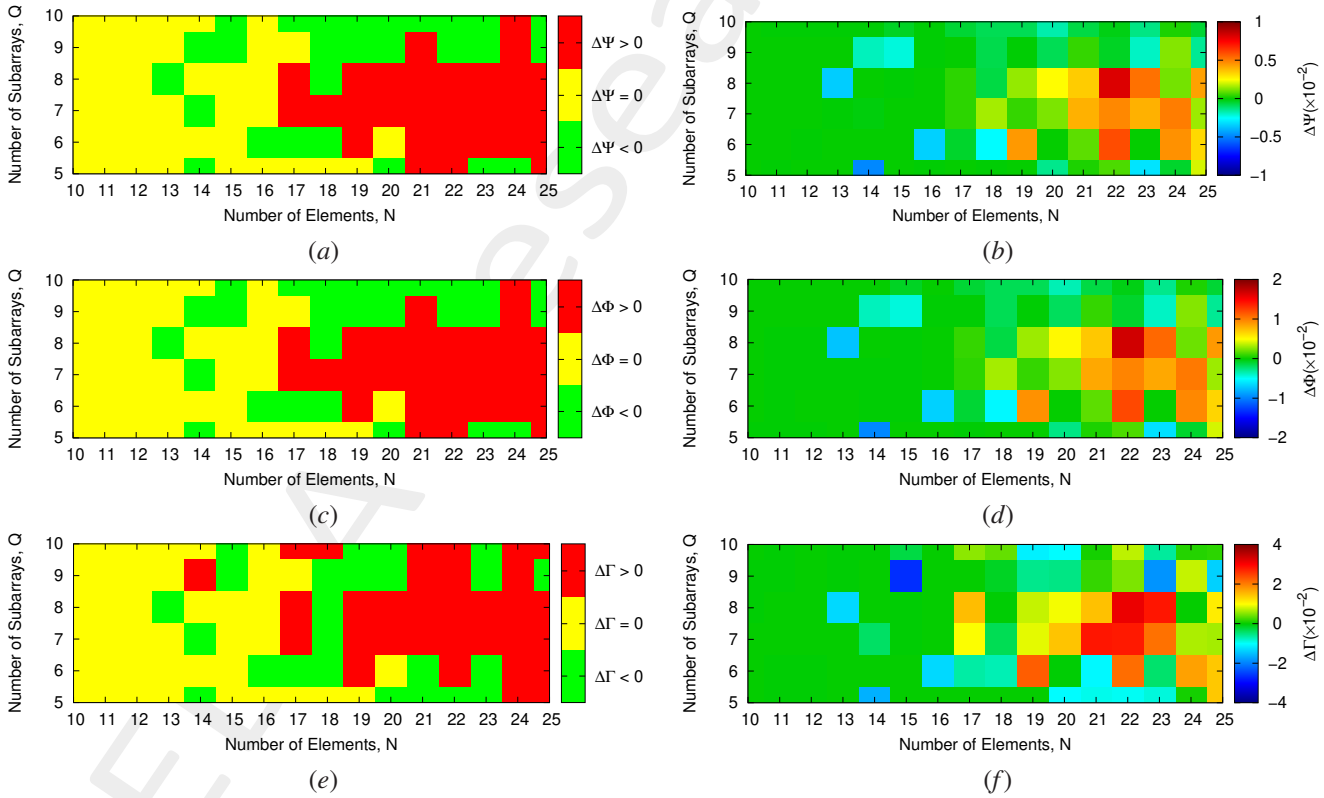

Figure 4: *Steered Pencil Beam Pattern, $\theta_0 = -10$ [deg], Performance Evaluation:* (*a*)(*c*)(*e*) 3 colors map and (*b*)(*d*) quantitative color map for the (*a*)(*b*)(*f*) difference excitation matching, (*c*)(*d*) difference field matching and (*e*)(*f*) difference power pattern matching.

N/Q	$N=10$	$N=11$	$N=12$	$N=13$	$N=14$	$N=15$	$N=16$	$N=17$
$Q=5$	N/A	N/A	N/A	N/A	1	N/A	N/A	N/A
$Q=6$	N/A	N/A	N/A	N/A	N/A	N/A	$[1:2]$ 1	1
$Q=7$	N/A	N/A	N/A	N/A	$[1:5]$	N/A	N/A	N/A
$Q=8$	N/A	N/A	N/A	$[1:2]$	N/A	N/A	N/A	N/A
$Q=9$	N/A	N/A	N/A	N/A	2	$1/[3:5]$	N/A	N/A
$Q=10$	N/A	N/A	N/A	N/A	N/A	$1/[3:5]$	N/A	$[1:5]$ $[2:5]$

N/Q	$N=18$	$N=19$	$N=20$	$N=21$	$N=22$	$N=23$	$N=24$	$N=25$
$Q=5$	N/A	N/A	2	N/A	N/A	$[2:5]$	$[2:5]$	N/A
$Q=6$	1	N/A	N/A	2	N/A	N/A	N/A	N/A
$Q=7$	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
$Q=8$	1	N/A	N/A	N/A	N/A	N/A	N/A	N/A
$Q=9$	1	1	2	N/A	$[2:5]$	$[2:5]$	N/A	$[2:5]$
$Q=10$	$[1:5]$ 1	1	2	2	$[2:5]$	$[1:5]$ $[2:5]$	N/A	$[2:5]$

Table II: *Steered Pencil Beam Pattern, $\theta_0 = -10$ [deg], Performance Evaluation:* Hilbert orders for which the proposed approach works better than the K-means algorithm in terms of excitations and field matching, in ***black*** the curve orders where the Hilbert Curve outoperforms the K-means and in ***green*** the cases where the best excitation matching is obtained.

Spectral LPM Sorting + Exhaustive Search vs. K-means

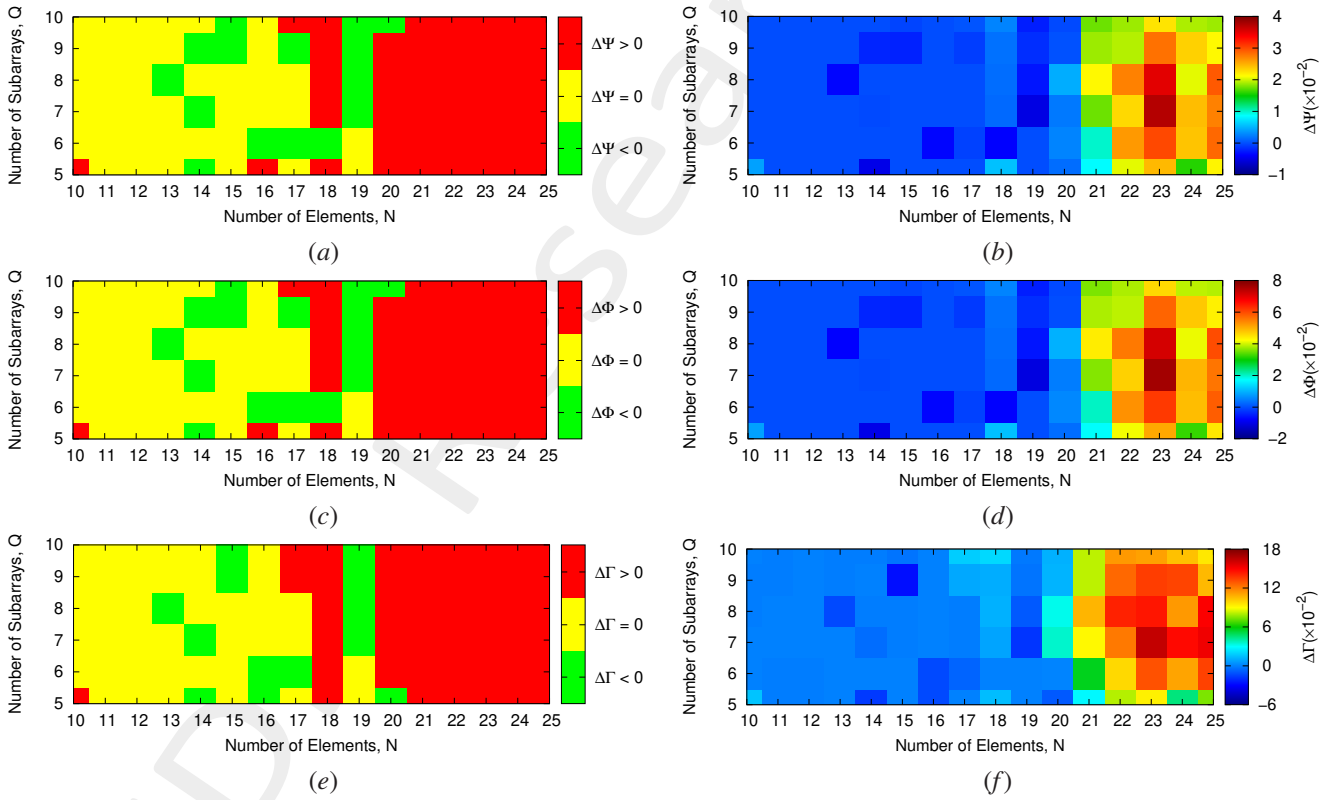

Figure 5: *Steered Pencil Beam Pattern, $\theta_0 = -10$ [deg], Performance Evaluation:* $(a)(c)(e)$ 3 colors map and $(b)(d)(f)$ quantitative color map for the $(a)(b)(e)$ difference excitation matching, $(c)(d)$ difference field matching and $(e)(f)$ difference power pattern matching.

1.1.2 Steered Pencil Beam Pattern, $\mathbf{N = [10,\ 25]}$, $\mathbf{Q = [5,\ 10]}$, $\boldsymbol{\theta_0 = 15}$**[deg]**

Test Case Description

Antenna configuration

- isotropic elements
- number of elements: $N = [10,\ 25]$
- distance between elements along x axis: $d_x = \lambda/2$

Target excitations

- $w_n = \alpha_n e^{j\varphi_n}$ with $n = 1, .., N$
- main lobe steering: $\theta_0 = 15$ [deg]
- Taylor pattern
  - nominal sidelobe level: $SLL_{ref} = -30$ [dB]
  - polinomial order: $\bar{n} = 7$

Sub-array generation

- number of clusters: $Q = [5,\ 10]$
- excitation matching strategies:
  - K-Means
  - Hilbert + Exhaustive Search
  - Moore + Exhaustive Search
  - Spectral LPM + Exhaustive Search

Hilbert Curve Sorting + Exhaustive Search vs. K-means

Analysis varying Hilbert order:

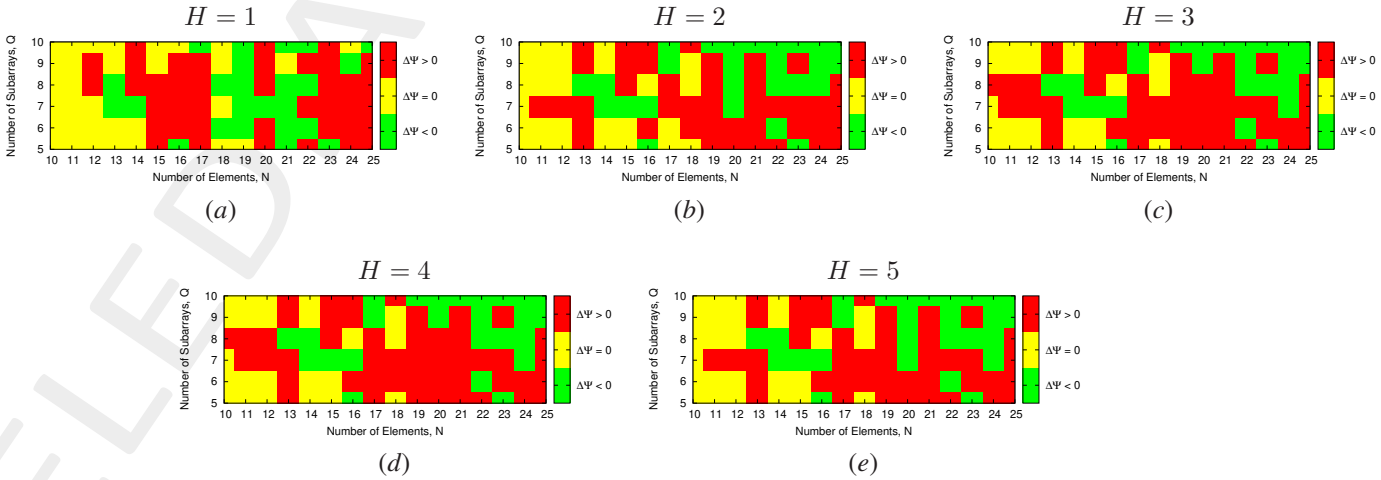

Figure 6: *Steered Pencil Beam Pattern, $\theta_0 = 15$ [deg], Performance Evaluation:* difference excitation matching evaluation, for (*a*) $H = 1$, (*b*) $H = 2$, (*c*) $H = 3$, (*d*) $H = 4$ and (*e*) $H = 5$, taking into account 4 starting points (*BOTTOM/TOP/RIGHT/LEFT*).

Resume:

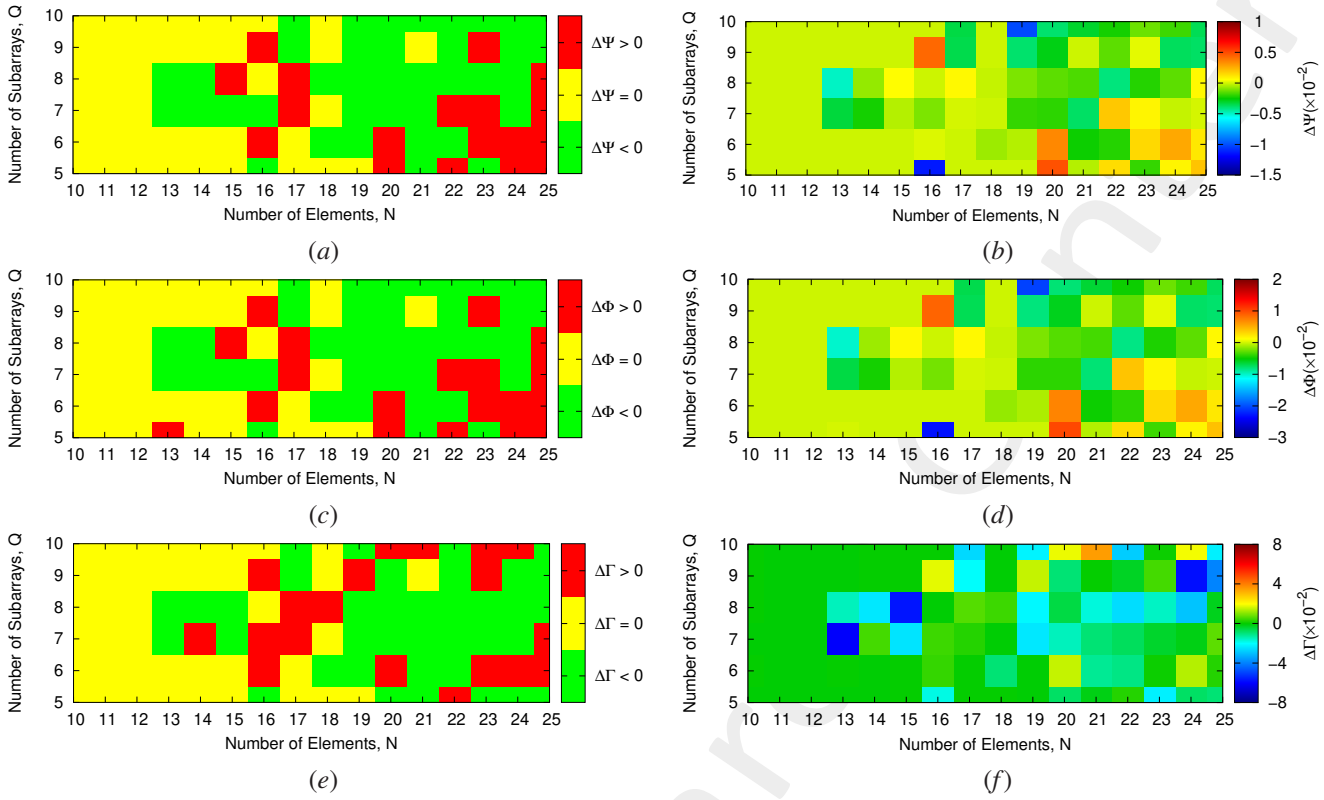

Figure 7: *Steered Pencil Beam Pattern, $\theta_0 = 15$ [deg], Performance Evaluation:* $(a)(c)$ 3 colors map and $(b)(d)$ quantitative color map for the $(a)(b)$ difference excitation matching, $(c)(d)$ difference field matching and $(e)(f)$ difference power pattern matching.

N/Q	$N=10$	$N=11$	$N=12$	$N=13$	$N=14$	$N=15$	$N=16$	$N=17$
$Q=5$	N/A	N/A	N/A	N/A	N/A	N/A	$[1:5]$ $[2:5]$	N/A
$Q=6$	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
$Q=7$	N/A	N/A	N/A	1	$[1:5]$ $[2:5]$	$[2:5]$	$[2:5]$	N/A
$Q=8$	N/A	N/A	N/A	$[1:5]$ 1	$[2:5]$	N/A	N/A	N/A
$Q=9$	N/A	N/A	N/A	N/A	N/A	N/A	N/A	$[2:5]$
$Q=10$	N/A	N/A	N/A	N/A	N/A	N/A	N/A	$[1:5]$ 2

N/Q	$N=18$	$N=19$	$N=20$	$N=21$	$N=22$	$N=23$	$N=24$	$N=25$
$Q=5$	N/A	N/A	N/A	1	N/A	$[1:5]$ 2, 5	N/A	N/A
$Q=6$	1	1	N/A	1	$[1:5]$ 1	N/A	N/A	N/A
$Q=7$	N/A	1	$[1:2]$, 5 2, 5	1	N/A	N/A	$[3:5]$	N/A
$Q=8$	1	1	2, 5	1	$[1:5]$ $[2:5]$	$[2:5]$ 3, 4	$[2:5]$	N/A
$Q=9$	N/A	1	$[2:5]$ 2, 5	N/A	$[2:5]$	N/A	$[1:5]$ 2	$[2:5]$ 2
$Q=10$	N/A	$[1:5]$ $[1:3]$	$[2:5]$	$[1:5]$ $[2:5]$	$[1:5]$ $[2:5]$	$[2:5]$	$[1:5]$ $[2:5]$	$[1:5]$ 2

Table III: *Steered Pencil Beam Pattern, $\theta_0 = 15$ [deg], Performance Evaluation:* Hilbert orders for which the proposed approach works better than the K-means algorithm in terms of excitations and field matching, in ***black*** the curve orders where the Hilbert Curve outperforms the K-means and in ***green*** the cases where the best excitation matching is obtained.

Moore Curve Sorting + Exhaustive Search vs. K-means

Analysis varying Hilbert order:

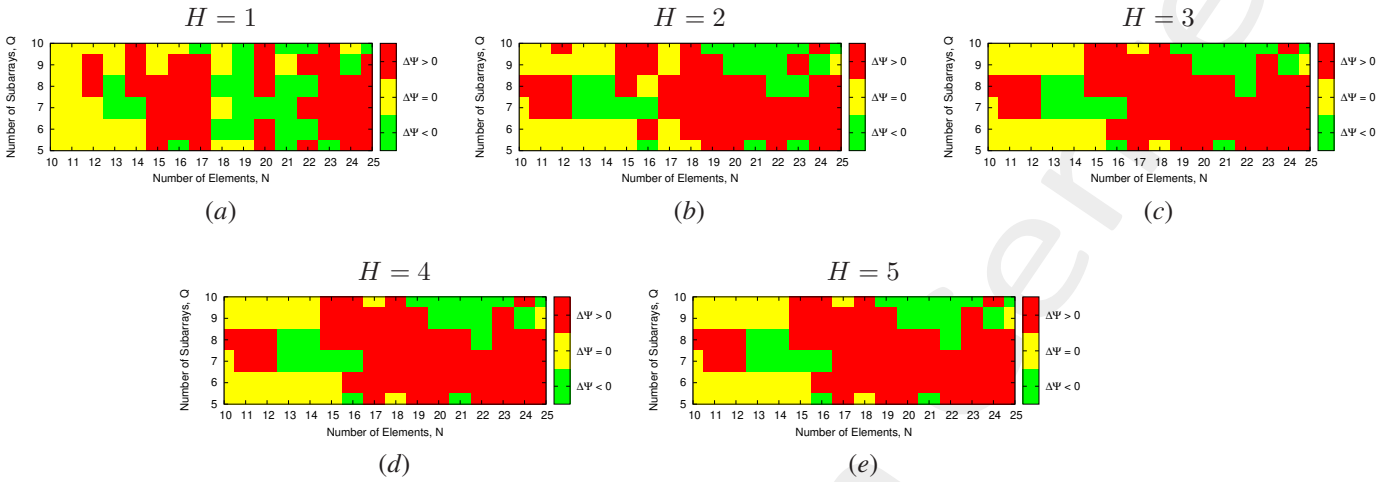

Figure 8: *Steered Pencil Beam Pattern, $\theta_0 = 15$ [deg], Performance Evaluation:* difference excitation matching evaluation, for (*a*) $H = 1$, (*b*) $H = 2$, (*c*) $H = 3$, (*d*) $H = 4$ and (*e*) $H = 5$, taking into account 4 starting points (*BOTTOM/TOP/RIGHT/LEFT*).

Resume:

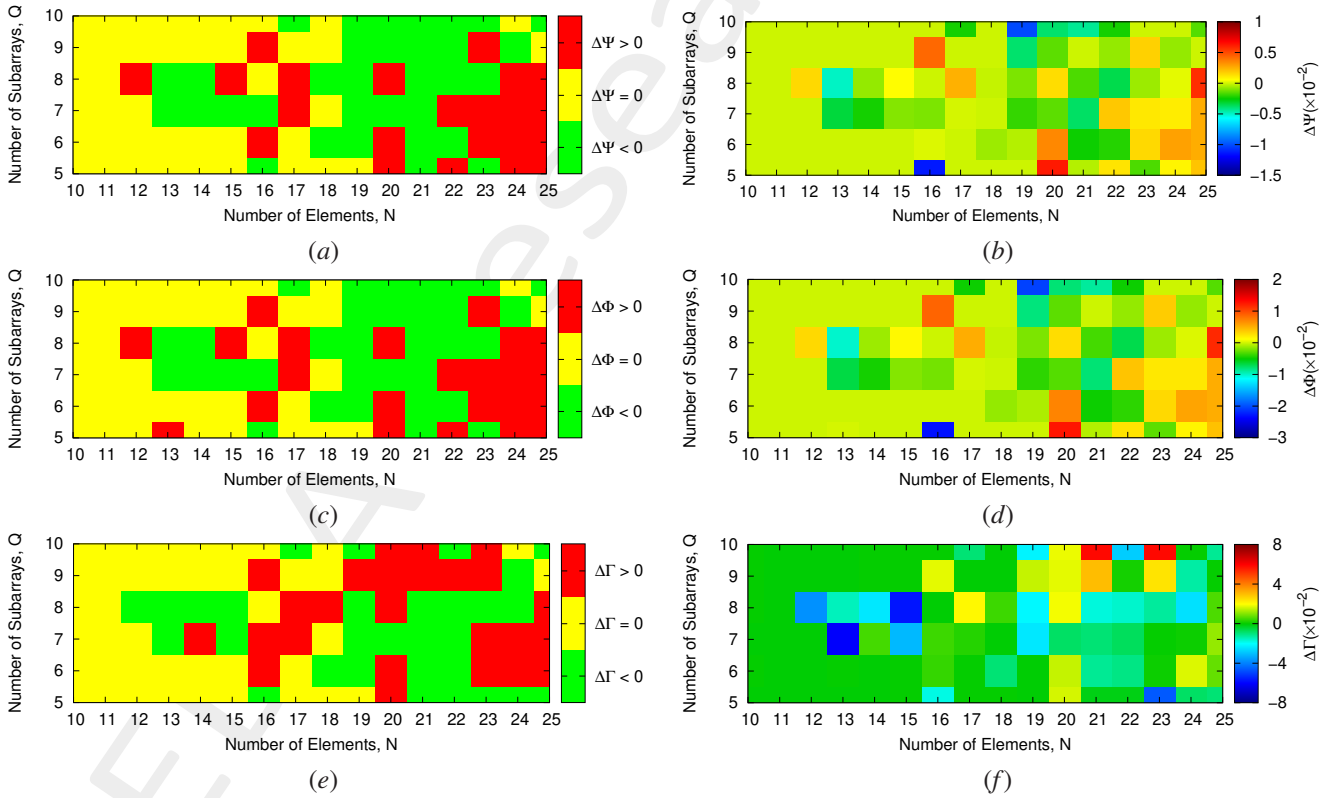

Figure 9: *Steered Pencil Beam Pattern, $\theta_0 = 15$ [deg], Performance Evaluation:* (*a*)(*c*) 3 colors map and (*b*)(*d*) quantitative color map for the (*a*)(*b*) difference excitation matching,(*c*)(*d*) difference field matching and (*e*)(*f*) difference power pattern matching.

N/Q	$N=10$	$N=11$	$N=12$	$N=13$	$N=14$	$N=15$	$N=16$	$N=17$
$Q=5$	N/A	N/A	N/A	N/A	N/A	N/A	[2 : 5]	N/A
$Q=6$	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
$Q=7$	N/A	N/A	N/A	[1 : 5]	[1 : 5]	[2 : 5] 2	[2 : 5]	N/A
$Q=8$	N/A	N/A	N/A	[1 : 5]	[2 : 5]	N/A	N/A	N/A
$Q=9$	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
$Q=10$	N/A	N/A	N/A	N/A	N/A	N/A	N/A	1

N/Q	$N=18$	$N=19$	$N=20$	$N=21$	$N=22$	$N=23$	$N=24$	$N=25$
$Q=5$	N/A	N/A	N/A	[1 : 5]	N/A	[1 : 2] 1	N/A	N/A
$Q=6$	1	1	N/A	1	1	N/A	N/A	N/A
$Q=7$	N/A	1	1	1	N/A	N/A	N/A	N/A
$Q=8$	1	1	N/A	1	[1 : 5] [2 : 5]	2	N/A	N/A
$Q=9$	N/A	1	[2 : 5]	[1 : 5] [2 : 5]	[2 : 5] 2	N/A	[1 : 5] [2 : 5]	N/A
$Q=10$	N/A	[1 : 5] [1 : 3]	[2 : 5]	[1 : 5] [2 : 5]	[1 : 5] [3 : 5]	[2 : 5]	N/A	[1 : 5] [2 : 5]

Table IV: *Steered Pencil Beam Pattern, $\theta_0 = 15$ [deg], Performance Evaluation:* Hilbert orders for which the proposed approach works better than the K-means algorithm in terms of excitations and field matching, in ***black*** the curve orders where the Hilbert Curve outoperforms the K-means and in ***green*** the cases where the best excitation matching is obtained.

Spectral LPM Sorting + Exhaustive Search vs. K-means

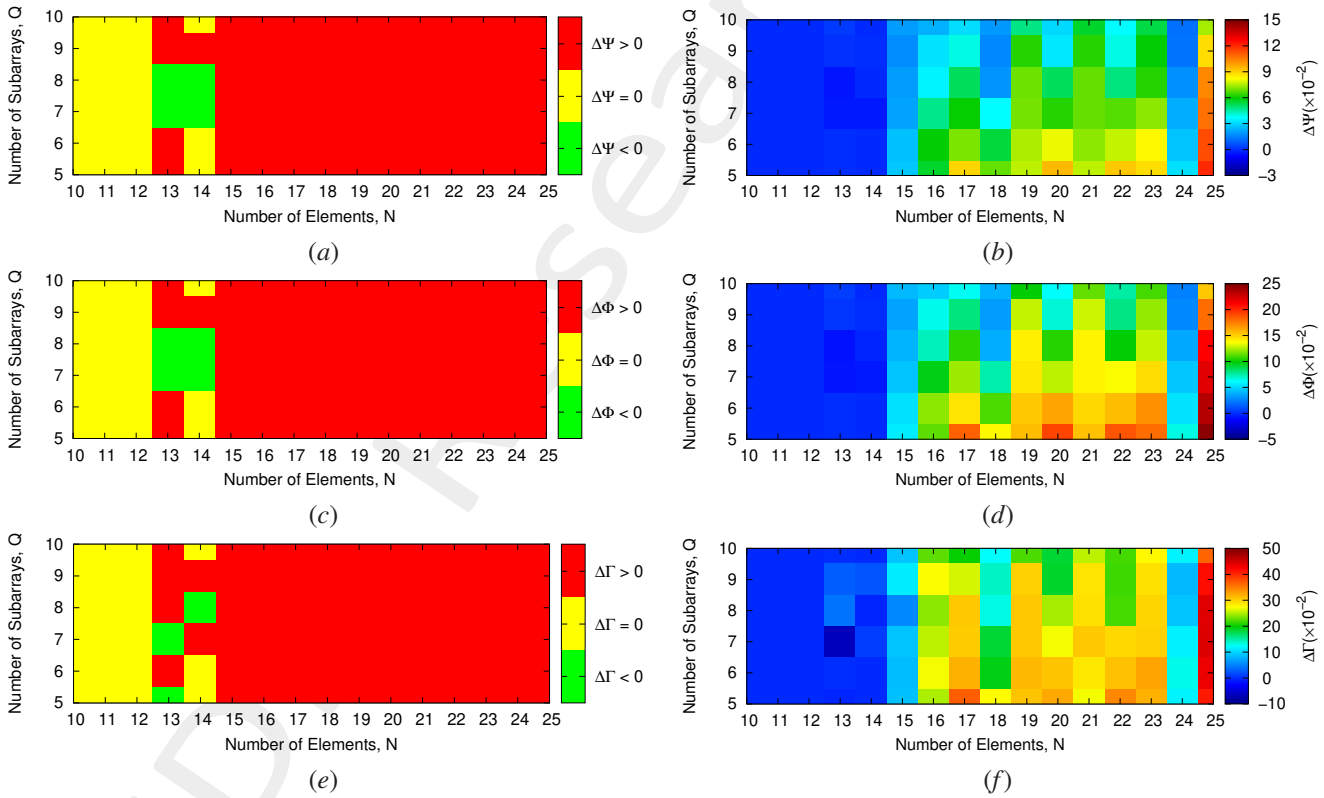

Figure 10: *Steered Pencil Beam Pattern, $\theta_0 = 15$ [deg], Performance Evaluation:* (*a*)(*c*) 3 colors map and (*b*)(*d*) quantitative color map for the (*a*)(*b*) difference excitation matching, (*c*)(*d*) difference field matching and (*e*)(*f*) difference power pattern matching.

1.1.3 Flat Top Pattern, $\mathbf{N} = [10,\ 25]$, $\mathbf{Q} = [5,\ 10]$

Test Case Description

Antenna configuration

- isotropic elements
- number of elements: $N = [10,\ 25]$
- distance between elements along x axis: $d_x = \lambda/2$

Target excitations

- $w_n = \alpha_n e^{j\varphi_n}$ with $n = 1, .., N$
- Flat top

Sub-array generation

- number of clusters: $Q = [5,\ 10]$
- excitation matching strategies:
  - K-Means
  - Hilbert + Exhaustive Search
  - Moore + Exhaustive Search
  - Spectral LPM + Exhaustive Search

Hilbert Curve Sorting + Exhaustive Search vs. K-means

Analysis varying Hilbert order:

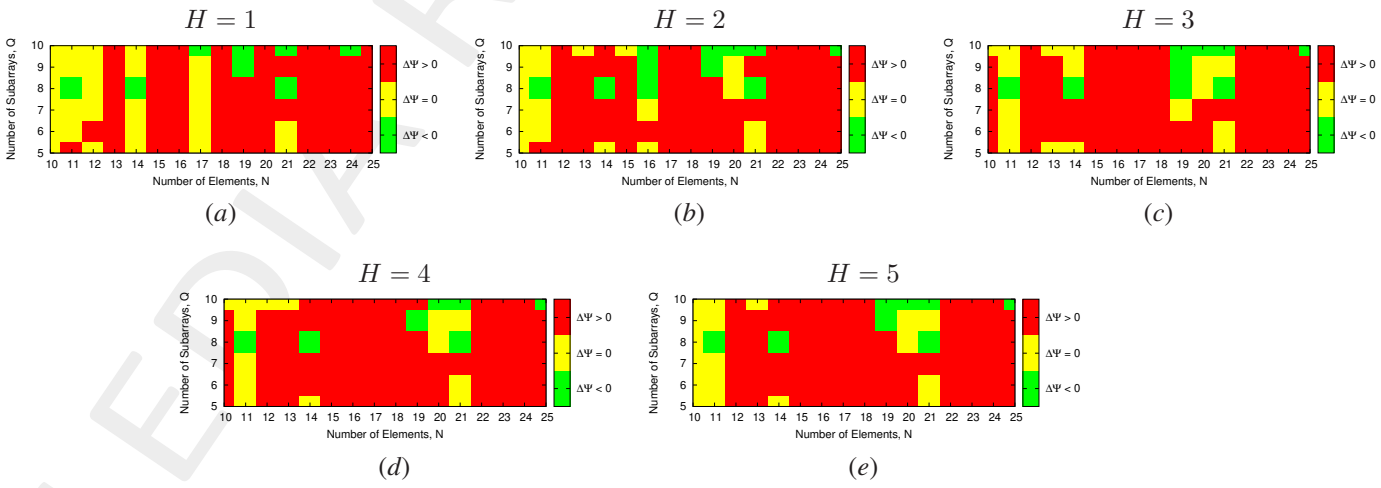

Figure 11: *Flat Top Pattern, Performance Evaluation:* difference excitation matching evaluation, for (*a*) $H = 1$, (*b*) $H = 2$, (*c*) $H = 3$, (*d*) $H = 4$ and (*e*) $H = 5$, taking into account 4 starting points (*BOTTOM/TOP/RIGHT/LEFT*).

Resume:

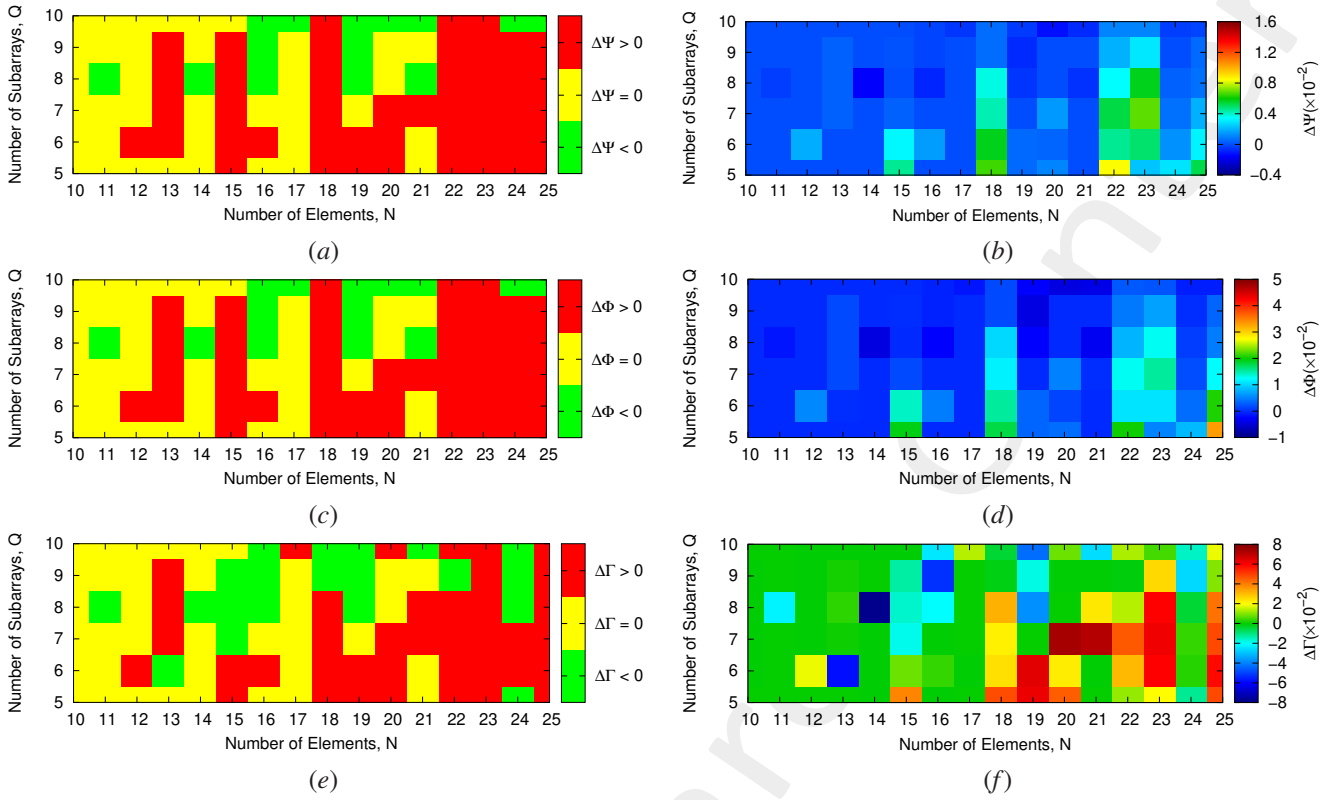

Figure 12: *Flat Top Pattern, Performance Evaluation:* (*a*)(*c*)(*e*) 3 colors map and (*b*)(*d*)(*f*) quantitative color map for the (*a*)(*b*) difference excitation matching, (*c*)(*d*) difference field matching and (*e*)(*f*) diffrence pattern matching.

N/Q	$N=10$	$N=11$	$N=12$	$N=13$	$N=14$	$N=15$	$N=16$	$N=17$
$Q=5$	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
$Q=6$	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
$Q=7$	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
$Q=8$	N/A	$[1:5]$	N/A	N/A	$[1:5]$ 1, 3	N/A	2	N/A
$Q=9$	N/A	N/A	N/A	N/A	N/A	N/A	2	N/A
$Q=10$	N/A	N/A	N/A	N/A	N/A	N/A	2	1

N/Q	$N=18$	$N=19$	$N=20$	$N=21$	$N=22$	$N=23$	$N=24$	$N=25$
$Q=5$	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
$Q=6$	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
$Q=7$	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
$Q=8$	N/A	3	N/A	$[1:5]$	N/A	N/A	N/A	N/A
$Q=9$	N/A	$[1:5]$ 3	N/A	N/A	N/A	N/A	N/A	N/A
$Q=10$	N/A	$[1:5]$ 3	$[2:5]$	$[1:5]$	N/A	N/A	1	$[2:5]$

Table V: *Flat Top Pattern, Performance Evaluation:* Hilbert orders for which the proposed approach works better than the K-means algorithm in terms of excitation and field matching, in ***black*** the curve orders where the Hilbert Curve outoperforms the K-means and in ***green*** the cases where the best excitation matching is obtained.

Moore Curve Sorting + Exhaustive Search vs. K-means

Analysis varying Hilbert order:

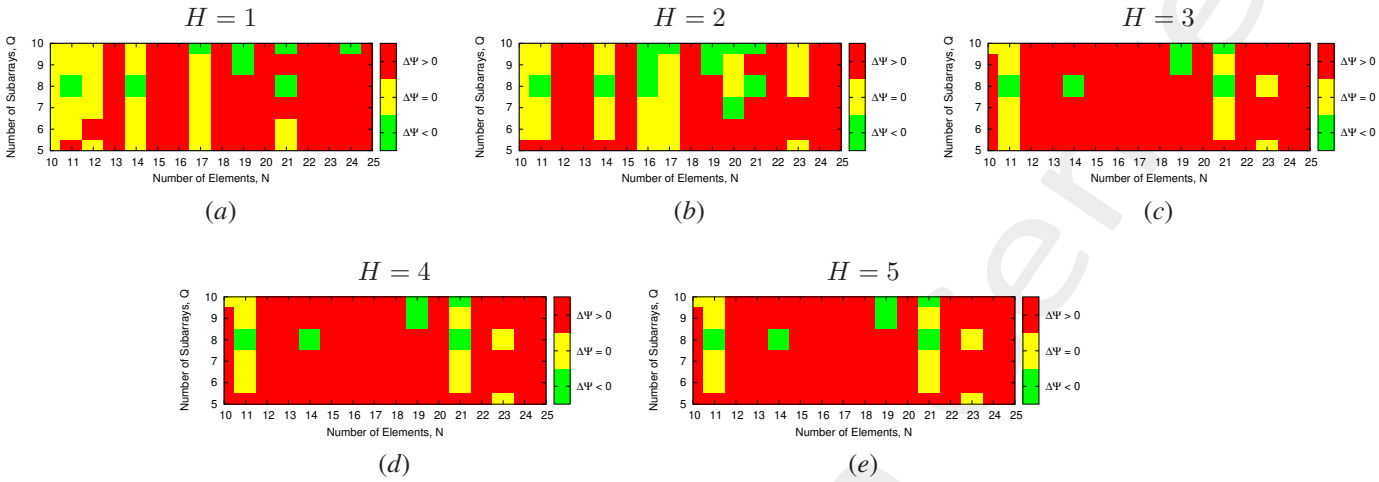

Figure 13: *Flat Top Pattern, Performance Evaluation:* difference excitation matching evaluation, for (a) $H = 1$, (b) $H = 2$, (c) $H = 3$, (d) $H = 4$ and (e) $H = 5$, taking into account 4 starting points (*BOTTOM/TOP/RIGHT/LEFT*).

Resume:

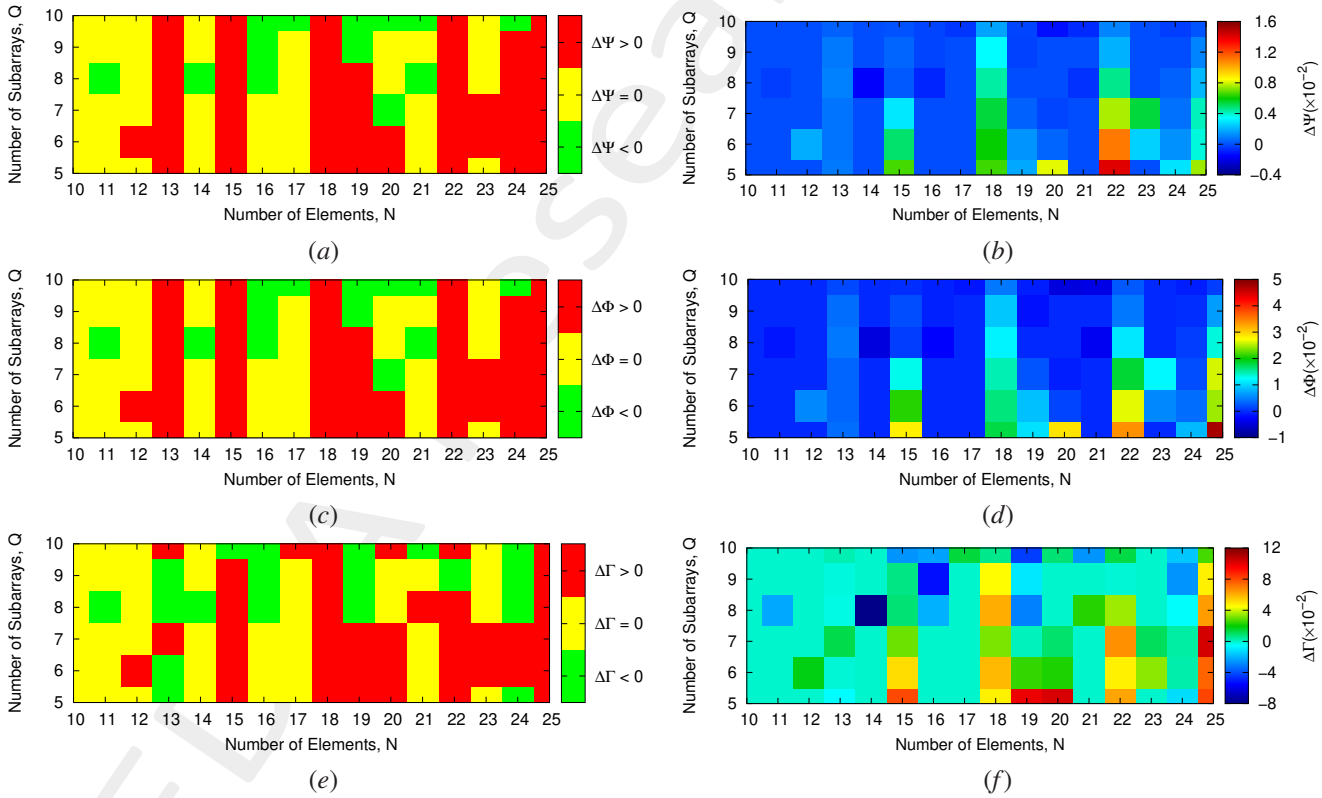

Figure 14: *Flat Top Pattern, Performance Evaluation:* $(a)(c)(e)$ 3 colors map and $(b)(d)(f)$ quantitative color map for the $(a)(b)$ difference excitation matching, $(c)(d)$ difference field matching and $(e)(f)$ difference power pattern matching.

N/Q	$N=10$	$N=11$	$N=12$	$N=13$	$N=14$	$N=15$	$N=16$	$N=17$
$Q=5$	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
$Q=6$	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
$Q=7$	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
$Q=8$	N/A	$[1:5]$	N/A	N/A	$[1:5]$ 1	N/A	2	N/A
$Q=9$	N/A	N/A	N/A	N/A	N/A	N/A	2	N/A
$Q=10$	N/A	N/A	N/A	N/A	N/A	N/A	2	$[1:2]$

N/Q	$N=18$	$N=19$	$N=20$	$N=21$	$N=22$	$N=23$	$N=24$	$N=25$
$Q=5$	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
$Q=6$	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
$Q=7$	N/A	N/A	2	N/A	N/A	N/A	N/A	N/A
$Q=8$	N/A	N/A	N/A	$[1:5]$ $[3:5]$	N/A	N/A	N/A	N/A
$Q=9$	N/A	$[1:5]$	N/A	N/A	N/A	N/A	N/A	N/A
$Q=10$	N/A	$[1:5]$	2	$[1:5]$	N/A	N/A	1	N/A

Table VI: *Flat Top Pattern, Performance Evaluation:* Hilbert orders for which the proposed approach works better than the K-means algorithm in terms of excitations and field matching, in ***black*** the curve orders where the Hilbert Curve outoperforms the K-means and in ***green*** the cases where the best excitation matching is obtained.

Spectral LPM Sorting + Exhaustive Search vs. K-means

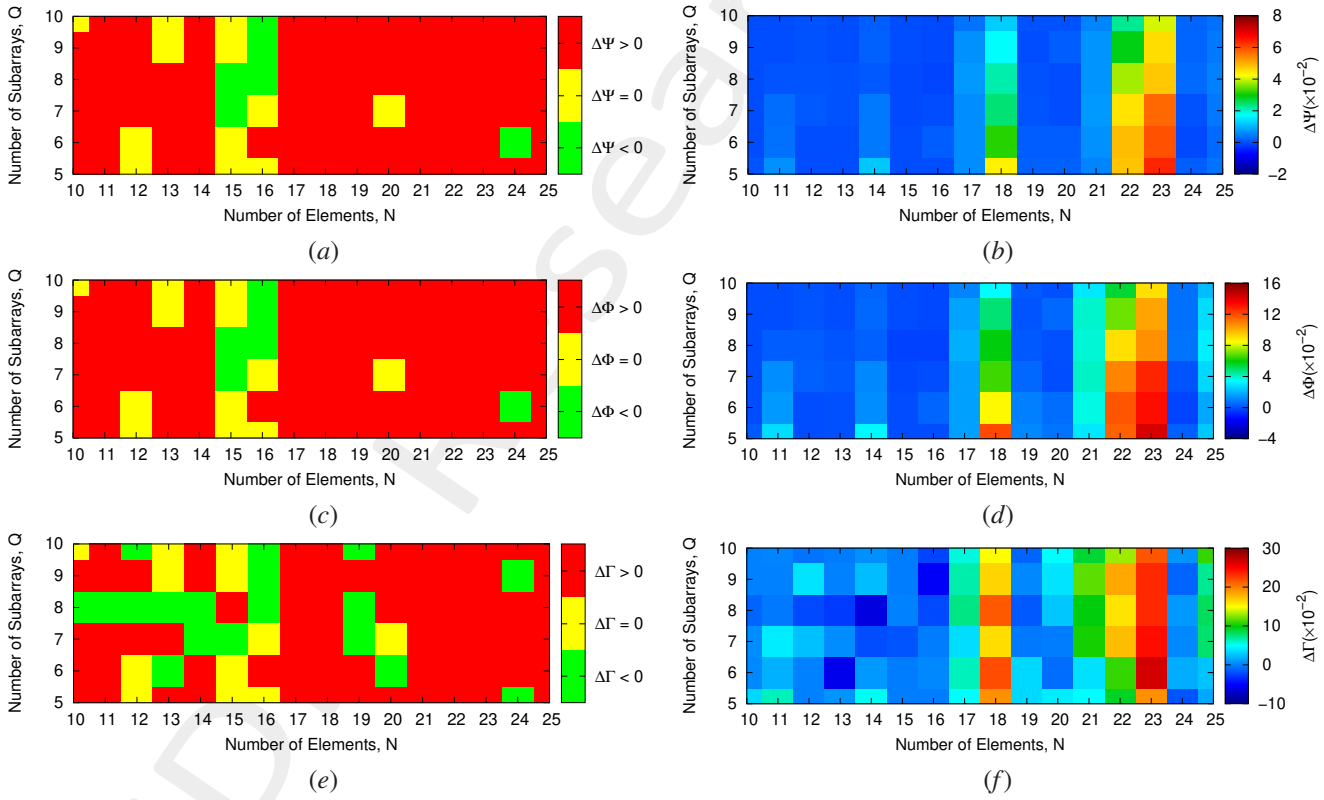

Figure 15: *Flat Top Pattern, Performance Evaluation:* (*a*)(*c*) 3 colors map and (*b*)(*d*) quantitative color map for the (*a*)(*b*) difference excitation matching, (*c*)(*d*) difference field matching and (*e*)(*f*) difference power pattern matching.

Observation: To further investigate the the potentialities of the Hilbert curve sorting, the complex target excitations can be rotated of an aribitrary angle and sorted according to the Hilbert Curve. In this way an improvement of the performance can arise.

1.1.4 Steered Pencil Beam Pattern, $\mathbf{N} = [10,\ 25]$, $\mathbf{Q} = [5,\ 10]$, $\theta_0 = -10$[deg] - rotated excitations: $\alpha = 45$[deg]

Test Case Description

Antenna configuration

- isotropic elements
- number of elements: $N = [10,\ 25]$
- distance between elements along x axis: $d_x = \lambda/2$

Target excitations

- $w_n = \alpha_n e^{j\varphi_n}$ with $n = 1, .., N$
- main lobe steering: $\theta_0 = -10$ [deg]
- Taylor pattern
  - nominal sidelobe level: $SLL_{ref} = -30$ [dB]
  - polinomial order: $\bar{n} = 7$

Sub-array generation

- number of clusters: $Q = [5,\ 10]$
- excitation matching strategies:
  - K-Means
  - Hilbert + Exhaustive Search

Hilbert Curve Sorting + Exhaustive Search vs. K-means

The following comparative assessment has been performed, computing five different orders of Hilbert curve and choosing four different starting points as done for the previous cases.

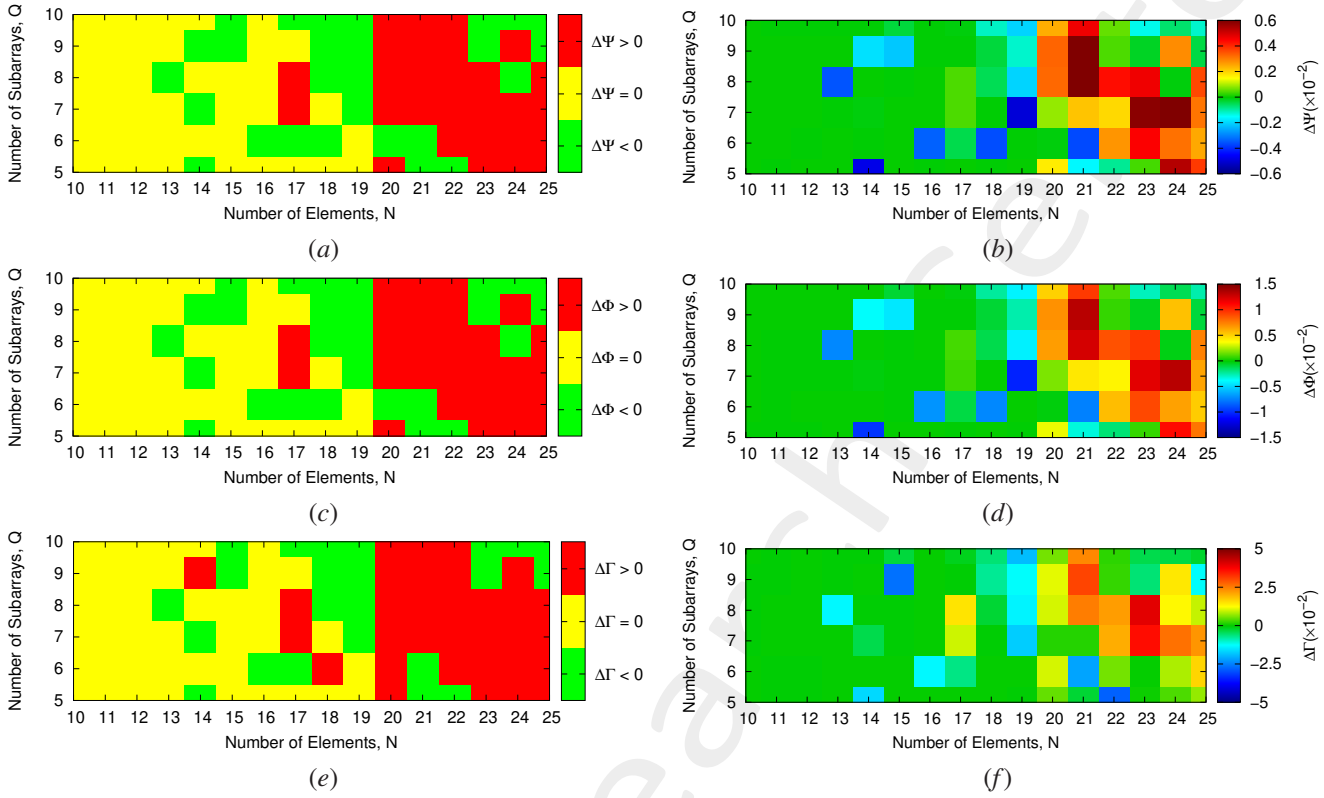

Figure 16: *Steered Pencil Beam Pattern, $\theta_0 = -10$ [deg] - rotated excitations 45[deg], Performance Evaluation:* $(a)(c)(e)$ 3 colors map and $(b)(d)(f)$ quantitative color map for the $(a)(b)$ difference excitation matching, $(c)(d)$ difference field matching and $(e)(f)$ difference power pattern matching.

N/Q	$N=10$	$N=11$	$N=12$	$N=13$	$N=14$	$N=15$	$N=16$	$N=17$
$Q=5$	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
$Q=6$	N/A	N/A	N/A	N/A	1	N/A	[1 : 5] 2	N/A
$Q=7$	N/A	N/A	N/A	N/A	[1 : 3]	N/A	N/A	N/A
$Q=8$	N/A	N/A	N/A	[1 : 5]	N/A	N/A	N/A	N/A
$Q=9$	N/A	N/A	N/A	N/A	[1 : 3] [1 : 2]	[1 : 5] [1 : 2]	N/A	N/A
$Q=10$	N/A	N/A	N/A	N/A	N/A	[1 : 5]	N/A	1

N/Q	$N=18$	$N=19$	$N=20$	$N=21$	$N=22$	$N=23$	$N=24$	$N=25$
$Q=5$	N/A	N/A	N/A	1	1	N/A	N/A	N/A
$Q=6$	1	N/A	1	[1 : 5] 1	N/A	N/A	N/A	N/A
$Q=7$	N/A	1	N/A	N/A	N/A	N/A	N/A	N/A
$Q=8$	[1 : 2]	[1 : 5]	N/A	N/A	N/A	N/A	[2 : 5]	N/A
$Q=9$	[1 : 2]	[1 : 5]	N/A	N/A	2	[1 : 5]	N/A	[1 : 5] 1
$Q=10$	[1 : 2]	[1 : 2] 1, [3 : 5]	N/A	N/A	2	[1 : 5]	[2 : 5] 1	[2 : 5]

Table VII: *Steered Pencil Beam Pattern, $\theta_0 = -10$ [deg] - rotated excitations 45[deg], Performance Evaluation:* Hilbert orders for which the proposed approach works better than the K-means algorithm in terms of excitations and field matching, in ***black*** the curve orders where the Hilbert Curve outoperforms the K-means and in ***green*** the cases where the best excitation matching is obtained.

1.1.5 Steered Pencil Beam Pattern, $\mathbf{N} = [10,\ 25]$, $\mathbf{Q} = [5,\ 10]$, $\theta_0 = -10$[deg] - rotated excitations: $\alpha = 135$[deg]

Test Case Description

Antenna configuration

- isotropic elements
- number of elements: $N = [10,\ 25]$
- distance between elements along x axis: $d_x = \lambda/2$

Target excitations

- $w_n = \alpha_n e^{j\varphi_n}$ with $n = 1, .., N$
- main lobe steering: $\theta_0 = -10$ [deg]
- Taylor pattern
  - nominal sidelobe level: $SLL_{ref} = -30$ [dB]
  - polinomial order: $\bar{n} = 7$

Sub-array generation

- number of clusters: $Q = [5,\ 10]$
- excitation matching strategies:
  - K-Means
  - Hilbert + Exhaustive Search

Hilbert Curve Sorting + Exhaustive Search vs. K-means

The following comparative assessment has been performed, computing five different orders of Hilbert curve and choosing four different starting points as done for the previous cases.

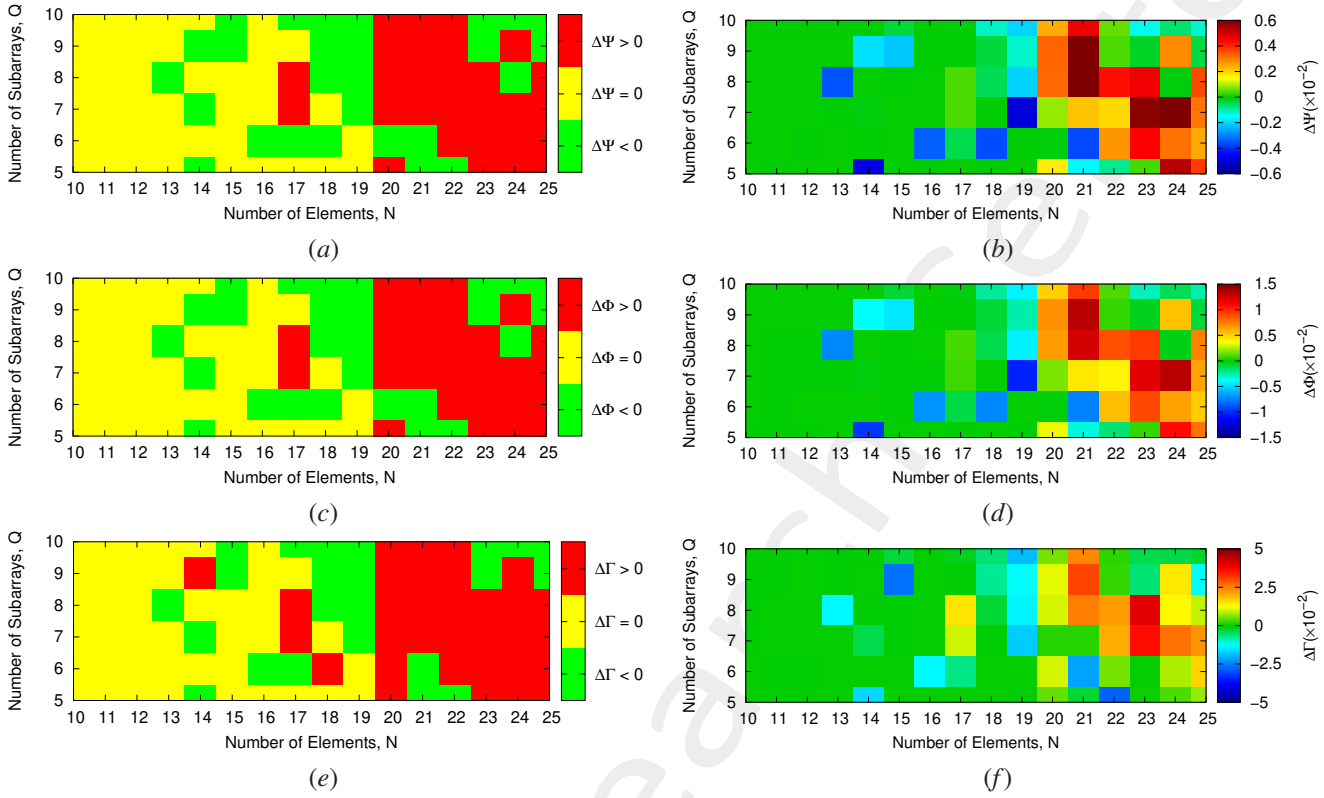

Figure 17: *Steered Pencil Beam Pattern, $\theta_0 = -10$ [deg] - rotated excitations 135[deg], Performance Evaluation:* (*a*)(*c*)(*e*) 3 colors map and (*b*)(*d*)(*f*) quantitative color map for the (*a*)(*b*) difference excitation matching, (*c*)(*d*) difference field matching and (*e*)(*f*) difference power pattern matching.

N/Q	$N=10$	$N=11$	$N=12$	$N=13$	$N=14$	$N=15$	$N=16$	$N=17$
$Q=5$	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
$Q=6$	N/A	N/A	N/A	N/A	1	N/A	[1 : 5] 2	N/A
$Q=7$	N/A	N/A	N/A	N/A	[1 : 3]	N/A	N/A	N/A
$Q=8$	N/A	N/A	N/A	[1 : 5]	N/A	N/A	N/A	N/A
$Q=9$	N/A	N/A	N/A	N/A	[1 : 3] [1 : 2]	[1 : 5] [1 : 2]	N/A	N/A
$Q=10$	N/A	N/A	N/A	N/A	N/A	[1 : 5]	N/A	1

N/Q	$N=18$	$N=19$	$N=20$	$N=21$	$N=22$	$N=23$	$N=24$	$N=25$
$Q=5$	N/A	N/A	N/A	1	1	N/A	N/A	N/A
$Q=6$	1	N/A	1	[1 : 5] 1	N/A	N/A	N/A	N/A
$Q=7$	N/A	1	N/A	N/A	N/A	N/A	N/A	N/A
$Q=8$	[1 : 2]	[1 : 5]	N/A	N/A	N/A	N/A	[2 : 5]	N/A
$Q=9$	[1 : 2]	[1 : 5]	N/A	N/A	2	[1 : 5]	N/A	[1 : 5] 1
$Q=10$	[1 : 2]	[1 : 2] 1, [3 : 5]	N/A	N/A	2	[1 : 5]	[2 : 5] 1	[2 : 5]

Table VIII: *Steered Pencil Beam Pattern, $\theta_0 = -10$ [deg] - rotated excitations 135[deg], Performance Evaluation:* Hilbert orders for which the proposed approach works better than the K-means algorithm in terms of excitations and field matching, in ***black*** the curve orders where the Hilbert Curve outoperforms the K-means and in ***green*** the cases where the best excitation matching is obtained.

Observation: From SubSubSec. 1.1.4 and SubSubSec. 1.1.5 it is immediately possible to understand that rotating the excitations with an angle α or with an angle $\alpha + 90$[deg] gives the same results. This can be easily explain taking into account how the analaysis is performed, i.e. the Hilbert curve is rotated of 0,90, 180 and 270[deg] to change the starting point. Therefore, different clustering solution can achieve only rotating the excitations with an angle $\alpha \in [0,\ 90]$ [deg].

In the following test cases a comparative assessment of the Hilbert Curve Sorting approach with respect to the K-means method. A same number of initializations has been performed. More in details, $R = 92$ different excitations rotations have been considered with a rotation angle $\alpha \in [0,\ 360]$[deg] for the Hilbert Curve Sorting and $R = 92$ different random seeds initialization have been taken into account for the K-means method. In this way the comparison is "fair".

Fig. 18 illustrates how the rotation of the target excitations can have a role in the improvement of the Hilbert Sorting approach. More in details the calculated sorted list changes choosing different rotation angles.

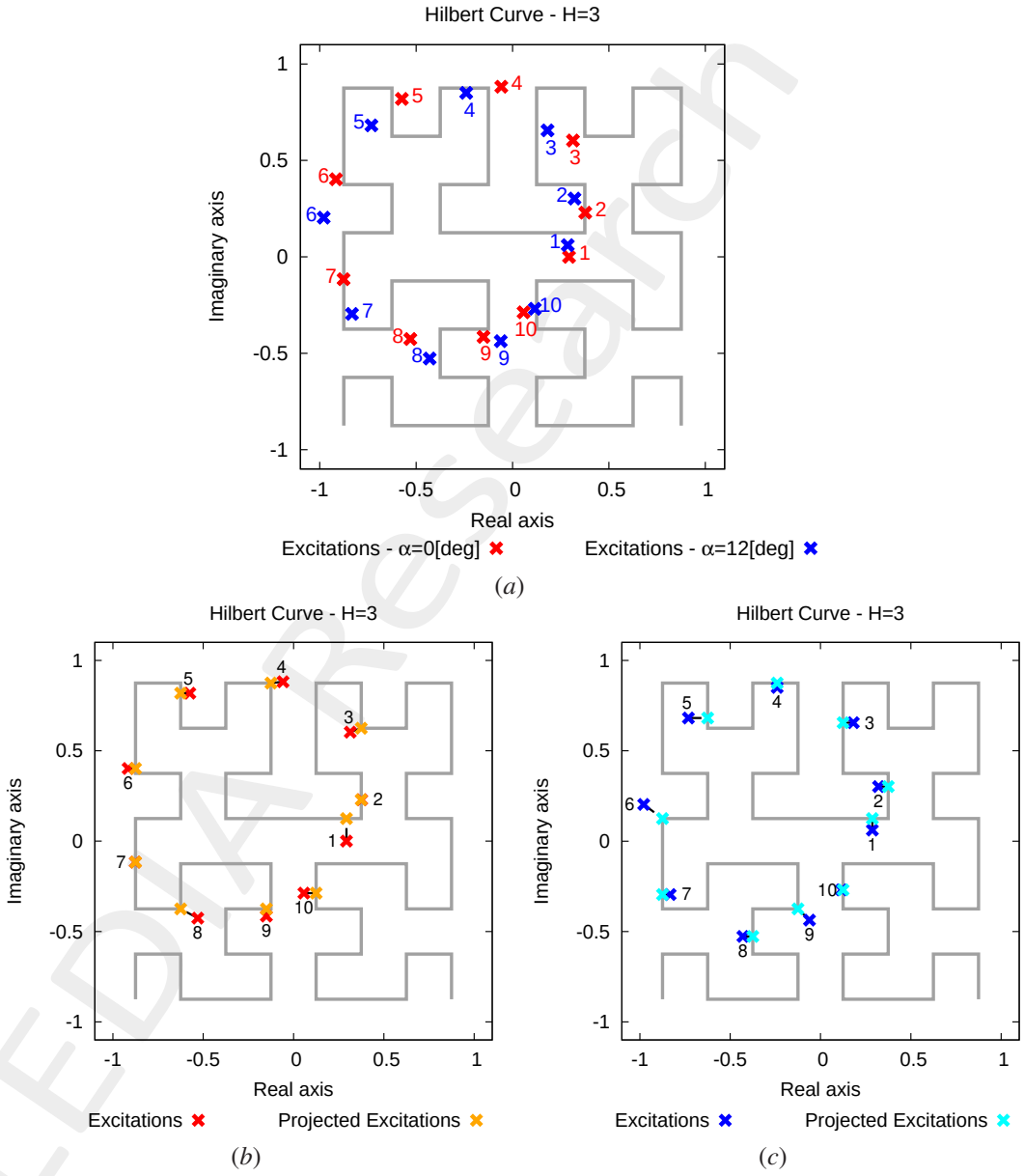

Figure 18: *Steered Pencil Beam Pattern, $\theta_0 = -10$ [deg]:* (a) target excitations (red crosses) and rotated excitations with an angle $\alpha = 12$[deg], (b) projection of the 2D complex target excitations and (c) projection of the 2D complex rotated target excitations along the Hilbert Curve of order $H = 3$, with a bottom starting point.

Rotation angle, α[deg]	*Sorted Indexes*									
$\alpha = 0$[deg]	9	8	7	6	5	4	1	2	3	10
$\alpha = 12$[deg]	8	9	7	6	5	4	1	2	3	10

Table IX: *Steered Pencil Beam Pattern, $\theta_0 = -10$ [deg]:* list of sorted excitations changing the rotation angle.

1.1.6 Steered Pencil Beam Pattern, $\mathbf{N = [10,\ 25]}$, $\mathbf{Q = [5,\ 10]}$, $\boldsymbol{\theta_0 = -10}$[deg] - rotated excitations $\boldsymbol{\alpha \in [0,\ 360]}$ [deg], $\boldsymbol{\Delta\alpha = 4}$[deg]

Test Case Description

Antenna configuration

- isotropic elements
- number of elements: $N = [10,\ 25]$
- distance between elements along x axis: $d_x = \lambda/2$

Target excitations

- $w_n = \alpha_n e^{j\varphi_n}$ with $n = 1, .., N$
- main lobe steering: $\theta_0 = -10$ [deg]
- Taylor pattern
  - nominal sidelobe level: $SLL_{ref} = -30$ [dB]
  - polinomial order: $\bar{n} = 7$

Sub-array generation

- number of clusters: $Q = [5,\ 10]$
- number realization: $R = 92$
- excitation matching strategies:
  - K-Means
    - initialization: $R = 92$ random seeds
  - Hilbert + Exhaustive Search:
    - initialization: $R = 92$ excitations rotations, $\alpha = [0,\ 360]$[deg]
    - rotation step: $\Delta\alpha = 4$[deg]

Hilbert Curve Sorting + Exhaustive Search vs. K-means

The following comparative assessment has been performed, computing five different orders of Hilbert curve.

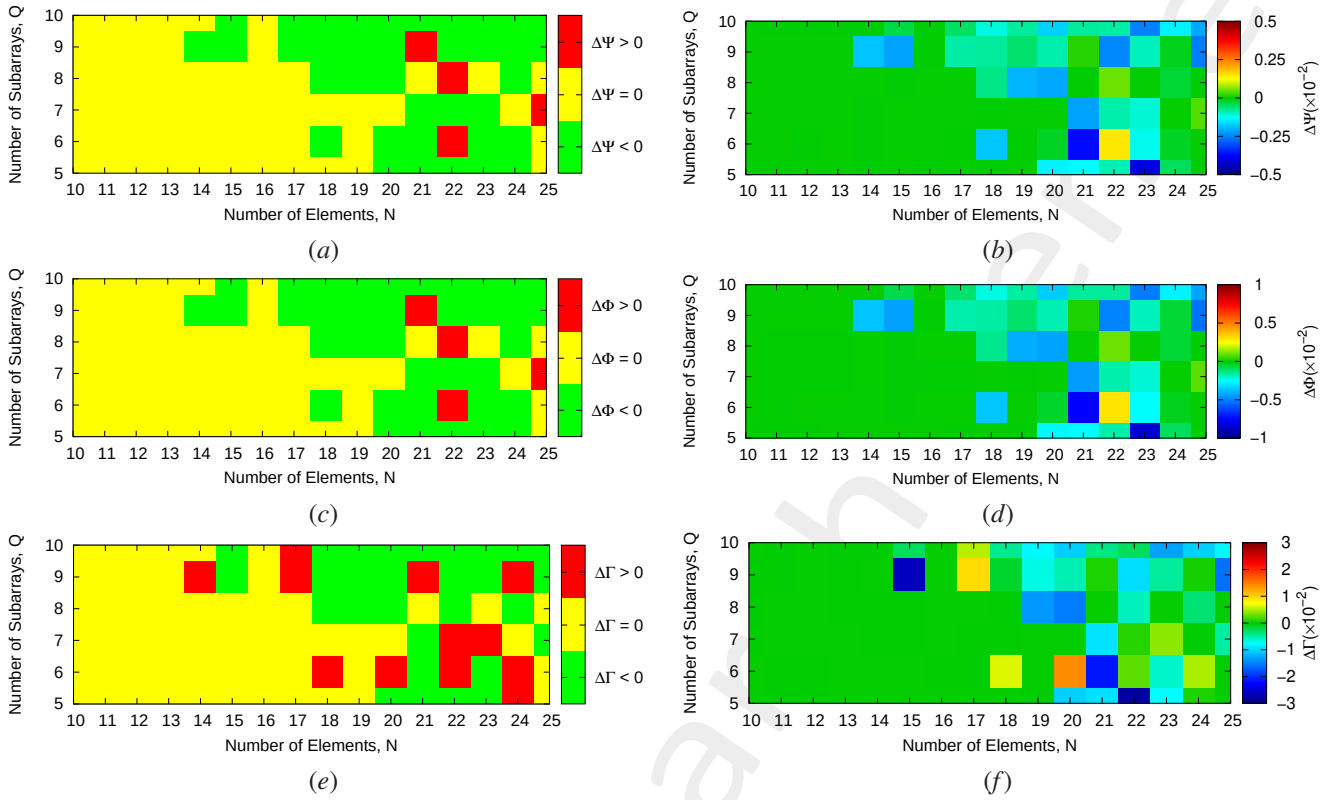

Figure 19: *Steered Pencil Beam Pattern,* $\theta_0 = -10$ *[deg] - rotated excitations* $[0,\ 360]$*[deg], Performance Evaluation:* $(a)(c)(e)$ 3 colors map and $(b)(d)(f)$ quantitative color map for the $(a)(b)$ difference excitation matching, $(c)(d)$ difference field matching and $(e)(f)$ difference power pattern matching.

In the following a comparative assessment of the performance in terms of power pattern and subarray clusters. More in details, three configurations have been considered. For case $N = 20$ and $Q = 6$ a slighty improvement has been reached by the Hilbert Sorting, while for case $N = 21$ and $Q = 6$ the greatest improvement has been obtained compared to the K-means technique. Instead, the third case ($N = 22$ and $Q = 6$) reports a configuration, where the K-means approach has outperformed the Hilbert Sorting technique.

Number elements $N = 20$**, Number Clusters** $Q = 6$

- difference excitation matching: $\Delta\Psi = -1.95 \times 10^{-4}$
- difference field matching: $\Delta\Phi = -4.04 \times 10^{-4}$
- difference power pattern matching: $\Delta\Gamma = 1.42 \times 10^{-2}$

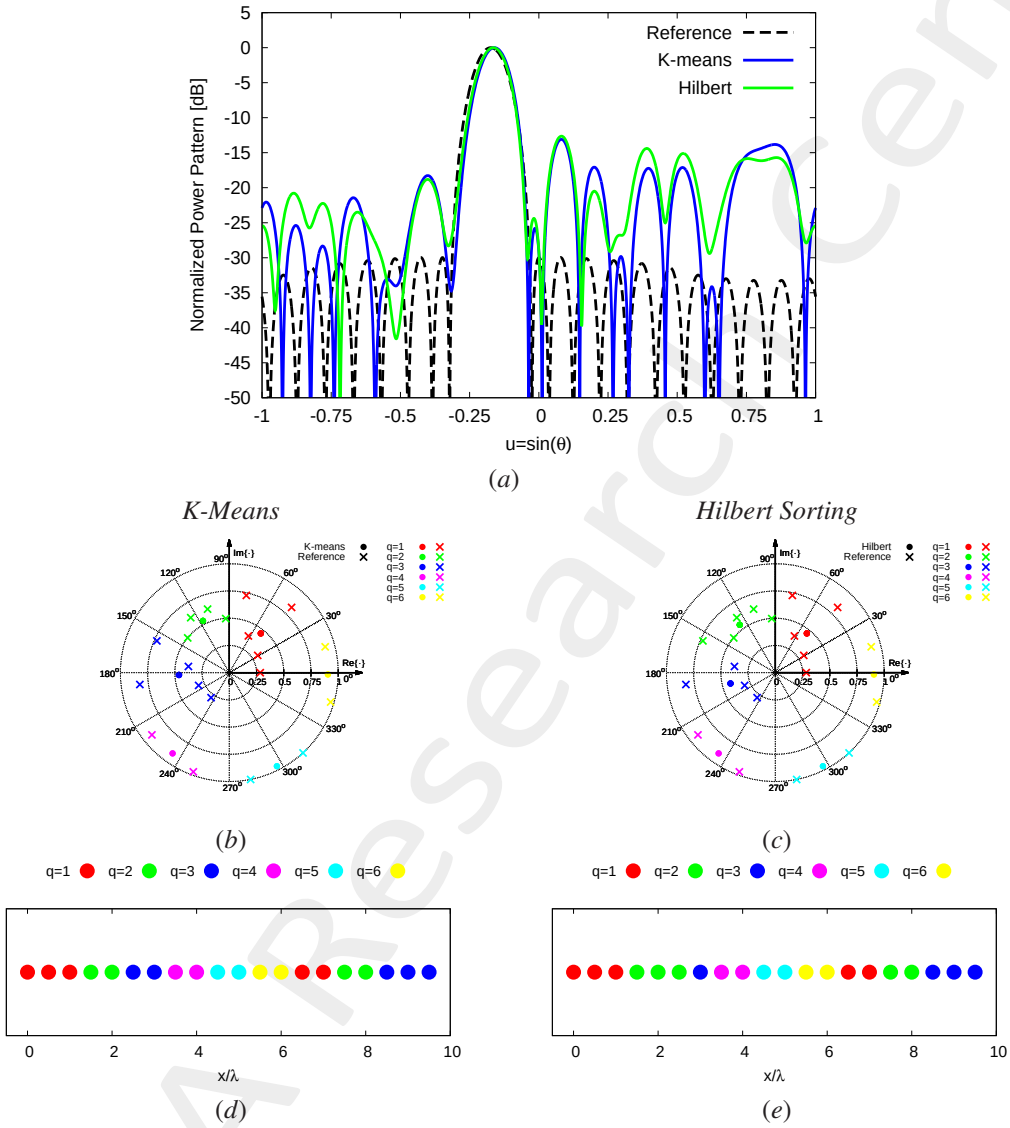

Figure 20: *Steered Pencil Beam* ($N = 20$, $Q = 6$, $\theta_0 = -10$ [deg]): plot of (a) the power pattern of the clusterized solutions computed with the K-Means (blue line) and the Hilbert Sorting (green line) together with the reference ones (dashed black line), (b)(c) representation of the reference and the sub-array excitations in the complex plane, and (d)(e) layout of the clustered array sythetized with (b)(d) the K-means and (c)(e) the Hilbert Sorting.

Method	Ψ	Φ	Γ	SLL [dB]	D [dBi]	$HPBW$ [u]
Hilbert Sorting	7.19×10^{-2}	1.459×10^{-1}	3.84×10^{-1}	-12.64	12.05	1.03×10^{-1}
K-Means	7.21×10^{-2}	1.463×10^{-1}	3.70×10^{-1}	-13.10	12.15	1.02×10^{-1}

Table X: *Steered Pencil Beam* ($N = 20$, $Q = 6$, $\theta_0 = -10$ [deg]): values of excitation matching index, Ψ, the field matching index, Φ, the power pattern matching index, Γ, the SLL, the directivity D and the half power beamwidth $HPBW$ obtained with Hilbert Sorting + ES and with the K-means.

Number elements $N = 21$, Number Clusters $Q = 6$

- difference excitation matching: $\Delta\Psi = -3.64 \times 10^{-3}$
- difference field matching: $\Delta\Phi = -7.50 \times 10^{-3}$
- difference power pattern matching: $\Delta\Gamma = -2.12 \times 10^{-2}$

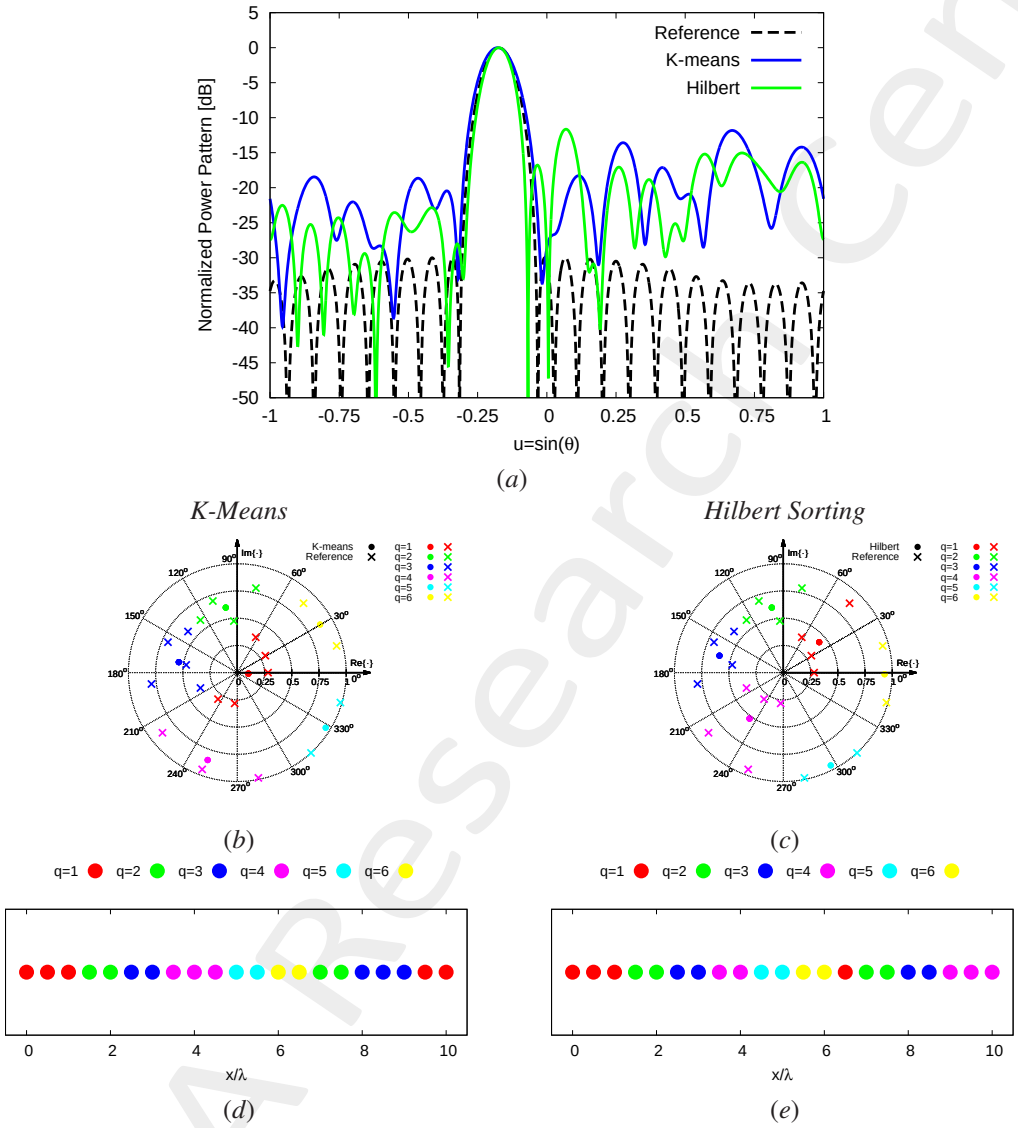

Figure 21: *Steered Pencil Beam* ($N = 21$, $Q = 6$, $\theta_0 = -10$ [deg]): plot of (a) the power pattern of the clusterized solutions computed with the K-Means (blue line) and the Hilbert Sorting (green line) together with the reference ones (dashed black line), (b)(c) representation of the reference and the sub-array excitations in the complex plane, and (d)(e) layout of the clustered array sythetized with (b)(d) the K-means and (c)(e) the Hilbert Sorting.

Method	Ψ	Φ	Γ	SLL [dB]	D [dBi]	$HPBW$ [u]
Hilbert Sorting	7.85×10^{-2}	1.607×10^{-1}	4.11×10^{-1}	-11.65	12.42	9.37×10^{-2}
K-Means	8.21×10^{-2}	1.682×10^{-1}	4.32×10^{-1}	-11.82	11.40	1.18×10^{-1}

Table XI: *Steered Pencil Beam* ($N = 21$, $Q = 6$, $\theta_0 = -10$ [deg]): values of excitation matching index, Ψ, the field matching index, Φ, the power pattern matching index, Γ, the SLL, the directivity D and the half power beamwidth $HPBW$ obtained with Hilbert Sorting + ES and with the K-means.

Number elements $N = 22$, Number Clusters $Q = 6$

- difference excitation matching: $\Delta\Psi = 1.45 \times 10^{-3}$
- difference field matching: $\Delta\Phi = 3.00 \times 10^{-3}$
- difference power pattern matching: $\Delta\Gamma = 2.80 \times 10^{-3}$

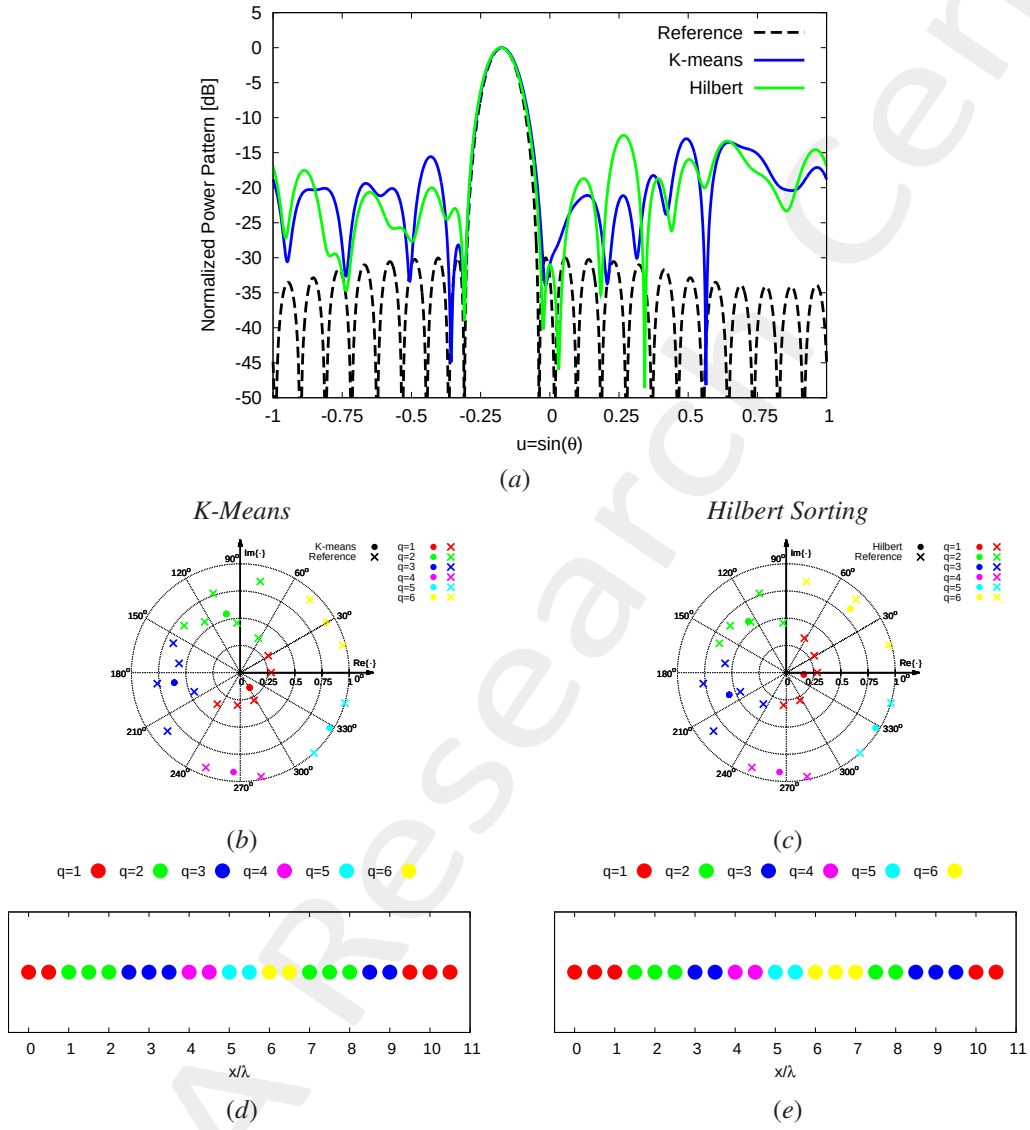

Figure 22: *Steered Pencil Beam* ($N = 22$, $Q = 6$, $\theta_0 = -10$ [deg]): plot of (a) the power pattern of the clusterized solutions computed with the K-Means (blue line) and the Hilbert Sorting (green line) together with the reference ones (dashed black line), (b)(c) representation of the reference and the sub-array excitations in the complex plane, and (d)(e) layout of the clustered array sythetized with (b)(d) the K-means and (c)(e) the Hilbert Sorting.

Method	Ψ	Φ	Γ	SLL [dB]	D [dBi]	$HPBW$ [u]
Hilbert Sorting	8.19×10^{-2}	1.667×10^{-1}	4.29×10^{-1}	-12.52	11.76	1.08×10^{-1}
K-Means	8.05×10^{-2}	1.637×10^{-1}	4.26×10^{-1}	-13.01	11.64	1.09×10^{-1}

Table XII: *Steered Pencil Beam* ($N = 22$, $Q = 6$, $\theta_0 = -10$ [deg]): values of excitation matching index, Ψ, the field matching index, Φ, the power pattern matching index, Γ, the SLL, the directivity D and the half power beamwidth $HPBW$ obtained with Hilbert Sorting + ES and with the K-means.

1.1.7 Steered Pencil Beam Pattern, $\mathbf{N = [10,\ 25]}$, $\mathbf{Q = [5,\ 10]}$, $\boldsymbol{\theta_0 = 10}$[deg] - rotated excitations $\boldsymbol{\alpha \in [0,\ 360]}$ [deg], $\boldsymbol{\Delta\alpha = 4}$[deg]

Test Case Description

Antenna configuration

- isotropic elements
- number of elements: $N = [10,\ 25]$
- distance between elements along x axis: $d_x = \lambda/2$

Target excitations

- $w_n = \alpha_n e^{j\varphi_n}$ with $n = 1, .., N$
- main lobe steering: $\theta_0 = 10$ [deg]
- Taylor pattern
  - nominal sidelobe level: $SLL_{ref} = -30$ [dB]
  - polinomial order: $\bar{n} = 7$

Sub-array generation

- number of clusters: $Q = [5,\ 10]$
- number realization: $R = 92$
- excitation matching strategies:
  - K-Means
    - initialization: $R = 92$ random seeds
  - Hilbert + Exhaustive Search:
    - initialization: $R = 92$ excitations rotations, $\alpha = [0,\ 360]$[deg]
    - rotation step: $\Delta\alpha = 4$[deg]

Hilbert Curve Sorting + Exhaustive Search vs. K-means

The following comparative assessment has been performed, computing five different orders of Hilbert curve.

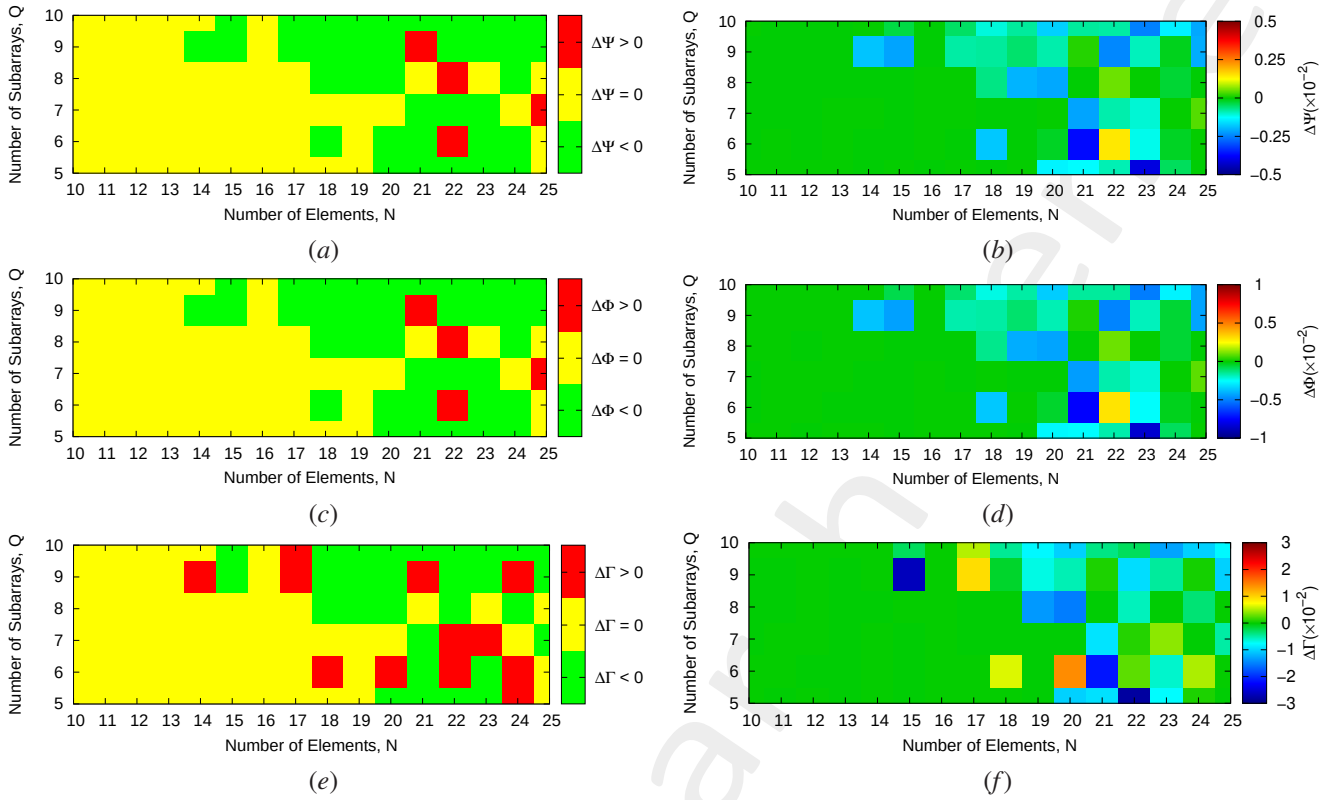

Figure 23: *Steered Pencil Beam Pattern,* $\theta_0 = 10$ *[deg] - rotated excitations* $[0, 360]$*[deg], Performance Evaluation:* $(a)(c)(e)$ 3 colors map and $(b)(d)(f)$ quantitative color map for the $(a)(b)$ difference excitation matching, $(c)(d)$ difference field matching and $(e)(f)$ difference power pattern matching.

In the following a comparative assessment of the performance in terms of power pattern and subarray clusters for some case of interest.

Number elements $N = 20$**, Number Clusters** $Q = 6$

- difference excitation matching: $\Delta\Psi = -1.95 \times 10^{-4}$
- difference field matching: $\Delta\Phi = -4.04 \times 10^{-4}$
- difference power pattern matching: $\Delta\Gamma = 1.42 \times 10^{-2}$

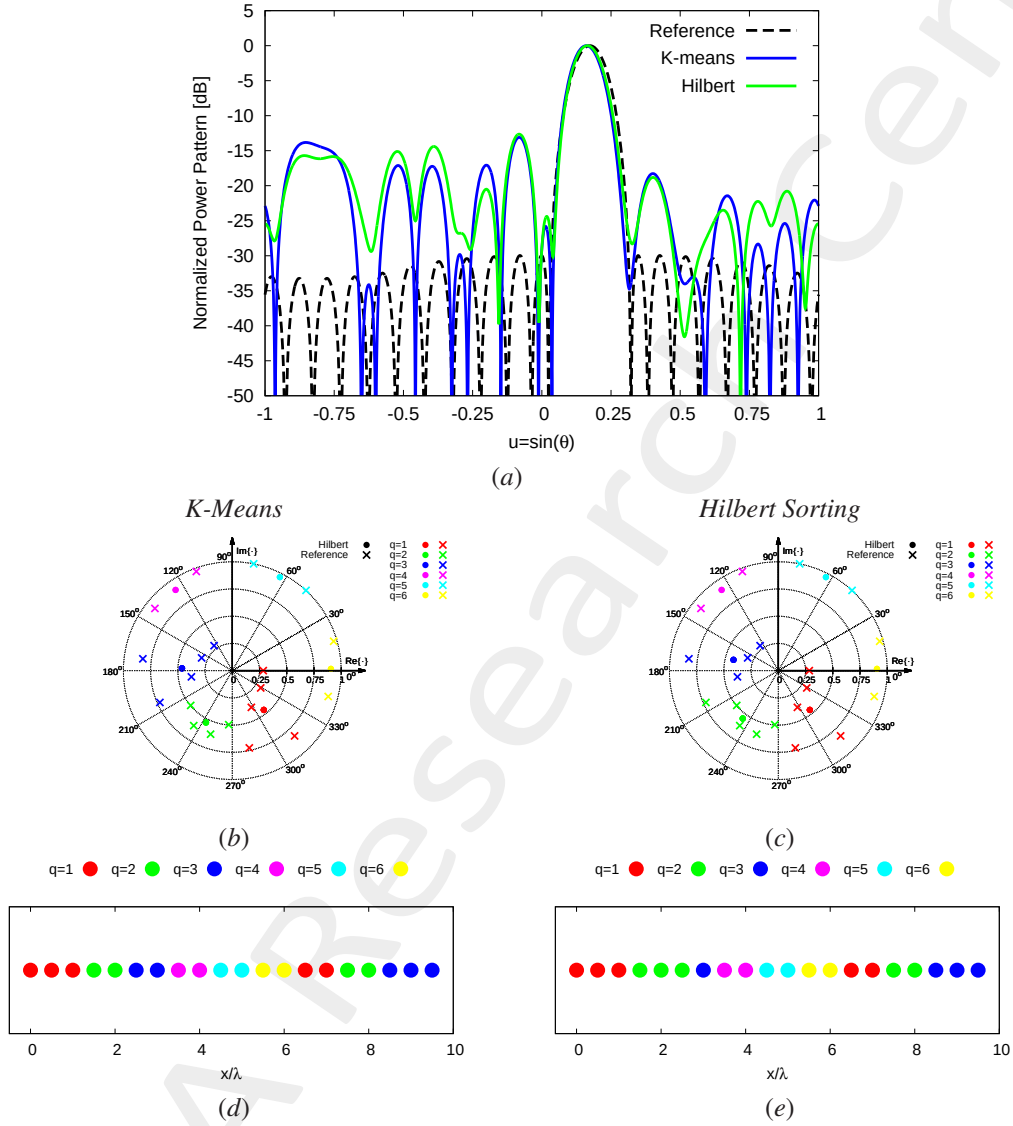

Figure 24: *Steered Pencil Beam* ($N = 20$, $Q = 6$, $\theta_0 = 10$ [deg]): plot of (a) the power pattern of the clusterized solutions computed with the K-Means (blue line) and the Hilbert Sorting (green line) together with the reference ones (dashed black line), (b)(c) representation of the reference and the sub-array excitations in the complex plane, and (d)(e) layout of the clustered array sythetized with (b)(d) the K-means and (c)(e) the Hilbert Sorting.

Method	Ψ	Φ	Γ	SLL [dB]	D [dBi]	$HPBW$ [u]
Hilbert Sorting	-7.19×10^{-2}	1.45×10^{-1}	3.84×10^{-1}	-12.64	12.05	1.03×10^{-1}
K-Means	7.21×10^{-2}	1.46×10^{-1}	3.70×10^{-1}	-13.10	12.15	1.02×10^{-1}

Table XIII: *Steered Pencil Beam* ($N = 20$, $Q = 6$, $\theta_0 = 10$ [deg]): values of excitation matching index, Ψ, the field matching index, Φ, the power pattern matching index, Γ, the SLL, the directivity D and the half power beamwidth $HPBW$ obtained with Hilbert Sorting + ES and with the K-means.

Number elements $N = 21$, Number Clusters $Q = 6$

- difference excitation matching: $\Delta\Psi = -3.64 \times 10^{-3}$
- difference field matching: $\Delta\Phi = -7.50 \times 10^{-3}$
- difference power pattern matching: $\Delta\Gamma = -2.12 \times 10^{-2}$

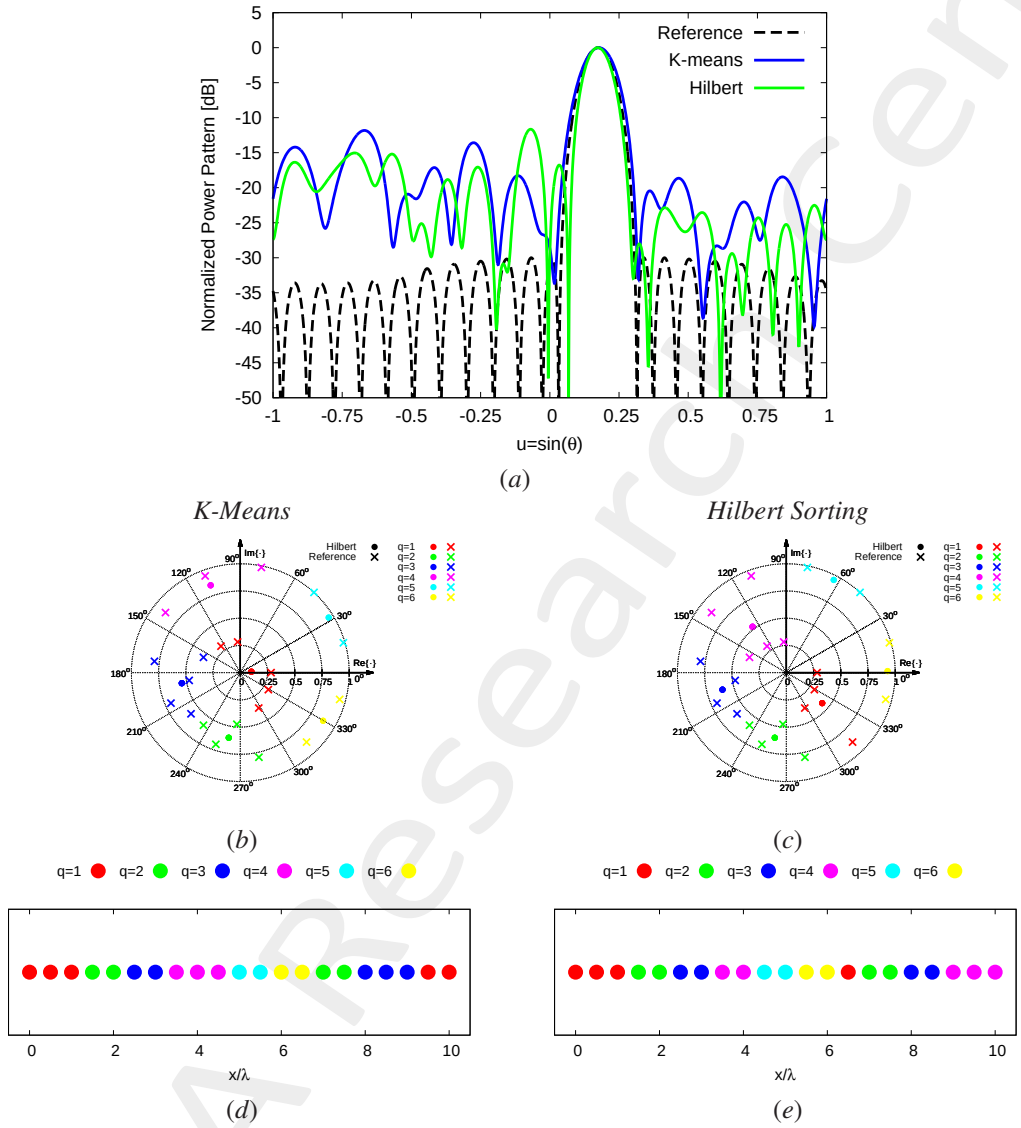

Figure 25: *Steered Pencil Beam* ($N = 21$, $Q = 6$, $\theta_0 = 10$ [deg]): plot of (a) the power pattern of the clusterized solutions computed with the K-Means (blue line) and the Hilbert Sorting (green line) together with the reference ones (dashed black line), (b)(c) representation of the reference and the sub-array excitations in the complex plane, and (d)(e) layout of the clustered array sythetized with (b)(d) the K-means and (c)(e) the Hilbert Sorting.

Method	Ψ	Φ	Γ	SLL [dB]	D [dBi]	$HPBW$ [u]
Hilbert Sorting	7.85×10^{-2}	1.61×10^{-1}	4.11×10^{-1}	-11.67	12.42	9.37×10^{-2}
K-Means	8.21×10^{-2}	1.68×10^{-1}	4.32×10^{-1}	-11.82	11.40	1.18×10^{-1}

Table XIV: *Steered Pencil Beam* ($N = 21$, $Q = 6$, $\theta_0 = 10$ [deg]): values of excitation matching index, Ψ, the field matching index, Φ, the power pattern matching index, Γ, the SLL, the directivity D and the half power beamwidth $HPBW$ obtained with Hilbert Sorting + ES and with the K-means.

Number elements $N = 22$**, Number Clusters** $Q = 6$

- difference excitation matching: $\Delta\Psi = 1.45 \times 10^{-3}$
- difference field matching: $\Delta\Phi = 3.00 \times 10^{-3}$
- difference power pattern matching: $\Delta\Gamma = 2.80 \times 10^{-3}$

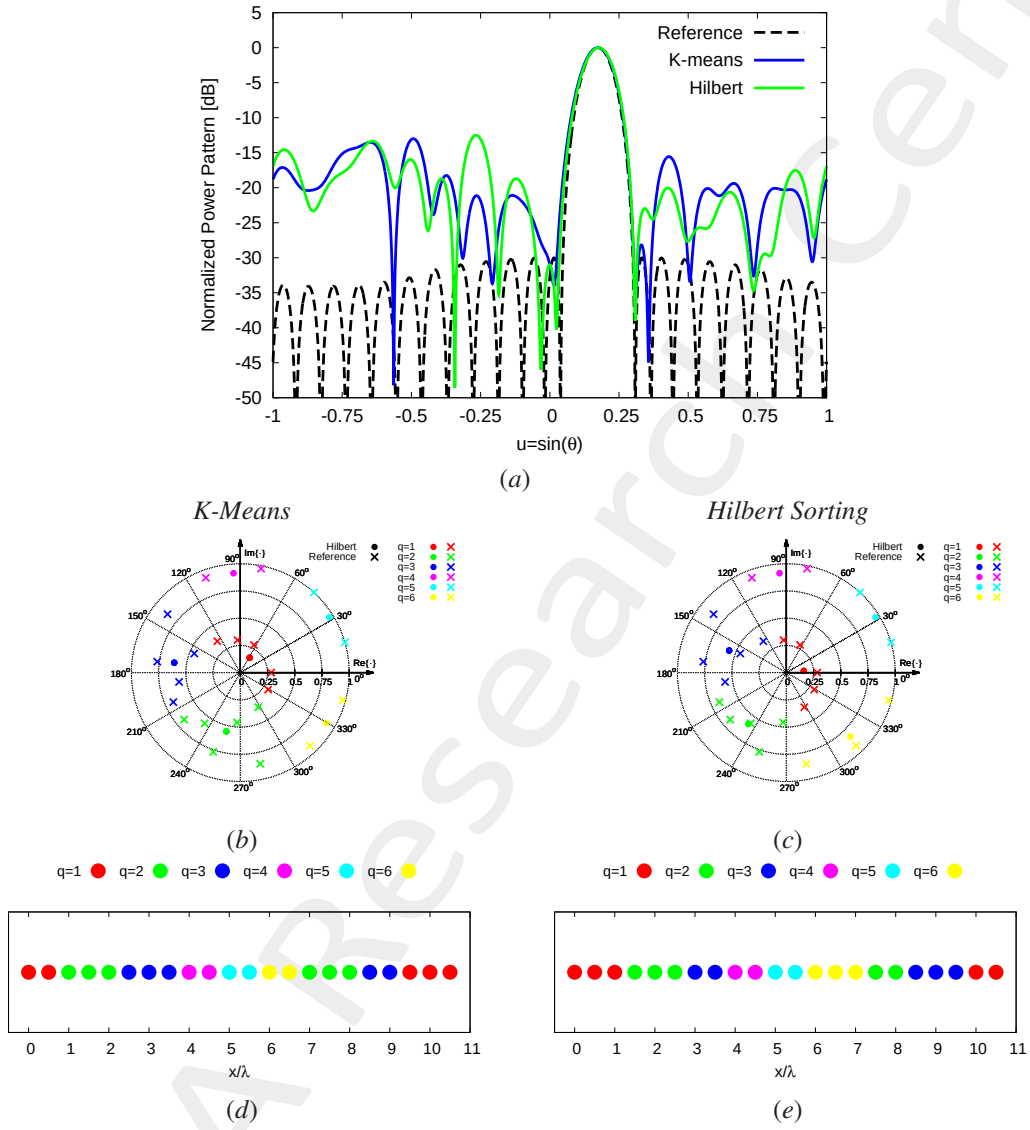

Figure 26: *Steered Pencil Beam* ($N = 22$, $Q = 6$, $\theta_0 = 10$ [deg]): plot of (a) the power pattern of the clusterized solutions computed with the K-Means (blue line) and the Hilbert Sorting (green line) together with the reference ones (dashed black line), (b)(c) representation of the reference and the sub-array excitations in the complex plane, and (d)(e) layout of the clustered array sythetized with (b)(d) the K-means and (c)(e) the Hilbert Sorting.

Method	Ψ	Φ	Γ	SLL [dB]	D [dBi]	$HPBW$ [u]
Hilbert Sorting	8.19×10^{-2}	1.67×10^{-1}	4.29×10^{-1}	-12.52	11.76	1.08×10^{-1}
K-Means	8.05×10^{-2}	1.64×10^{-1}	4.26×10^{-1}	-13.01	11.75	1.09×10^{-1}

Table XV: *Steered Pencil Beam* ($N = 22$, $Q = 6$, $\theta_0 = 10$ [deg]): values of excitation matching index, Ψ, the field matching index, Φ, the power pattern matching index, Γ, the SLL, the directivity D and the half power beamwidth $HPBW$ obtained with Hilbert Sorting + ES and with the K-means.

1.1.8 Steered Pencil Beam Pattern, $\mathbf{N = [10,\ 25]}$, $\mathbf{Q = [5,\ 10]}$, $\boldsymbol{\theta_0 = 30}$[deg] - rotated excitations $\boldsymbol{\alpha \in [0,\ 360]}$ [deg], $\boldsymbol{\Delta\alpha = 4}$[deg]

Test Case Description

Antenna configuration

- isotropic elements
- number of elements: $N = [10,\ 25]$
- distance between elements along x axis: $d_x = \lambda/2$

Target excitations

- $w_n = \alpha_n e^{j\varphi_n}$ with $n = 1, .., N$
- main lobe steering: $\theta_0 = 30$ [deg]
- Taylor pattern
  - nominal sidelobe level: $SLL_{ref} = -30$ [dB]
  - polinomial order: $\bar{n} = 7$

Sub-array generation

- number of clusters: $Q = [5,\ 10]$
- number realization: $R = 92$
- excitation matching strategies:
  - K-Means
    - initialization: $R = 92$ random seeds
  - Hilbert + Exhaustive Search:
    - initialization: $R = 92$ excitations rotations, $\alpha = [0,\ 360]$[deg]
    - rotation step: $\Delta\alpha = 4$[deg]

Hilbert Curve Sorting + Exhaustive Search vs. K-means

The following comparative assessment has been performed, computing five different orders of Hilbert curve.

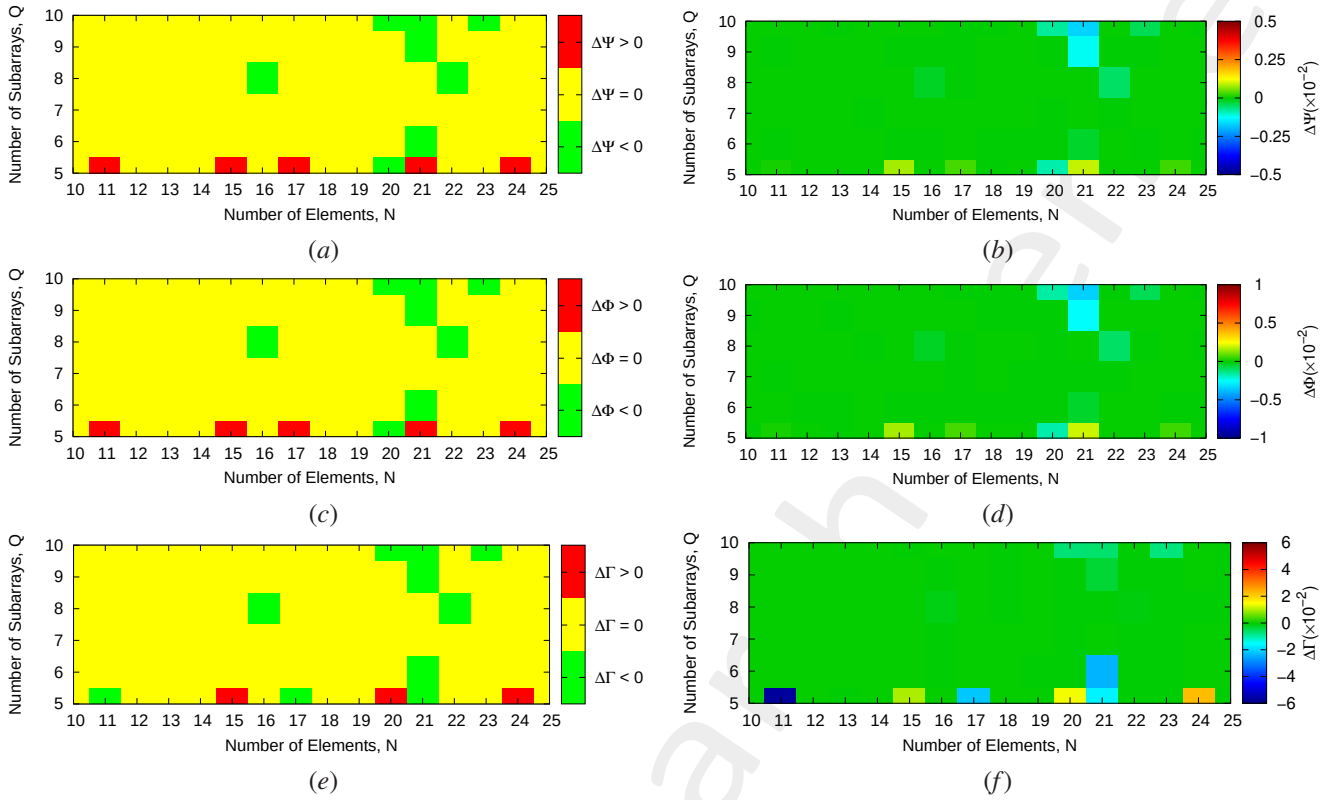

Figure 27: *Steered Pencil Beam Pattern,* $\theta_0 = 30$ *[deg] - rotated excitations* $[0, 360]$*[deg], Performance Evaluation:* (*a*)(*c*)(*e*) 3 colors map and (*b*)(*d*)(*f*) quantitative color map for the (*a*)(*b*) difference excitation matching, (*c*)(*d*) difference field matching and (*e*)(*f*) difference power pattern matching.

In the following a comparative assessment of the performance in terms of power pattern and subarray clusters. More in details, three configurations have been considered. For case $N = 20$ and $Q = 5$ an improvement has been obtained compared to the K-means technique, while for case $N = 21$ and $Q = 5$ a worse excitation matching has been reached by the Hilbert Sorting. Instead, the third case ($N = 22$ and $Q = 5$) reports a configuration, where the two clustering approaches reach the same results in terms of excitation and field matching.

Number elements $N = 20$, Number Clusters $Q = 5$

- difference excitation matching: $\Delta\Psi = -8.28 \times 10^{-4}$
- difference field matching: $\Delta\Phi = -1.67 \times 10^{-3}$
- difference power pattern matching: $\Delta\Gamma = 1.41 \times 10^{-2}$

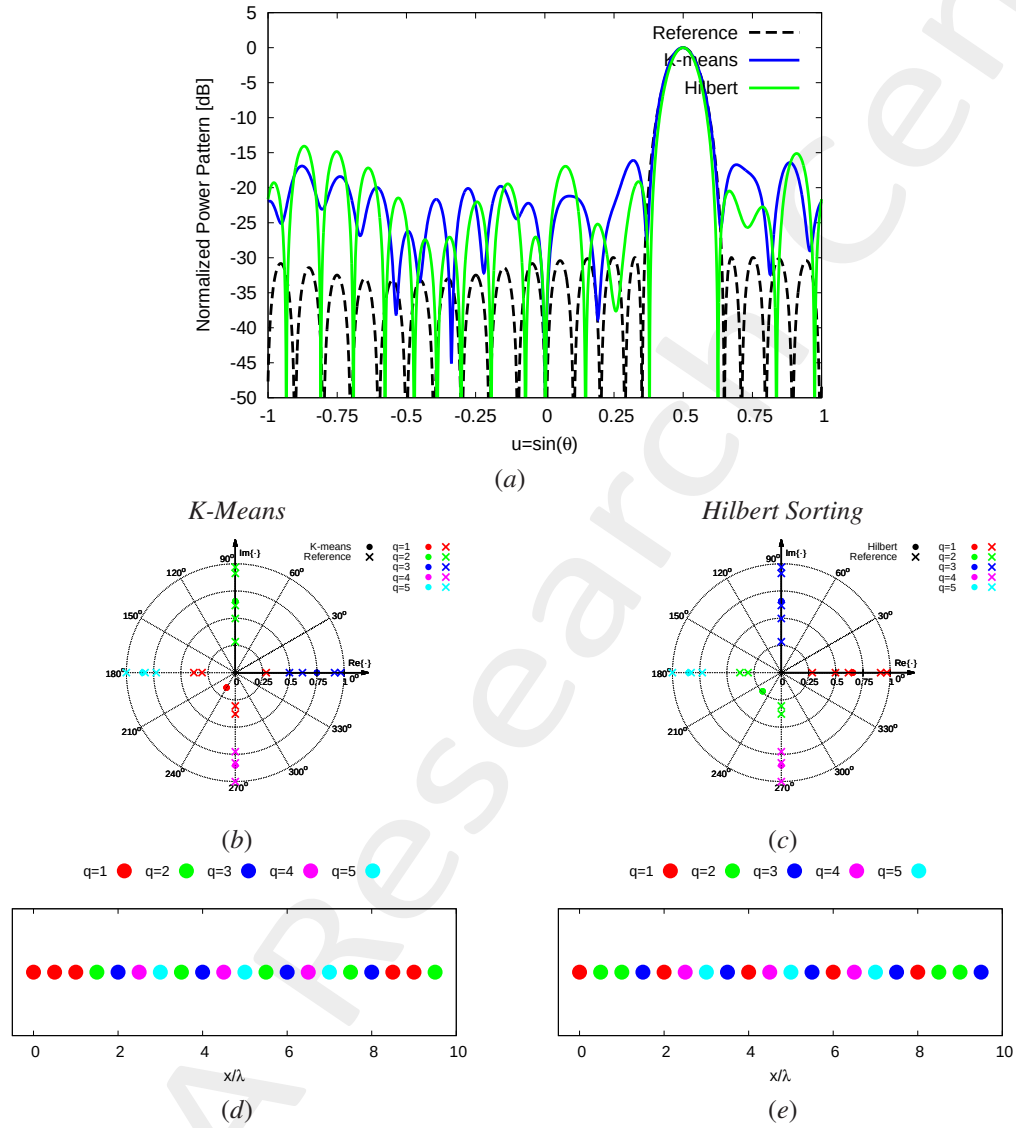

Figure 28: *Steered Pencil Beam* ($N = 20$, $Q = 5$, $\theta_0 = 30$ [deg]): plot of (a) the power pattern of the clusterized solutions computed with the K-Means (blue line) and the Hilbert Sorting (green line) together with the reference ones (dashed black line), (b)(c) representation of the reference and the sub-array excitations in the complex plane, and (d)(e) layout of the clustered array sythetized with (b)(d) the K-means and (c)(e) the Hilbert Sorting.

Method	Ψ	Φ	Γ	SLL [dB]	D [dBi]	$HPBW$ [u]
Hilbert Sorting	4.87×10^{-2}	9.884×10^{-2}	2.85×10^{-1}	14.06	12.35	1.01×10^{-1}
K-Means	4.95×10^{-2}	1.005×10^{-1}	2.71×10^{-1}	16.10	11.91	1.12×10^{-1}

Table XVI: *Steered Pencil Beam* ($N = 20$, $Q = 5$, $\theta_0 = 30$ [deg]): values of excitation matching index, Ψ, the field matching index, Φ, the power pattern matching index, Γ, the SLL, the directivity D and the half power beamwidth $HPBW$ obtained with Hilbert Sorting + ES and with the K-means.

Number elements $N = 21$, Number Clusters $Q = 5$

- difference excitation matching: $\Delta\Psi = 9.71 \times 10^{-4}$
- difference field matching: $\Delta\Phi = 1.99 \times 10^{-3}$
- difference power pattern matching: $\Delta\Gamma = -1.60 \times 10^{-1}$

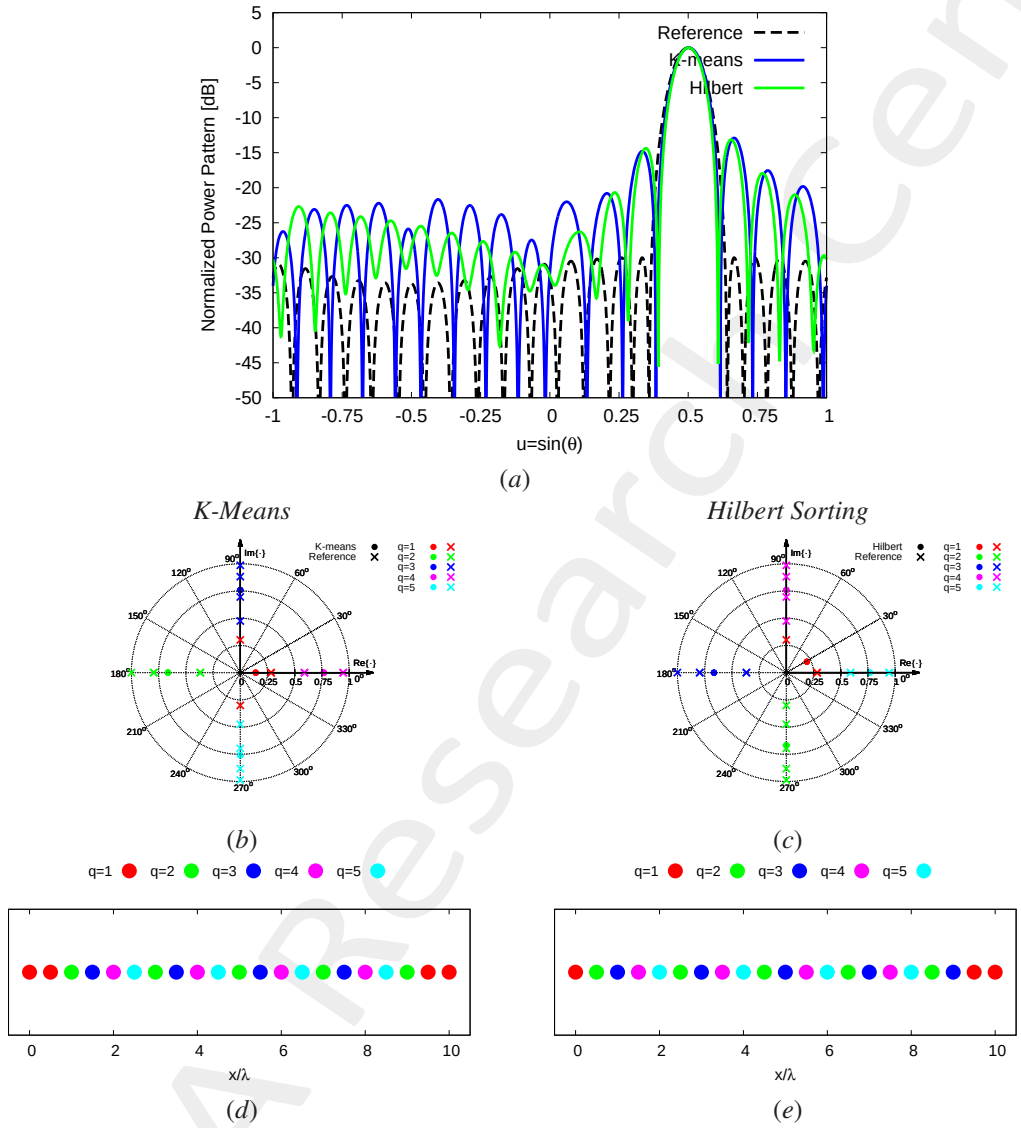

Figure 29: *Steered Pencil Beam* ($N = 21$, $Q = 5$, $\theta_0 = 30$ [deg]): plot of (a) the power pattern of the clusterized solutions computed with the K-Means (blue line) and the Hilbert Sorting (green line) together with the reference ones (dashed black line), (b)(c) representation of the reference and the sub-array excitations in the complex plane, and (d)(e) layout of the clustered array sythetized with (b)(d) the K-means and (c)(e) the Hilbert Sorting.

Method	Ψ	Φ	Γ	SLL [dB]	D [dBi]	$HPBW$ [u]
Hilbert Sorting	4.92×10^{-2}	1.008×10^{-2}	2.30×10^{-1}	-13.14	12.79	9.48×10^{-2}
K-Means	4.82×10^{-2}	9.881×10^{-2}	2.46×10^{-1}	-12.90	12.45	1.01×10^{-1}

Table XVII: *Steered Pencil Beam* ($N = 21$, $Q = 5$, $\theta_0 = 30$ [deg]): values of excitation matching index, Ψ, the field matching index, Φ, the power pattern matching index, Γ, the SLL, the directivity D and the half power beamwidth $HPBW$ obtained with Hilbert Sorting + ES and with the K-means.

Number elements $N = 22$, Number Clusters $Q = 5$

- difference excitation matching: $\Delta\Psi = 7.45 \times 10^{-9}$
- difference field matching: $\Delta\Phi = 5.22 \times 10^{-7}$
- difference power pattern matching: $\Delta\Gamma = 0.00$

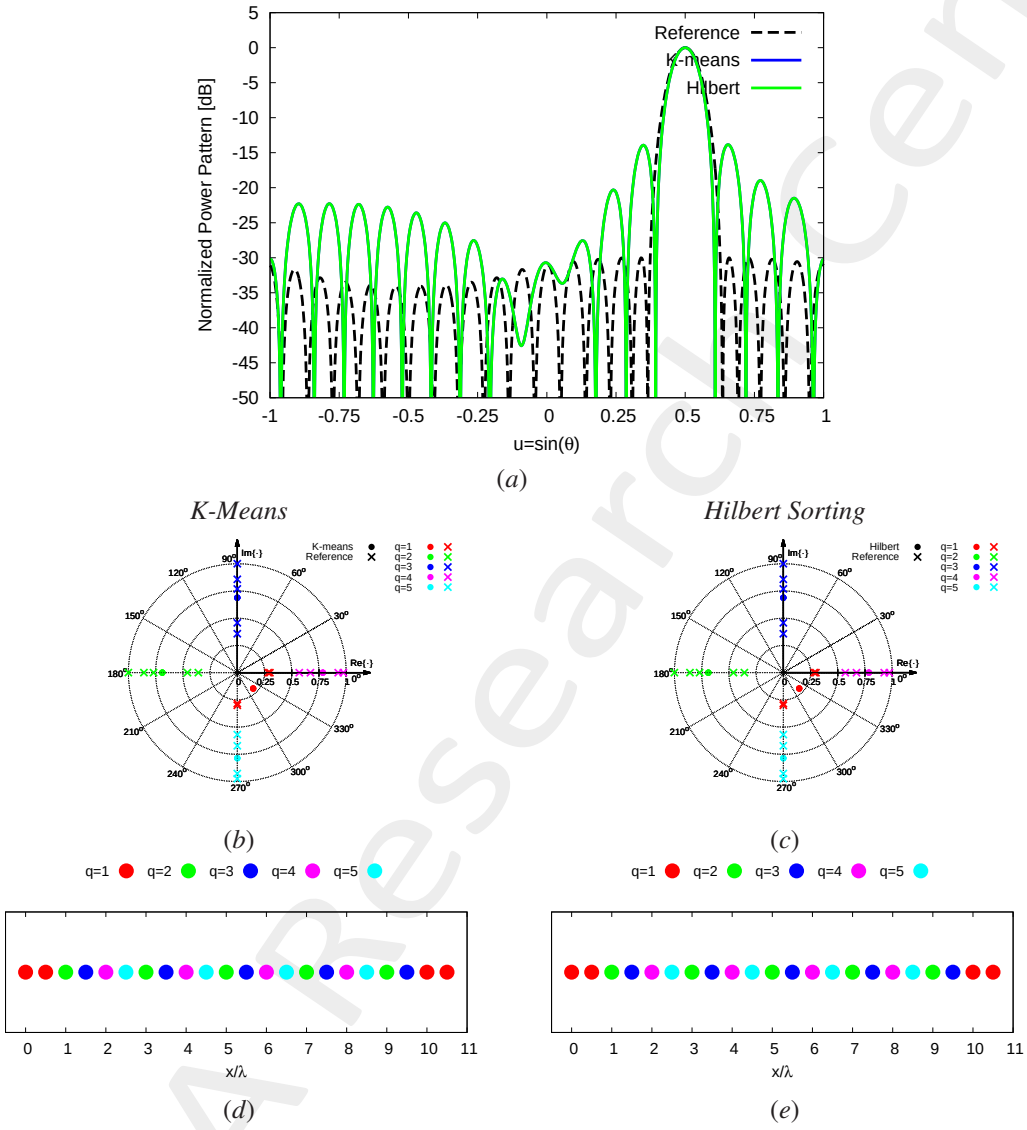

Figure 30: *Steered Pencil Beam* ($N = 22$, $Q = 5$, $\theta_0 = 30$ [deg]): plot of (a) the power pattern of the clusterized solutions computed with the K-Means (blue line) and the Hilbert Sorting (green line) together with the reference ones (dashed black line), (b)(c) representation of the reference and the sub-array excitations in the complex plane, and (d)(e) layout of the clustered array sythetized with (b)(d) the K-means and (c)(e) the Hilbert Sorting.

Method	Ψ	Φ	Γ	SLL [dB]	D [dBi]	$HPBW$ [u]
Hilbert Sorting	4.50×10^{-2}	9.153×10^{-2}	2.21×10^{-1}	-13.85	12.83	9.39×10^{-2}
K-Means	4.50×10^{-2}	9.153×10^{-2}	2.21×10^{-1}	-13.85	12.83	9.39×10^{-2}

Table XVIII: *Steered Pencil Beam* ($N = 22$, $Q = 5$, $\theta_0 = 30$ [deg]): values of excitation matching index, Ψ, the field matching index, Φ, the power pattern matching index, Γ, the SLL, the directivity D and the half power beamwidth $HPBW$ obtained with Hilbert Sorting + ES and with the K-means.

1.1.9 Steered Pencil Beam Pattern, $\mathbf{N = [10,\ 25]}$, $\mathbf{Q = [5,\ 10]}$, $\boldsymbol{\theta_0 = 60}$[deg] - rotated excitations $\boldsymbol{\alpha \in [0,\ 360]}$ [deg], $\boldsymbol{\Delta\alpha = 4}$[deg]

Test Case Description

Antenna configuration

- isotropic elements
- number of elements: $N = [10,\ 25]$
- distance between elements along x axis: $d_x = \lambda/2$

Target excitations

- $w_n = \alpha_n e^{j\varphi_n}$ with $n = 1, .., N$
- main lobe steering: $\theta_0 = 60$ [deg]
- Taylor pattern
  - nominal sidelobe level: $SLL_{ref} = -30$ [dB]
  - polinomial order: $\bar{n} = 7$

Sub-array generation

- number of clusters: $Q = [5,\ 10]$
- number realization: $R = 92$
- excitation matching strategies:
  - K-Means
    - initialization: $R = 92$ random seeds
  - Hilbert + Exhaustive Search:
    - initialization: $R = 92$ excitations rotations, $\alpha = [0,\ 360]$[deg]
    - rotation step: $\Delta\alpha = 4$[deg]

Hilbert Curve Sorting + Exhaustive Search vs. K-means

The following comparative assessment has been performed, computing five different orders of Hilbert curve.

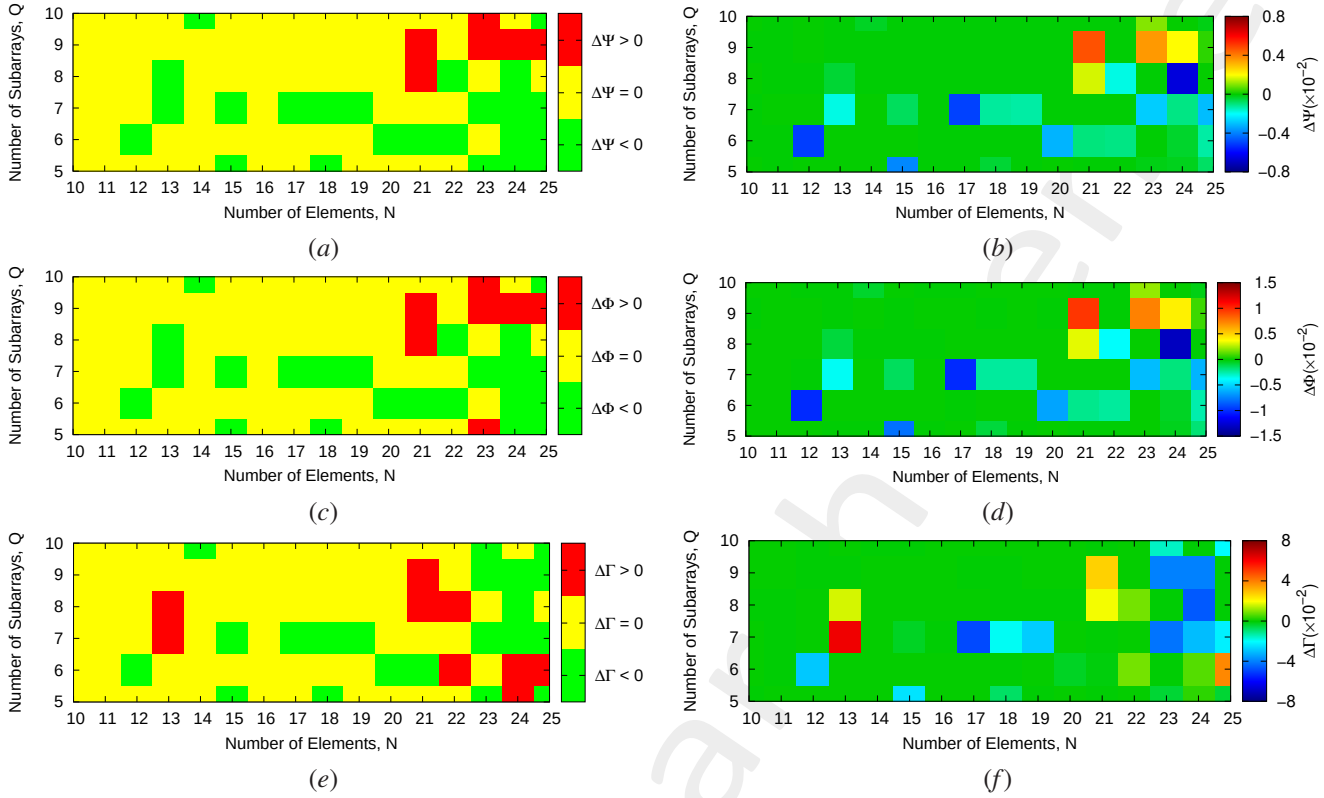

Figure 31: *Steered Pencil Beam Pattern,* $\theta_0 = 60$ *[deg] - rotated excitations* $[0, 360]$*[deg], Performance Evaluation:* $(a)(c)(e)$ 3 colors map and $(b)(d)(f)$ quantitative color map for the $(a)(b)$ difference excitation matching, $(c)(d)$ difference field matching and $(e)(f)$ difference power pattern matching.

In the following a comparative assessment of the performance in terms of power pattern and subarray clusters for some case of interest.

Number elements $N = 21$, Number Clusters $Q = 9$

- difference excitation matching: $\Delta\Psi = 4.70 \times 10^{-3}$
- difference field matching: $\Delta\Phi = 9.62 \times 10^{-3}$
- difference power pattern matching: $\Delta\Gamma = 2.70 \times 10^{-2}$

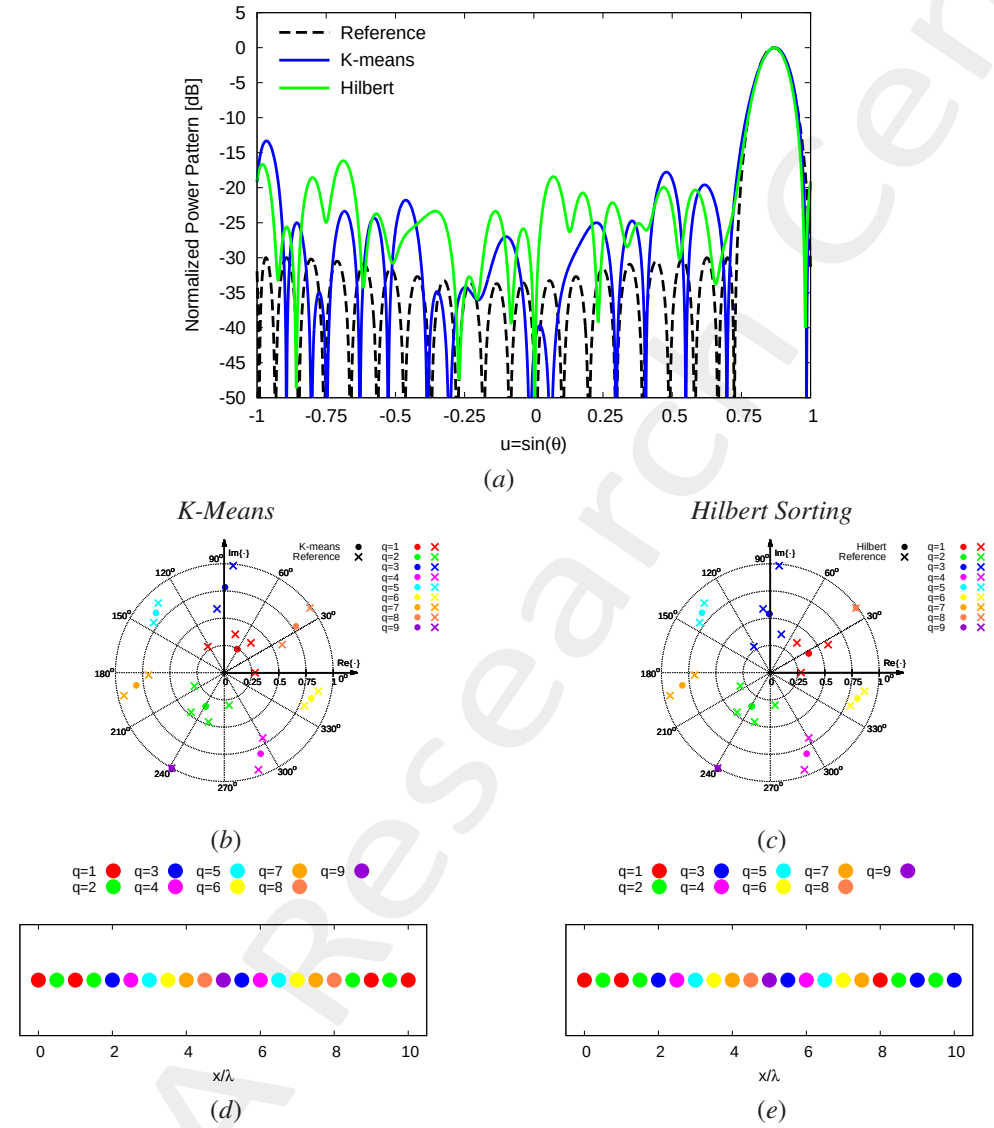

Figure 32: *Steered Pencil Beam* ($N = 21$, $Q = 9$, $\theta_0 = 60$ [deg]): plot of (a) the power pattern of the clusterized solutions computed with the K-Means (blue line) and the Hilbert Sorting (green line) together with the reference ones (dashed black line), (b)(c) representation of the reference and the sub-array excitations in the complex plane, and (d)(e) layout of the clustered array sythetized with (b)(d) the K-means and (c)(e) the Hilbert Sorting.

Method	Ψ	Φ	Γ	SLL [dB]	D [dBi]	$HPBW$ [u]
Hilbert Sorting	3.41×10^{-2}	7.00×10^{-2}	2.01×10^{-1}	-16.15	12.28	1.01×10^{-1}
K-Means	2.94×10^{-2}	6.04×10^{-2}	1.74×10^{-1}	-13.32	12.16	1.09×10^{-1}

Table XIX: *Steered Pencil Beam* ($N = 21$, $Q = 9$, $\theta_0 = 60$ [deg]): values of excitation matching index, Ψ, the field matching index, Φ, the power pattern matching index, Γ, the SLL, the directivity D and the half power beamwidth $HPBW$ obtained with Hilbert Sorting + ES and with the K-means.

Number elements $N = 22$, Number Clusters $Q = 8$

- difference excitation matching: $\Delta\Psi = -1.78 \times 10^{-3}$
- difference field matching: $\Delta\Phi = -3.64 \times 10^{-3}$
- difference power pattern matching: $\Delta\Gamma = 9.17 \times 10^{-3}$

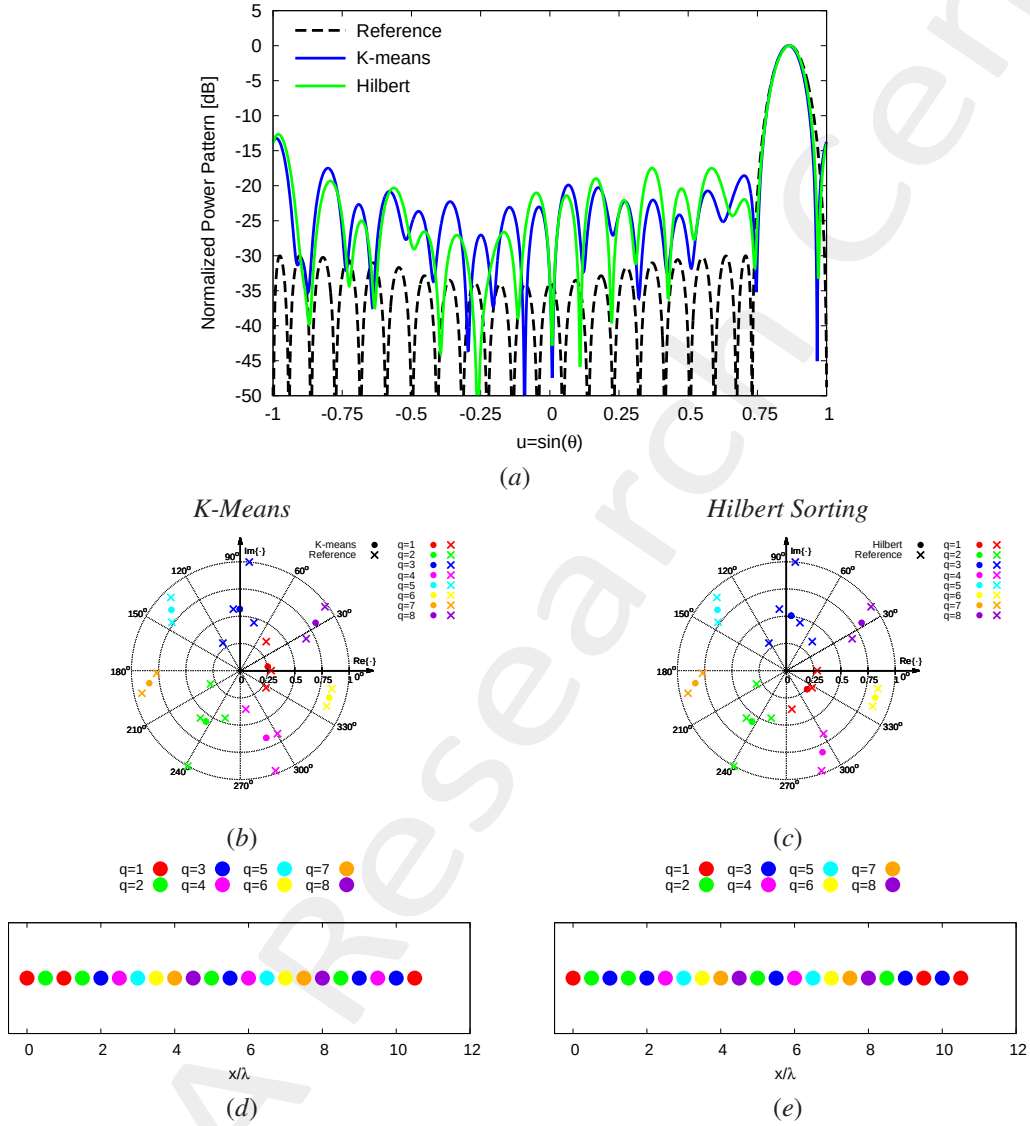

Figure 33: *Steered Pencil Beam* ($N = 22$, $Q = 8$, $\theta_0 = 60$ [deg]): plot of (a) the power pattern of the clusterized solutions computed with the K-Means (blue line) and the Hilbert Sorting (green line) together with the reference ones (dashed black line), (b)(c) representation of the reference and the sub-array excitations in the complex plane, and (d)(e) layout of the clustered array sythetized with (b)(d) the K-means and (c)(e) the Hilbert Sorting.

Method	Ψ	Φ	Γ	SLL [dB]	D [dBi]	$HPBW$ [u]
Hilbert Sorting	4.98×10^{-2}	1.02×10^{-1}	2.64×10^{-1}	-12.65	12.39	9.50×10^{-2}
K-Means	5.15×10^{-2}	1.05×10^{-1}	2.55×10^{-1}	-13.23	12.55	9.28×10^{-2}

Table XX: *Steered Pencil Beam* ($N = 22$, $Q = 8$, $\theta_0 = 60$ [deg]): values of excitation matching index, Ψ, the field matching index, Φ, the power pattern matching index, Γ, the SLL, the directivity D and the half power beamwidth $HPBW$ obtained with Hilbert Sorting + ES and with the K-means.

Number elements $N = 24$**, Number Clusters** $Q = 8$

- difference excitation matching: $\Delta\Psi = -6.51 \times 10^{-3}$
- difference field matching: $\Delta\Phi = -1.31 \times 10^{-2}$
- difference power pattern matching: $\Delta\Gamma = -4.60 \times 10^{-2}$

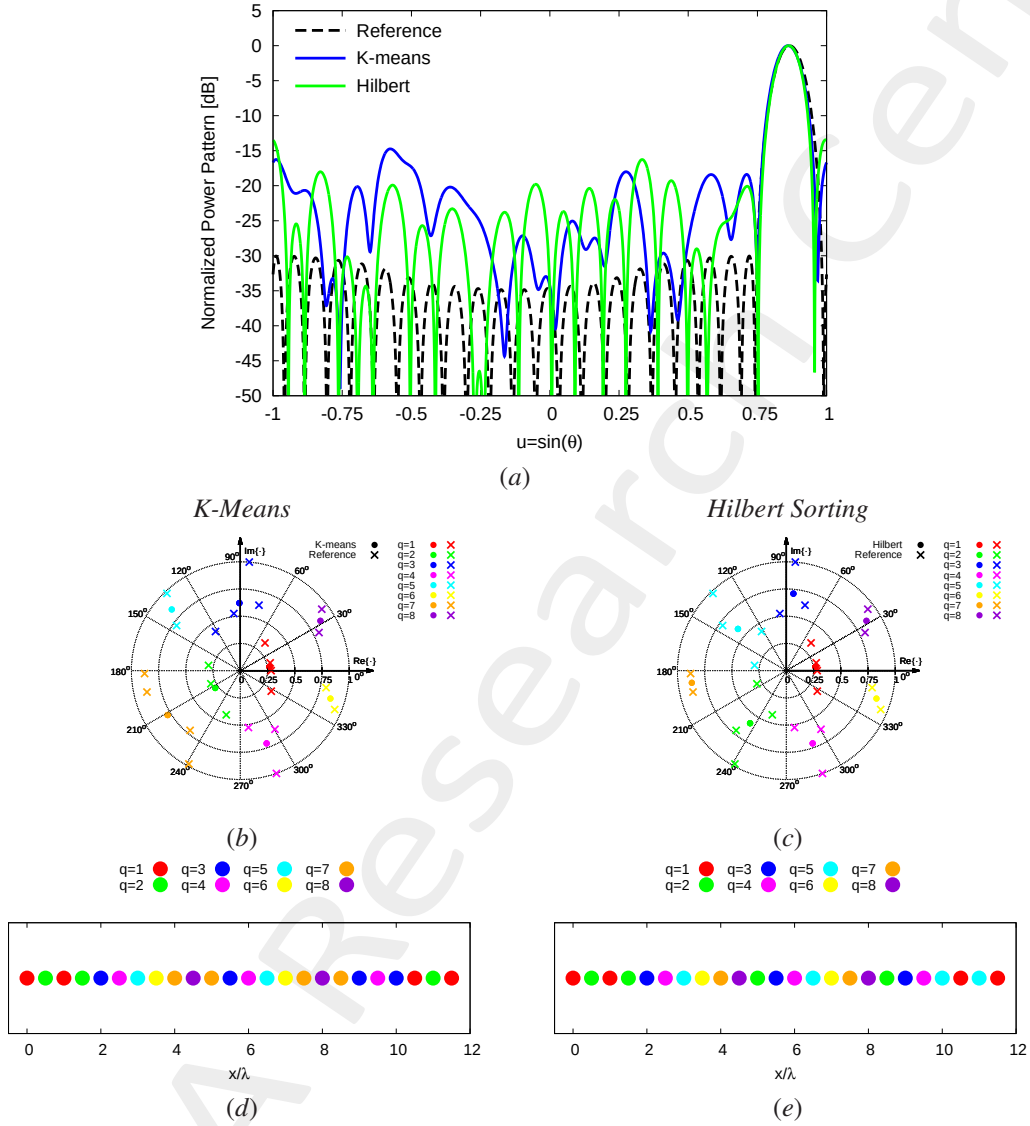

Figure 34: *Steered Pencil Beam* ($N = 24$, $Q = 8$, $\theta_0 = 60$ [deg]): plot of (a) the power pattern of the clusterized solutions computed with the K-Means (blue line) and the Hilbert Sorting (green line) together with the reference ones (dashed black line), (b)(c) representation of the reference and the sub-array excitations in the complex plane, and (d)(e) layout of the clustered array sythetized with (b)(d) the K-means and (c)(e) the Hilbert Sorting.

Method	Ψ	Φ	Γ	SLL [dB]	D [dBi]	$HPBW$ [u]
Hilbert Sorting	5.00×10^{-2}	1.02×10^{-1}	2.54×10^{-1}	-13.45	12.59	8.53×10^{-2}
K-Means	5.65×10^{-2}	1.15×10^{-1}	3.00×10^{-1}	-14.74	12.31	$9.02 \times 10 - 2$

Table XXI: *Steered Pencil Beam* ($N = 24$, $Q = 8$, $\theta_0 = 60$ [deg]): values of excitation matching index, Ψ, the field matching index, Φ, the power pattern matching index, Γ, the SLL, the directivity D and the half power beamwidth $HPBW$ obtained with Hilbert Sorting + ES and with the K-means.

More information on the topics of this document can be found in the following list of references.

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