Multi-Resolution Imaging through a Novel Bayesian Compressive Sensing Approach

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Abstract

In this work, a multi-resolution strategy is proposed for improving the reconstruction capabilities of standard Bayesian compressive sensing (*BCS*) when dealing with the imaging of sparse targets. Towards this end, a customized relevance vector machine (*RVM*) solver is derived and implemented in order to exploit the progressively acquired information about the scatterer shape and location within the imaged domain.

Some numerical results are shown to validate the effectiveness of the proposed imaging technique.

1 Numerical Results

1.1 Rhombus, $D = 1.5\lambda$

Test Case Description

Direct solver:

- Side of the investigation domain: $L = 6.0\lambda$
- Cubic domain divided in $\sqrt{D} \times \sqrt{D}$ cells
- Number of cells for the direct solver: D = 1600 (discretization = $\lambda/10$)

Investigation domain:

- Cubic domain divided in $\sqrt{N} \times \sqrt{N}$ cells
- Number of cells for the inversion:
 - First Step IMSA: $N^{(1)} = 100$ (discretization = $\lambda/10$)
 - Following Steps IMSA: $N^{(i)}$ not fixed, defined according to the estimated RoI $\mathcal{D}^{(i)}$

Measurement domain:

- Total number of measurements: M = 60
- Measurement points placed on circles of radius $\rho = 4.5\lambda$

Sources:

- Plane waves
- Number of views: V = 60; $\theta_{inc}^v = 0^\circ + (v 1) \times (360/V)$
- Amplitude: A = 1.0
- Frequency: $F = 300 \text{ MHz} (\lambda = 1)$

Background:

- $\varepsilon_r = 1.0$
- $\sigma = 0 \, [\text{S/m}]$

Scatterer

- Rhombus, $D = 1.5\lambda$
- $\varepsilon_r \in \{1.01, 1.02, 1.04, 1.05, 1.06, 1.08, 1.10, 1.15, 1.20\}$
- $\sigma = 0 [S/m]$



Figure 1: Rhombus, $D = 1.5\lambda$, $\tau = 0.02$ - (a) Actual profile and (b)-(o) IMSA-BCS reconstructed profiles for (b)(e)(h)(m) SNR = 20 [dB], (c)(f)(i)(n) SNR = 10 [dB] and (d)(g)(l)(o) SNR = 5 [dB] at the step (b)-(d) S = 1, (e)-(g) S = 2, (h)-(l) S = 3 and (m)-(o) S = 4.

	SNR = 50dB				
	S = 1	S=2	S=3	S = 4	
ξ_{tot}	1.05×10^{-3}	4.63×10^{-4}	4.48×10^{-4}	4.48×10^{-4}	
ξ_{int}	1.09×10^{-2}	7.65×10^{-3}	7.80×10^{-3}	7.80×10^{-3}	
ξ_{ext}	6.69×10^{-4}	$1.82 imes 10^{-4}$	1.62×10^{-4}	1.62×10^{-4}	
		SNR =	= 20 <i>dB</i>		
	S = 1	S=2	S=3	S = 4	
ξ_{tot}	1.05×10^{-3}	4.66×10^{-4}	4.44×10^{-4}	4.44×10^{-4}	
ξ_{int}	1.08×10^{-2}	7.19×10^{-3}	7.59×10^{-3}	$7.59 imes 10^{-3}$	
ξ_{ext}	$6.65 imes 10^{-4}$	2.03×10^{-4}	$1.66 imes 10^{-4}$	1.66×10^{-4}	
		SNR =	= 10 <i>dB</i>		
	S = 1	SNR = S = 2	= 10dB $S = 3$	S = 4	
ξtot	S = 1 1.09×10^{-3}	$SNR = $ $S = 2$ 4.87×10^{-4}	= $10dB$ S = 3 4.04×10^{-4}	$S = 4$ 4.04×10^{-4}	
ξ_{tot} ξ_{int}	S = 1 1.09×10^{-3} 1.08×10^{-2}	$SNR = $ $S = 2$ 4.87×10^{-4} 7.14×10^{-3}	= $10dB$ S = 3 4.04×10^{-4} 6.67×10^{-3}	S = 4 4.04×10^{-4} 6.67×10^{-3}	
$\frac{\xi_{tot}}{\xi_{int}}$	S = 1 1.09 × 10 ⁻³ 1.08 × 10 ⁻² 6.99 × 10 ⁻⁴	SNR = S = 2 4.87×10^{-4} 7.14×10^{-3} 2.26×10^{-4}	= $10dB$ S = 3 4.04×10^{-4} 6.67×10^{-3} 1.59×10^{-4}	S = 4 4.04×10^{-4} 6.67×10^{-3} 1.59×10^{-4}	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{ext}}$	$S = 1$ 1.09×10^{-3} 1.08×10^{-2} 6.99×10^{-4}	SNR = S = 2 4.87×10^{-4} 7.14×10^{-3} 2.26×10^{-4} SNR =	= $10dB$ S = 3 4.04×10^{-4} 6.67×10^{-3} 1.59×10^{-4} = $5dB$	$S = 4$ 4.04×10^{-4} 6.67×10^{-3} 1.59×10^{-4}	
$\frac{\xi_{tot}}{\xi_{int}}$	S = 1 1.09×10^{-3} 1.08×10^{-2} 6.99×10^{-4} S = 1	SNR = S = 2 4.87×10^{-4} 7.14×10^{-3} 2.26×10^{-4} SNR = S = 2	= $10dB$ S = 3 4.04×10^{-4} 6.67×10^{-3} 1.59×10^{-4} = $5dB$ S = 3	$S = 4$ 4.04×10^{-4} 6.67×10^{-3} 1.59×10^{-4} $S = 4$	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{tot}}$	$S = 1$ 1.09×10^{-3} 1.08×10^{-2} 6.99×10^{-4} $S = 1$ 1.18×10^{-3}	SNR = S = 2 4.87×10^{-4} 7.14×10^{-3} 2.26×10^{-4} SNR = S = 2 5.45×10^{-4}	= $10dB$ S = 3 4.04×10^{-4} 6.67×10^{-3} 1.59×10^{-4} = $5dB$ S = 3 3.88×10^{-4}	$S = 4$ 4.04×10^{-4} 6.67×10^{-3} 1.59×10^{-4} $S = 4$ 3.88×10^{-4}	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{tot}}$	$S = 1$ 1.09×10^{-3} 1.08×10^{-2} 6.99×10^{-4} $S = 1$ 1.18×10^{-3} 1.10×10^{-2}	$SNR = S = 2$ 4.87×10^{-4} 7.14×10^{-3} 2.26×10^{-4} $SNR = S$ $S = 2$ 5.45×10^{-4} 7.40×10^{-3}	= 10dB S = 3 4.04×10^{-4} 6.67×10^{-3} 1.59×10^{-4} = 5dB S = 3 3.88×10^{-4} 6.03×10^{-3}	$S = 4$ 4.04×10^{-4} 6.67×10^{-3} 1.59×10^{-4} $S = 4$ 3.88×10^{-4} 6.03×10^{-3}	

Table I: *Rhombus*, $D = 1.5\lambda$, $\tau = 0.20$ - Reconstruction errors: total (ξ_{tot}) , internal (ξ_{int}) and external (ξ_{ext}) errors.

	SNR = 50dB				
	S = 1	S = 2	S=3	S = 4	
$L^{(S)}$	6.00	2.40	1.50	1.50	
$N^{(S)}$	100	148	148	148	
$Q^{(S)}$	100	64	25	25	
		SNR =	= 20 <i>dB</i>		
	S = 1	S=2	S=3	S = 4	
$L^{(S)}$	6.00	2.40	1.50	1.50	
$N^{(S)}$	100	148	148	148	
$Q^{(S)}$	100	64	25	25	
		SNR =	= 10 dB		
	S = 1	S=2	S=3	S = 4	
$L^{(S)}$	6.00	3.00	1.50	1.50	
$N^{(S)}$	100	175	175	175	
$Q^{(S)}$	100	100	25	25	
	SNR = 5dB				
	S = 1	S=2	S=3	S = 4	
$L^{(S)}$	6.00	3.00	1.50	1.50	
$N^{(S)}$	100	175	175	175	
$Q^{(S)}$	100	100	25	25	

Table II: Rhombus, $D = 1.5\lambda$, $\tau = 0.02$ - Investigation domain parameters: restricted investigation domain size $L^{(S)}$, total number of cells $N^{(S)}$ and number of cells within the restricted domain size $Q^{(S)}$.



Figure 2: Rhombus, $D = 1.5\lambda$, $\tau = 0.05$ - (a) Actual profile and (b)-(o) IMSA-BCS reconstructed profiles for (b)(e)(h)(m) SNR = 20 [dB], (c)(f)(i)(n) SNR = 10 [dB] and (d)(g)(l)(o) SNR = 5 [dB] at the step (b)-(d) S = 1, (e)-(g) S = 2, (h)-(l) S = 3 and (m)-(o) S = 4.

	SNR = 50dB				
	S = 1	S=2	S=3	S = 4	
ξ_{tot}	3.01×10^{-3}	1.35×10^{-3}	1.12×10^{-3}	1.12×10^{-3}	
ξ_{int}	2.39×10^{-2}	1.75×10^{-2}	1.65×10^{-2}	1.65×10^{-2}	
ξ_{ext}	2.12×10^{-3}	6.98×10^{-4}	5.13×10^{-4}	$5.13 imes 10^{-4}$	
		SNR =	= 20 <i>dB</i>		
	S = 1	S=2	S = 3	S = 4	
ξ_{tot}	3.04×10^{-3}	1.31×10^{-3}	1.09×10^{-3}	1.09×10^{-3}	
ξ_{int}	2.45×10^{-2}	1.69×10^{-2}	1.53×10^{-2}	1.53×10^{-2}	
ξ_{ext}	2.12×10^{-3}	6.81×10^{-4}	5.27×10^{-4}	5.27×10^{-4}	
30000					
3000		SNR =	= 10 <i>dB</i>		
30000	S = 1	SNR = S = 2	= 10dB $S = 3$	S = 4	
ξtot	S = 1 3.12×10^{-3}	$SNR = $ $S = 2$ 1.39×10^{-3}	= $10dB$ S = 3 9.67×10^{-4}	S = 4 9.67 × 10 ⁻⁴	
ξ _{tot} ξ _{int}	S = 1 3.12×10^{-3} 2.51×10^{-2}	$SNR = $ $S = 2$ 1.39×10^{-3} 1.53×10^{-2}	= $10dB$ S = 3 9.67×10^{-4} 1.30×10^{-2}	S = 4 9.67 × 10 ⁻⁴ 1.30 × 10 ⁻²	
$\frac{\xi_{tot}}{\xi_{int}}$	S = 1 3.12 × 10 ⁻³ 2.51 × 10 ⁻² 2.14 × 10 ⁻³	SNR = S = 2 1.39×10^{-3} 1.53×10^{-2} 7.93×10^{-4}	= $10dB$ S = 3 9.67×10^{-4} 1.30×10^{-2} 4.81×10^{-4}	S = 4 9.67 × 10 ⁻⁴ 1.30 × 10 ⁻² 4.81 × 10 ⁻⁴	
$\frac{\xi_{tot}}{\xi_{int}}$	S = 1 3.12 × 10 ⁻³ 2.51 × 10 ⁻² 2.14 × 10 ⁻³	SNR = S = 2 1.39×10^{-3} 1.53×10^{-2} 7.93×10^{-4} SNR =	= 10dB S = 3 9.67×10^{-4} 1.30×10^{-2} 4.81×10^{-4} = 5dB	S = 4 9.67 × 10 ⁻⁴ 1.30 × 10 ⁻² 4.81 × 10 ⁻⁴	
$\frac{\xi_{tot}}{\xi_{int}}$	S = 1 3.12 × 10 ⁻³ 2.51 × 10 ⁻² 2.14 × 10 ⁻³ S = 1	SNR = S = 2 1.39×10^{-3} 1.53×10^{-2} 7.93×10^{-4} SNR = S = 2	= $10dB$ S = 3 9.67×10^{-4} 1.30×10^{-2} 4.81×10^{-4} = $5dB$ S = 3	S = 4 9.67 × 10 ⁻⁴ 1.30 × 10 ⁻² 4.81 × 10 ⁻⁴ S = 4	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{tot}}{\xi_{ext}}$	S = 1 3.12 × 10 ⁻³ 2.51 × 10 ⁻² 2.14 × 10 ⁻³ $S = 1$ 3.30 × 10 ⁻³	SNR = S = 2 1.39×10^{-3} 1.53×10^{-2} 7.93×10^{-4} SNR = S = 2 1.48×10^{-3}	= 10dB S = 3 9.67×10^{-4} 1.30×10^{-2} 4.81×10^{-4} = 5dB S = 3 8.71×10^{-4}	S = 4 9.67 × 10 ⁻⁴ 1.30 × 10 ⁻² 4.81 × 10 ⁻⁴ S = 4 8.71 × 10 ⁻⁴	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{tot}}{\xi_{tot}}$ $\frac{\xi_{tot}}{\xi_{int}}$	S = 1 3.12 × 10 ⁻³ 2.51 × 10 ⁻² 2.14 × 10 ⁻³ $S = 1$ 3.30 × 10 ⁻³ 2.61 × 10 ⁻²	SNR = S = 2 1.39×10^{-3} 1.53×10^{-2} 7.93×10^{-4} SNR = SNR = 1.48×10^{-3} 1.50×10^{-2}	$= 10dB$ $S = 3$ 9.67×10^{-4} 1.30×10^{-2} 4.81×10^{-4} $= 5dB$ $S = 3$ 8.71×10^{-4} 1.01×10^{-2}	S = 4 9.67 × 10 ⁻⁴ 1.30 × 10 ⁻² 4.81 × 10 ⁻⁴ S = 4 8.71 × 10 ⁻⁴ 1.01 × 10 ⁻²	

Table III: *Rhombus*, $D = 1.5\lambda$, $\tau = 0.20$ - Reconstruction errors: total (ξ_{tot}) , internal (ξ_{int}) and external (ξ_{ext}) errors.

	SNR = 50dB				
	S = 1	S = 2	S=3	S = 4	
$L^{(S)}$	6.00	3.00	1.50	1.50	
$N^{(S)}$	100	175	175	175	
$Q^{(S)}$	100	100	25	25	
		SNR =	= 20 <i>dB</i>		
	S = 1	S=2	S=3	S = 4	
$L^{(S)}$	6.00	3.00	1.50	1.50	
$N^{(S)}$	100	175	175	175	
$Q^{(S)}$	100	100	25	25	
		SNR =	= 10 dB		
	S = 1	S=2	S=3	S = 4	
$L^{(S)}$	6.00	3.60	1.50	1.50	
$N^{(S)}$	100	208	208	208	
$Q^{(S)}$	100	144	25	25	
	SNR = 5dB				
	S = 1	S=2	S=3	S = 4	
$L^{(S)}$	6.00	3.60	1.50	1.50	
$N^{(S)}$	100	208	208	208	
$Q^{(S)}$	100	144	25	25	

Table IV: *Rhombus*, $D = 1.5\lambda$, $\tau = 0.05$ - Investigation domain parameters: restricted investigation domain size $L^{(S)}$, total number of cells $N^{(S)}$ and number of cells within the restricted domain size $Q^{(S)}$.



Figure 3: Rhombus, $D = 1.5\lambda$, $\tau = 0.10$ - (a) Actual profile and (b)-(o) IMSA-BCS reconstructed profiles for (b)(e)(h) SNR = 20 [dB], (c)(f)(i) SNR = 10 [dB] and (d)(g)(l) SNR = 5 [dB] at the step (b)-(d) S = 1, (e)-(g) S = 2, and (h)-(l) S = 3.

	SNR = 50 dB				
	S = 1	S=2	S=3	S = 4	
ξ_{tot}	6.37×10^{-3}	3.09×10^{-3}	2.10×10^{-3}	2.10×10^{-3}	
ξ_{int}	4.77×10^{-2}	3.29×10^{-2}	2.72×10^{-2}	2.72×10^{-2}	
ξ_{ext}	4.40×10^{-3}	$1.73 imes 10^{-3}$	9.86×10^{-4}	9.86×10^{-4}	
		SNR =	= 20 <i>dB</i>		
	S = 1	S=2	S=3	S = 4	
ξ_{tot}	6.52×10^{-3}	3.08×10^{-3}	2.18×10^{-3}	2.18×10^{-3}	
ξ_{int}	4.86×10^{-2}	3.27×10^{-2}	2.81×10^{-2}	2.81×10^{-2}	
ξ_{ext}	4.48×10^{-3}	1.71×10^{-3}	1.01×10^{-3}	1.01×10^{-3}	
		SNR =	= 10 <i>dB</i>		
	S = 1	SNR = S = 2	= 10dB $S = 3$	S = 4	
ξ_{tot}	S = 1 6.51×10^{-3}	$SNR = $ $S = 2$ 3.14×10^{-3}	= $10dB$ S = 3 2.05×10^{-3}	$S = 4$ 2.05×10^{-3}	
ξ_{tot} ξ_{int}	S = 1 6.51×10^{-3} 4.86×10^{-2}	SNR = S = 2 3.14×10^{-3} 3.10×10^{-2}	= $10dB$ S = 3 2.05×10^{-3} 2.29×10^{-2}	S = 4 2.05 × 10 ⁻³ 2.29 × 10 ⁻²	
$\frac{\xi_{tot}}{\xi_{int}}$	S = 1 6.51×10^{-3} 4.86×10^{-2} 4.46×10^{-3}	SNR = S = 2 3.14×10^{-3} 3.10×10^{-2} 1.76×10^{-3}	= $10dB$ S = 3 2.05×10^{-3} 2.29×10^{-2} 1.02×10^{-3}	S = 4 2.05 × 10 ⁻³ 2.29 × 10 ⁻² 1.02 × 10 ⁻³	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{ext}}$	S = 1 6.51×10^{-3} 4.86×10^{-2} 4.46×10^{-3}	SNR = S = 2 3.14×10^{-3} 3.10×10^{-2} 1.76×10^{-3} SNR =	= $10dB$ S = 3 2.05×10^{-3} 2.29×10^{-2} 1.02×10^{-3} = $5dB$	S = 4 2.05 × 10 ⁻³ 2.29 × 10 ⁻² 1.02 × 10 ⁻³	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{ext}}$	S = 1 6.51×10^{-3} 4.86×10^{-2} 4.46×10^{-3} S = 1	SNR = S = 2 3.14×10^{-3} 3.10×10^{-2} 1.76×10^{-3} SNR = S = 2	= $10dB$ S = 3 2.05×10^{-3} 2.29×10^{-2} 1.02×10^{-3} = $5dB$ S = 3	S = 4 2.05 × 10 ⁻³ 2.29 × 10 ⁻² 1.02 × 10 ⁻³ S = 4	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{tot}}$	S = 1 6.51×10^{-3} 4.86×10^{-2} 4.46×10^{-3} S = 1 7.15×10^{-3}	SNR = S = 2 3.14×10^{-3} 3.10×10^{-2} 1.76×10^{-3} SNR = S = 2 3.65×10^{-3}	= $10dB$ S = 3 2.05×10^{-3} 2.29×10^{-2} 1.02×10^{-3} = $5dB$ S = 3 2.40×10^{-3}	S = 4 2.05 × 10 ⁻³ 2.29 × 10 ⁻² 1.02 × 10 ⁻³ $S = 4$ 1.88 × 10 ⁻³	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{tot}}$	S = 1 6.51×10^{-3} 4.86×10^{-2} 4.46×10^{-3} S = 1 7.15×10^{-3} 5.01×10^{-2}	SNR = S = 2 3.14×10^{-3} 3.10×10^{-2} 1.76×10^{-3} SNR = S = 2 3.65×10^{-3} 3.35×10^{-2}	= 10dB S = 3 2.05×10^{-3} 2.29×10^{-2} 1.02×10^{-3} = 5dB S = 3 2.40×10^{-3} 2.29×10^{-2}	S = 4 2.05 × 10 ⁻³ 2.29 × 10 ⁻² 1.02 × 10 ⁻³ $S = 4$ 1.88 × 10 ⁻³ 1.95 × 10 ⁻²	

Table V: *Rhombus*, $D = 1.5\lambda$, $\tau = 0.20$ - Reconstruction errors: total (ξ_{tot}) , internal (ξ_{int}) and external (ξ_{ext}) errors.

	SNR = 50dB				
	S = 1	S = 2	S=3	S = 4	
$L^{(S)}$	6.00	3.60	1.50	1.50	
$N^{(S)}$	100	208	208	208	
$Q^{(S)}$	100	144	25	25	
		SNR =	= 20 dB		
	S = 1	S=2	S=3	S = 4	
$L^{(S)}$	6.00	3.60	1.50	1.50	
$N^{(S)}$	100	208	208	208	
$Q^{(S)}$	100	144	25	25	
		SNR =	= 10 dB		
	S = 1	S=2	S=3	S = 4	
$L^{(S)}$	6.00	3.60	1.50	1.50	
$N^{(S)}$	100	208	208	208	
$Q^{(S)}$	100	144	25	25	
	SNR = 5dB				
	S = 1	S=2	S=3	S = 4	
$L^{(S)}$	6.00	3.60	1.80	1.50	
$N^{(S)}$	100	208	208	208	
$Q^{(S)}$	100	144	36	25	

Table VI: *Rhombus*, $D = 1.5\lambda$, $\tau = 0.05$ - Investigation domain parameters: restricted investigation domain size $L^{(S)}$, total number of cells $N^{(S)}$ and number of cells within the restricted domain size $Q^{(S)}$.



Figure 4: Rhombus, $D = 1.5\lambda$, $\tau = 0.15$ - (a) Actual profile and (b)-(o) IMSA-BCS reconstructed profiles for (b)(e)(h) SNR = 20 [dB], (c)(f)(i) SNR = 10 [dB] and (d)(g)(l) SNR = 5 [dB] at the step (b)-(d) S = 1, (e)-(g) S = 2, and (h)-(l) S = 3.

	SNR = 50 dB				
	S = 1	S=2	S=3	S = 4	
ξ_{tot}	9.65×10^{-3}	6.53×10^{-3}	5.42×10^{-3}	3.51×10^{-3}	
ξ_{int}	6.97×10^{-2}	7.15×10^{-2}	6.00×10^{-2}	4.27×10^{-2}	
ξ_{ext}	$6.50 imes 10^{-3}$	3.64×10^{-3}	$3.02 imes 10^{-3}$	1.52×10^{-3}	
		SNR =	= 20 <i>dB</i>		
	S = 1	S=2	S=3	S = 4	
ξ_{tot}	9.72×10^{-3}	6.42×10^{-3}	$5.35 imes 10^{-3}$	3.53×10^{-3}	
ξ_{int}	6.83×10^{-2}	6.85×10^{-2}	5.81×10^{-2}	4.09×10^{-2}	
ξ_{ext}	6.63×10^{-3}	3.54×10^{-3}	$2.96 imes 10^{-3}$	1.62×10^{-3}	
50000					
		SNR =	= 10 <i>dB</i>		
	S = 1	SNR = S = 2	= 10dB $S = 3$	S = 4	
ξtot	S = 1 9.95 × 10 ⁻³	$SNR = $ $S = 2$ 5.75×10^{-3}	= $10dB$ S = 3 3.91×10^{-3}	S = 4 3.06×10^{-3}	
ξ_{tot} ξ_{int}	S = 1 9.95×10^{-3} 7.07×10^{-2}	SNR = 0 S = 2 5.75×10^{-3} 5.83×10^{-2}	= $10dB$ S = 3 3.91×10^{-3} 3.94×10^{-2}	S = 4 3.06×10^{-3} 3.18×10^{-2}	
$\frac{\xi_{tot}}{\xi_{int}}$	S = 1 9.95 × 10 ⁻³ 7.07 × 10 ⁻² 6.81 × 10 ⁻³	SNR = S = 2 5.75×10^{-3} 5.83×10^{-2} 3.14×10^{-3}	= $10dB$ S = 3 3.91×10^{-3} 3.94×10^{-2} 2.01×10^{-3}	S = 4 3.06 × 10 ⁻³ 3.18 × 10 ⁻² 1.38 × 10 ⁻³	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{ext}}$	S = 1 9.95 × 10 ⁻³ 7.07 × 10 ⁻² 6.81 × 10 ⁻³	SNR = S = 2 5.75×10^{-3} 5.83×10^{-2} 3.14×10^{-3} SNR =	= $10dB$ S = 3 3.91×10^{-3} 3.94×10^{-2} 2.01×10^{-3} = $5dB$	S = 4 3.06 × 10 ⁻³ 3.18 × 10 ⁻² 1.38 × 10 ⁻³	
$\frac{\xi_{tot}}{\xi_{int}}$	S = 1 9.95×10^{-3} 7.07×10^{-2} 6.81×10^{-3} S = 1	SNR = S = 2 5.75×10^{-3} 5.83×10^{-2} 3.14×10^{-3} SNR = S = 2	= $10dB$ S = 3 3.91×10^{-3} 3.94×10^{-2} 2.01×10^{-3} = $5dB$ S = 3	S = 4 3.06×10^{-3} 3.18×10^{-2} 1.38×10^{-3} S = 4	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{tot}}$	S = 1 9.95 × 10 ⁻³ 7.07 × 10 ⁻² 6.81 × 10 ⁻³ S = 1 1.25 × 10 ⁻²	SNR = S = 2 5.75×10^{-3} 5.83×10^{-2} 3.14×10^{-3} SNR = S = 2 6.78×10^{-3}	= $10dB$ S = 3 3.91×10^{-3} 3.94×10^{-2} 2.01×10^{-3} = $5dB$ S = 3 5.43×10^{-3}	S = 4 3.06 × 10 ⁻³ 3.18 × 10 ⁻² 1.38 × 10 ⁻³ $S = 4$ 3.39 × 10 ⁻³	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{tot}}$	S = 1 9.95 × 10 ⁻³ 7.07 × 10 ⁻² 6.81 × 10 ⁻³ $S = 1$ 1.25 × 10 ⁻² 7.61 × 10 ⁻²	SNR = S = 2 5.75×10^{-3} 5.83×10^{-2} 3.14×10^{-3} SNR = S = 2 6.78×10^{-3} 5.72×10^{-2}	= 10dB S = 3 3.91×10^{-3} 3.94×10^{-2} 2.01×10^{-3} = 5dB S = 3 5.43×10^{-3} 5.37×10^{-2}	S = 4 3.06 × 10 ⁻³ 3.18 × 10 ⁻² 1.38 × 10 ⁻³ $S = 4$ 3.39 × 10 ⁻³ 3.40 × 10 ⁻²	

Table VII: *Rhombus*, $D = 1.5\lambda$, $\tau = 0.20$ - Reconstruction errors: total (ξ_{tot}) , internal (ξ_{int}) and external (ξ_{ext}) errors.

	SNR = 50dB				
	S = 1	S = 2	S = 3	S = 4	
$L^{(S)}$	6.00	3.60	2.10	1.50	
$N^{(S)}$	100	208	208	208	
$Q^{(S)}$	100	144	49	25	
		SNR =	= 20 dB		
	S = 1	S=2	S = 3	S = 4	
$L^{(S)}$	6.00	3.60	2.10	1.50	
$N^{(S)}$	100	208	208	208	
$Q^{(S)}$	100	144	49	25	
		SNR =	= 10 dB		
	S = 1	S=2	S = 3	S = 4	
$L^{(S)}$	6.00	3.60	1.80	1.50	
$N^{(S)}$	100	208	208	208	
$Q^{(S)}$	100	144	36	25	
	SNR = 5dB				
	S = 1	S=2	S = 3	S = 4	
$L^{(S)}$	6.00	3.60	2.10	1.50	
$N^{(S)}$	100	208	208	208	
$Q^{(S)}$	100	144	49	25	

Table VIII: Rhombus, $D = 1.5\lambda$, $\tau = 0.05$ - Investigation domain parameters: restricted investigation domain size $L^{(S)}$, total number of cells $N^{(S)}$ and number of cells within the restricted domain size $Q^{(S)}$.



Figure 5: Rhombus, $D = 1.5\lambda$, $\tau = 0.20$ - (a) Actual profile and (b)-(o) IMSA-BCS reconstructed profiles for (b)(e)(h) SNR = 20 [dB], (c)(f)(i) SNR = 10 [dB] and (d)(g)(l) SNR = 5 [dB] at the step (b)-(d) S = 1, (e)-(g) S = 2, and (h)-(l) S = 3.

	SNR = 50 dB				
	S = 1	S=2	S=3	S = 4	
ξ_{tot}	1.29×10^{-2}	9.07×10^{-3}	7.31×10^{-3}	4.83×10^{-3}	
ξ_{int}	9.23×10^{-2}	9.49×10^{-2}	7.80×10^{-2}	4.18×10^{-2}	
ξ_{ext}	8.61×10^{-3}	4.89×10^{-3}	3.86×10^{-3}	2.16×10^{-3}	
		SNR =	= 20 <i>dB</i>		
	S = 1	S=2	S = 3	S = 4	
ξ_{tot}	1.31×10^{-2}	9.14×10^{-3}	7.67×10^{-3}	4.86×10^{-3}	
ξ_{int}	9.19×10^{-2}	9.64×10^{-2}	8.29×10^{-2}	4.42×10^{-2}	
ξ_{ext}	8.60×10^{-3}	4.92×10^{-3}	$4.14 imes 10^{-3}$	2.13×10^{-3}	
		SNR =	= 10 <i>dB</i>		
	S = 1	SNR = S = 2	= 10dB $S = 3$	S = 4	
ξtot	S = 1 1.45 × 10 ⁻²	$SNR = $ $S = 2$ 9.40×10^{-3}	= $10dB$ S = 3 7.98×10^{-3}	$S = 4$ 4.56×10^{-3}	
$\frac{\xi_{tot}}{\xi_{int}}$	S = 1 1.45×10^{-2} 9.64×10^{-2}	SNR = S = 2 9.40×10^{-3} 9.49×10^{-2}	= $10dB$ S = 3 7.98×10^{-3} 8.43×10^{-2}	S = 4 4.56×10^{-3} 3.97×10^{-2}	
$\frac{\xi_{tot}}{\xi_{int}}$	S = 1 1.45×10^{-2} 9.64×10^{-2} 9.53×10^{-3}	SNR = S = 2 9.40×10^{-3} 9.49×10^{-2} 5.01×10^{-3}	= $10dB$ S = 3 7.98×10^{-3} 8.43×10^{-2} 4.34×10^{-3}	S = 4 4.56×10^{-3} 3.97×10^{-2} 1.95×10^{-3}	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{ext}}$	S = 1 1.45×10^{-2} 9.64×10^{-2} 9.53×10^{-3}	SNR = S = 2 9.40×10^{-3} 9.49×10^{-2} 5.01×10^{-3} SNR	= $10dB$ S = 3 7.98×10^{-3} 8.43×10^{-2} 4.34×10^{-3} = $5dB$	$S = 4$ 4.56×10^{-3} 3.97×10^{-2} 1.95×10^{-3}	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{ext}}$	$S = 1$ 1.45×10^{-2} 9.64×10^{-2} 9.53×10^{-3} $S = 1$	SNR = S = 2 9.40×10^{-3} 9.49×10^{-2} 5.01×10^{-3} SNR S = 2	= $10dB$ S = 3 7.98×10^{-3} 8.43×10^{-2} 4.34×10^{-3} = $5dB$ S = 3	S = 4 4.56×10^{-3} 3.97×10^{-2} 1.95×10^{-3} S = 4	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{tot}}$	$S = 1$ 1.45×10^{-2} 9.64×10^{-2} 9.53×10^{-3} $S = 1$ 1.84×10^{-2}	SNR = S = 2 9.40×10^{-3} 9.49×10^{-2} 5.01×10^{-3} SNR = S = 2 1.21×10^{-2}	= $10dB$ S = 3 7.98×10^{-3} 8.43×10^{-2} 4.34×10^{-3} = $5dB$ S = 3 9.75×10^{-3}	$S = 4$ 4.56×10^{-3} 3.97×10^{-2} 1.95×10^{-3} $S = 4$ 5.47×10^{-3}	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{tot}}$	$S = 1$ 1.45×10^{-2} 9.64×10^{-2} 9.53×10^{-3} $S = 1$ 1.84×10^{-2} 9.86×10^{-2}	SNR = S = 2 9.40×10^{-3} 9.49×10^{-2} 5.01×10^{-3} SNR S = 2 1.21×10^{-2} 9.05×10^{-2}	= $10dB$ S = 3 7.98×10^{-3} 8.43×10^{-2} 4.34×10^{-3} = $5dB$ S = 3 9.75×10^{-3} 8.26×10^{-2}	$S = 4$ 4.56×10^{-3} 3.97×10^{-2} 1.95×10^{-3} $S = 4$ 5.47×10^{-3} 5.71×10^{-2}	

Table IX: *Rhombus*, $D = 1.5\lambda$, $\tau = 0.20$ - Reconstruction errors: total (ξ_{tot}) , internal (ξ_{int}) and external (ξ_{ext}) errors.

	SNR = 50dB				
	S = 1	S = 2	S=3	S = 4	
$L^{(S)}$	6.00	3.60	2.10	1.50	
$N^{(S)}$	100	208	208	208	
$Q^{(S)}$	100	144	49	25	
		SNR =	= 20 dB		
	S = 1	S=2	S=3	S = 4	
$L^{(S)}$	6.00	3.60	2.10	1.50	
$N^{(S)}$	100	208	208	208	
$Q^{(S)}$	100	144	49	25	
		SNR =	= 10 dB		
	S = 1	S=2	S=3	S = 4	
$L^{(S)}$	6.00	3.60	2.10	1.50	
$N^{(S)}$	100	208	208	208	
$Q^{(S)}$	100	144	49	25	
	SNR = 5dB				
	S = 1	S=2	S=3	S = 4	
$L^{(S)}$	6.00	3.60	2.10	1.50	
$N^{(S)}$	100	208	208	208	
$Q^{(S)}$	100	144	49	25	

Table X: Rhombus, $D = 1.5\lambda$, $\tau = 0.20$ - Investigation domain parameters: restricted investigation domain size $L^{(S)}$, total number of cells $N^{(S)}$ and number of cells within the restricted domain size $Q^{(S)}$.

1.1.6 Rhombus, $D = 1.5\lambda$, $\tau = 0.20$ - IMSA-BCS multi-resolution grids



Figure 6: Rhombus, $D = 1.5\lambda$, $\tau = 0.20$ - Example of IMSA-BCS multi-resolution grids for (a)(d) SNR = 20 [dB], (b)(e) SNR = 10 [dB] and (c)(f) SNR = 5 [dB] at the step (a)-(c) S = 1 and (d)-(f) S = 2, 3.4.



Figure 7: *Rhombus*, $D = 1.5\lambda$ - Reconstruction errors vs. τ : (a) total error, (b) internal error and (c) external error.



Figure 8: *Rhombus*, $D = 1.5\lambda$ - Reconstruction errors vs. *SNR*: (a) total error, (b) internal error and (c) external error.



Figure 9: Rhombus, $D = 1.5\lambda$ - Reconstruction errors vs. IMSA step, S: (a)(b) total error, (c)(d) internal error and (e)(f) external error for $(a)(c)(e) \tau = 0.1$ and $(b)(d)(f) \tau = 0.2$.



Figure 10: Rhombus, $D = 1.5\lambda$ - Reconstruction errors vs. IMSA step, S: (a)(b) total error, (c)(d) internal error and (e)(f) external error for (a)(c)(e) SNR = 10dB and (b)(d)(f) SNR = 5dB.

1.2 Porous Object

Test Case Description

Direct solver:

- Side of the investigation domain: $L = 6.0\lambda$
- Cubic domain divided in $\sqrt{D} \times \sqrt{D}$ cells
- Number of cells for the direct solver: D = 1600 (discretization = $\lambda/10$)

Investigation domain:

- Cubic domain divided in $\sqrt{N} \times \sqrt{N}$ cells
- Number of cells for the inversion:
 - First Step IMSA: $N^{(1)} = 100$ (discretization = $\lambda/10$)
 - Following Steps IMSA: $N^{(i)}$ not fixed, defined according to the estimated RoI $\mathcal{D}^{(i)}$

Measurement domain:

- Total number of measurements: M = 60
- Measurement points placed on circles of radius $\rho = 4.5\lambda$

Sources:

- Plane waves
- Number of views: V = 60; $\theta_{inc}^v = 0^\circ + (v 1) \times (360/V)$
- Amplitude: A = 1.0
- Frequency: $F = 300 \text{ MHz} (\lambda = 1)$

Background:

- $\varepsilon_r = 1.0$
- $\sigma = 0 [S/m]$

$\mathbf{S} \mathbf{c} \mathbf{a} \mathbf{t} \mathbf{t} \mathbf{e} \mathbf{r} \mathbf{e} \mathbf{r}$

- Porous object
- $\varepsilon_r \in \{1.01, 1.02, 1.04, 1.05, 1.06, 1.08, 1.10, 1.15, 1.20\}$
- $\sigma = 0 [S/m]$



Figure 11: Porous Object, $\tau = 0.02$ - (a) Actual profile and (b)-(o) IMSA-BCS reconstructed profiles for (b)(e)(h)(m) SNR = 20 [dB], (c)(f)(i)(n) SNR = 10 [dB] and (d)(g)(l)(o) SNR = 5 [dB] at the step (b)-(d) S = 1, (e)-(g) S = 2, (h)-(l) S = 3 and (m)-(o) S = 4.

	SNR = 50 dB					
	S = 1	S=2	S=3	S = 4		
ξ_{tot}	8.60×10^{-4}	3.52×10^{-4}	3.21×10^{-4}	3.21×10^{-4}		
ξ_{int}	1.27×10^{-2}	7.39×10^{-3}	7.20×10^{-3}	7.20×10^{-3}		
ξ_{ext}	4.88×10^{-4}	1.34×10^{-4}	1.08×10^{-4}	1.08×10^{-4}		
		SNR =	= 20 <i>dB</i>			
	S = 1	S=2	S = 3	S = 4		
ξ_{tot}	8.73×10^{-4}	3.73×10^{-4}	3.41×10^{-4}	3.41×10^{-4}		
ξ_{int}	1.29×10^{-2}	7.61×10^{-3}	$7.59 imes 10^{-3}$	7.59×10^{-3}		
ξ_{ext}	4.96×10^{-4}	1.49×10^{-4}	1.16×10^{-4}	1.16×10^{-4}		
5000						
		SNR =	= 10 <i>dB</i>			
	S = 1	SNR = S = 2	= 10dB $S = 3$	S = 4		
ξtot	S = 1 8.87 × 10 ⁻⁴	$SNR = $ $S = 2$ 3.79×10^{-4}	= $10dB$ S = 3 3.19×10^{-4}	S = 4 3.19×10^{-4}		
ξ_{tot} ξ_{int}	S = 1 8.87 × 10 ⁻⁴ 1.28 × 10 ⁻²	SNR = S = 2 3.79×10^{-4} 7.40×10^{-3}	= $10dB$ S = 3 3.19×10^{-4} 6.82×10^{-3}	S = 4 3.19×10^{-4} 6.82×10^{-3}		
$\frac{\xi_{tot}}{\xi_{int}}$	S = 1 8.87 × 10 ⁻⁴ 1.28 × 10 ⁻² 5.11 × 10 ⁻⁴	SNR = S = 2 3.79×10^{-4} 7.40×10^{-3} 1.61×10^{-4}	= $10dB$ S = 3 3.19×10^{-4} 6.82×10^{-3} 1.18×10^{-4}	S = 4 3.19 × 10 ⁻⁴ 6.82 × 10 ⁻³ 1.18 × 10 ⁻⁴		
$\frac{\xi_{tot}}{\xi_{int}}$	S = 1 8.87 × 10 ⁻⁴ 1.28 × 10 ⁻² 5.11 × 10 ⁻⁴	SNR = S = 2 3.79×10^{-4} 7.40×10^{-3} 1.61×10^{-4} SNR =	= $10dB$ S = 3 3.19×10^{-4} 6.82×10^{-3} 1.18×10^{-4} = $5dB$	S = 4 3.19 × 10 ⁻⁴ 6.82 × 10 ⁻³ 1.18 × 10 ⁻⁴		
$\frac{\xi_{tot}}{\xi_{int}}$	S = 1 8.87×10^{-4} 1.28×10^{-2} 5.11×10^{-4} S = 1	SNR = S = 2 3.79×10^{-4} 7.40×10^{-3} 1.61×10^{-4} SNR = S = 2	= $10dB$ S = 3 3.19×10^{-4} 6.82×10^{-3} 1.18×10^{-4} = $5dB$ S = 3	S = 4 3.19×10^{-4} 6.82×10^{-3} 1.18×10^{-4} S = 4		
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{tot}}$	S = 1 8.87×10^{-4} 1.28×10^{-2} 5.11×10^{-4} S = 1 9.44×10^{-4}	SNR = S = 2 3.79×10^{-4} 7.40×10^{-3} 1.61×10^{-4} SNR = S = 2 4.50×10^{-4}	= $10dB$ S = 3 3.19×10^{-4} 6.82×10^{-3} 1.18×10^{-4} = $5dB$ S = 3 3.40×10^{-4}	S = 4 3.19 × 10 ⁻⁴ 6.82 × 10 ⁻³ 1.18 × 10 ⁻⁴ $S = 4$ 2.89 × 10 ⁻⁴		
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{tot}}$	$S = 1$ 8.87×10^{-4} 1.28×10^{-2} 5.11×10^{-4} $S = 1$ 9.44×10^{-4} 1.30×10^{-2}	SNR = S = 2 3.79×10^{-4} 7.40×10^{-3} 1.61×10^{-4} SNR = SNR = 4.50×10^{-4} 8.14×10^{-3}	= 10dB S = 3 3.19×10^{-4} 6.82×10^{-3} 1.18×10^{-4} = 5dB S = 3 3.40×10^{-4} 6.39×10^{-3}	S = 4 3.19 × 10 ⁻⁴ 6.82 × 10 ⁻³ 1.18 × 10 ⁻⁴ $S = 4$ 2.89 × 10 ⁻⁴ 5.67 × 10 ⁻³		

Table XI: Porous Object, $\tau = 0.02$ - Reconstruction errors: total (ξ_{tot}) , internal (ξ_{int}) and external (ξ_{ext}) errors.

	SNR = 50dB					
	S = 1	S=2	S=3	S = 4		
$L^{(S)}$	6.00	1.50	1.50	1.50		
$N^{(S)}$	100	175	175	175		
$Q^{(S)}$	100	100	25	25		
		SNR =	= 20 dB			
	S = 1	S=2	S=3	S = 4		
$L^{(S)}$	6.00	1.50	1.50	1.50		
$N^{(S)}$	100	175	175	175		
$Q^{(S)}$	100	100	25	25		
		SNR =	= 10 dB			
	S = 1	S=2	S=3	S = 4		
$L^{(S)}$	6.00	1.50	1.50	1.50		
$N^{(S)}$	100	175	175	175		
$Q^{(S)}$	100	100	25	25		
		SNR = 5dB				
	S = 1	S=2	S=3	S = 4		
$L^{(S)}$	6.00	1.50	1.50	1.50		
$N^{(S)}$	100	208	208	208		
$Q^{(S)}$	100	144	36	25		

Table XII: Porous Object, $\tau = 0.02$ - Investigation domain parameters: restricted investigation domain size $L^{(S)}$, total number of cells $N^{(S)}$ and number of cells within the restricted domain size $Q^{(S)}$.



Figure 12: Porous Object, $\tau = 0.05$ - (a) Actual profile and (b)-(o) IMSA-BCS reconstructed profiles for (b)(e)(h)(m) SNR = 20 [dB], (c)(f)(i)(n) SNR = 10 [dB] and (d)(g)(l)(o) SNR = 5 [dB] at the step (b)-(d) S = 1, (e)-(g) S = 2, (h)-(l) S = 3 and (m)-(o) S = 4.

	SNR = 50dB				
	S = 1	S=2	S=3	S = 4	
ξ_{tot}	2.64×10^{-3}	1.08×10^{-3}	9.33×10^{-4}	8.96×10^{-4}	
ξ_{int}	2.94×10^{-2}	1.69×10^{-2}	1.57×10^{-2}	1.50×10^{-2}	
ξ_{ext}	1.76×10^{-3}	$5.81 imes 10^{-4}$	4.68×10^{-4}	4.49×10^{-4}	
		SNR =	= 20 <i>dB</i>		
	S = 1	S=2	S=3	S = 4	
ξ_{tot}	2.67×10^{-3}	1.07×10^{-3}	8.87×10^{-4}	8.87×10^{-4}	
ξ_{int}	2.95×10^{-2}	1.65×10^{-2}	1.47×10^{-2}	1.47×10^{-2}	
ξ_{ext}	$1.78 imes 10^{-3}$	5.81×10^{-4}	4.51×10^{-4}	4.51×10^{-4}	
		SNR =	= 10 <i>dB</i>		
	S = 1	SNR = S = 2	= 10dB $S = 3$	S = 4	
ξ _{tot}	S = 1 2.68 × 10 ⁻³	$SNR =$ $S = 2$ 1.13×10^{-3}	= $10dB$ S = 3 9.14×10^{-4}	S = 4 7.91 × 10 ⁻⁴	
ξ_{tot} ξ_{int}	S = 1 2.68 × 10 ⁻³ 2.91 × 10 ⁻²	SNR = 0 S = 2 1.13×10^{-3} 1.61×10^{-2}	= $10dB$ S = 3 9.14×10^{-4} 1.40×10^{-2}	S = 4 7.91 × 10 ⁻⁴ 1.26 × 10 ⁻²	
$\frac{\xi_{tot}}{\xi_{int}}$	S = 1 2.68 × 10 ⁻³ 2.91 × 10 ⁻² 1.78 × 10 ⁻³	SNR = S = 2 1.13×10^{-3} 1.61×10^{-2} 6.30×10^{-4}	= $10dB$ S = 3 9.14×10^{-4} 1.40×10^{-2} 4.89×10^{-4}	S = 4 7.91 × 10 ⁻⁴ 1.26 × 10 ⁻² 4.14 × 10 ⁻⁴	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{ext}}$	S = 1 2.68 × 10 ⁻³ 2.91 × 10 ⁻² 1.78 × 10 ⁻³	SNR = S = 2 1.13×10^{-3} 1.61×10^{-2} 6.30×10^{-4} SNR =	= $10dB$ S = 3 9.14×10^{-4} 1.40×10^{-2} 4.89×10^{-4} = $5dB$	S = 4 7.91 × 10 ⁻⁴ 1.26 × 10 ⁻² 4.14 × 10 ⁻⁴	
$\frac{\xi_{tot}}{\xi_{int}}$	S = 1 2.68 × 10 ⁻³ 2.91 × 10 ⁻² 1.78 × 10 ⁻³ S = 1	SNR = S = 2 1.13×10^{-3} 1.61×10^{-2} 6.30×10^{-4} SNR = S = 2	= $10dB$ S = 3 9.14×10^{-4} 1.40×10^{-2} 4.89×10^{-4} = $5dB$ S = 3	S = 4 7.91 × 10 ⁻⁴ 1.26 × 10 ⁻² 4.14 × 10 ⁻⁴ S = 4	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{tot}}$	S = 1 2.68 × 10 ⁻³ 2.91 × 10 ⁻² 1.78 × 10 ⁻³ $S = 1$ 2.84 × 10 ⁻³	SNR = S = 2 1.13×10^{-3} 1.61×10^{-2} 6.30×10^{-4} SNR = S = 2 1.42×10^{-3}	= $10dB$ S = 3 9.14×10^{-4} 1.40×10^{-2} 4.89×10^{-4} = $5dB$ S = 3 8.95×10^{-4}	S = 4 7.91 × 10 ⁻⁴ 1.26 × 10 ⁻² 4.14 × 10 ⁻⁴ $S = 4$ 8.95 × 10 ⁻⁴	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{tot}}$	S = 1 2.68 × 10 ⁻³ 2.91 × 10 ⁻² 1.78 × 10 ⁻³ $S = 1$ 2.84 × 10 ⁻³ 3.00 × 10 ⁻²	SNR = S = 2 1.13×10^{-3} 1.61×10^{-2} 6.30×10^{-4} SNR = SNR = 1.42×10^{-3} 2.00×10^{-2}	= 10dB S = 3 9.14×10^{-4} 1.40×10^{-2} 4.89×10^{-4} = 5dB S = 3 8.95×10^{-4} 1.25×10^{-2}	S = 4 7.91 × 10 ⁻⁴ 1.26 × 10 ⁻² 4.14 × 10 ⁻⁴ $S = 4$ 8.95 × 10 ⁻⁴ 1.25 × 10 ⁻²	

Table XIII: Porous Object, $\tau = 0.05$ - Reconstruction errors: total (ξ_{tot}) , internal (ξ_{int}) and external (ξ_{ext}) errors.

$\begin{array}{ c c c c c c } SNR = 50dB \\ \hline S = 1 & S = 2 & S = 3 & S = 4 \\ \hline L^{(S)} & 6.00 & 1.50 & 1.50 & 1.50 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline Q^{(S)} & 100 & 144 & 36 & 25 \\ \hline SNR = 20dB \\ \hline S = 1 & S = 2 & S = 3 & S = 4 \\ \hline L^{(S)} & 6.00 & 1.50 & 1.50 & 1.50 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline Q^{(S)} & 100 & 144 & 25 & 25 \\ \hline SNR = 10dB \\ \hline S = 1 & S = 2 & S = 3 & S = 4 \\ \hline L^{(S)} & 6.00 & 1.50 & 1.50 & 1.50 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline Q^{(S)} & 100 & 144 & 36 & 25 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline Q^{(S)} & 100 & 144 & 36 & 25 \\ \hline SNR = 5dB \\ \hline S = 1 & S = 2 & S = 3 & S = 4 \\ \hline L^{(S)} & 6.00 & 1.80 & 1.80 & 1.80 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline O^{(S)} & 100 & 208 & 208 & 208 \\ \hline O^{(S)} & 100 & 208 & 208 & 208 \\ \hline O^{(S)} & 100 & 208 & 208 & 208 \\ \hline O^{(S)} & 100 & 208 & 208 & 208 \\ \hline \end{array}$						
$S=1$ $S=2$ $S=3$ $S=4$ $L^{(S)}$ 6.00 1.50 1.50 1.50 $N^{(S)}$ 100 208 208 208 $Q^{(S)}$ 100 144 36 25 $S=1$ $S=2$ $S=3$ $S=4$ $L^{(S)}$ 6.00 1.50 1.50 1.50 $N^{(S)}$ 100 208 208 208 $Q^{(S)}$ 100 208 208 208 $Q^{(S)}$ 100 208 208 208 $Q^{(S)}$ 100 144 25 25 $S=1$ $S=2$ $S=3$ $S=4$ $L^{(S)}$ 6.00 1.50 1.50 1.50 $N^{(S)}$ 100 208 208 208 $Q^{(S)}$ 100 144 36 25 $S=1$ $S=2$ $S=3$ $S=4$ $L^{(S)}$ 6.00 1.80 <t< th=""><th></th><th colspan="4">SNR = 50 dB</th></t<>		SNR = 50 dB				
$\begin{array}{c c c c c c c c c } L^{(S)} & 6.00 & 1.50 & 1.50 & 1.50 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline Q^{(S)} & 100 & 144 & 36 & 25 \\ \hline & SNR = 20dB \\ \hline & S=1 & S=2 & S=3 & S=4 \\ \hline L^{(S)} & 6.00 & 1.50 & 1.50 & 1.50 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline & SNR = 10dB \\ \hline & S=1 & S=2 & S=3 & S=4 \\ \hline L^{(S)} & 6.00 & 1.50 & 1.50 & 1.50 \\ \hline & SSNR = 10dB \\ \hline & S=1 & S=2 & S=3 & S=4 \\ \hline L^{(S)} & 6.00 & 1.50 & 1.50 & 1.50 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline & SNR = 5dB \\ \hline & S=1 & S=2 & S=3 & S=4 \\ \hline L^{(S)} & 6.00 & 1.80 & 1.80 & 1.80 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline & N^{(S)} & 100 & 208 & 208 & 208 \\ \hline & N^{(S)} & 100 & 208 & 208 & 208 \\ \hline & N^{(S)} & 100 & 144 & 36 & 25 \\ \hline \end{array}$		S = 1	S=2	S=3	S = 4	
$N^{(S)}$ 100 208 208 208 $Q^{(S)}$ 100 144 36 25 $S=1$ $S=2$ $S=3$ $S=4$ $L^{(S)}$ 6.00 1.50 1.50 1.50 $N^{(S)}$ 100 208 208 208 $Q^{(S)}$ 100 208 208 208 $S=1$ $S=2$ $S=3$ $S=4$ $L^{(S)}$ 6.00 1.50 1.50 1.50 $N^{(S)}$ 100 208 208 208 $Q^{(S)}$ 100 144 36 25 $S=1$ $S=2$ $S=3$ $S=4$ $L^{(S)}$ 6.00 1.80 1.80 1.80 $N^{(S)}$ 100 208 208 208	$L^{(S)}$	6.00	1.50	1.50	1.50	
$Q^{(S)}$ 100 144 36 25 $S = 1$ $S = 2$ $S = 3$ $S = 4$ $L^{(S)}$ 6.00 1.50 1.50 1.50 $N^{(S)}$ 100 208 208 208 $Q^{(S)}$ 100 144 25 25 $Q^{(S)}$ 100 144 25 25 $S = 1$ $S = 2$ $S = 3$ $S = 4$ $L^{(S)}$ 6.00 1.50 1.50 1.50 $N^{(S)}$ 100 208 208 208 $Q^{(S)}$ 100 144 36 25 $S = 1$ $S = 2$ $S = 3$ $S = 4$ $L^{(S)}$ 6.00 1.80 1.80 1.80 $N^{(S)}$ 100 208 208 208	$N^{(S)}$	100	208	208	208	
$\begin{array}{ c c c c c c } SNR = 20dB \\ \hline S = 1 & S = 2 & S = 3 & S = 4 \\ \hline L^{(S)} & 6.00 & 1.50 & 1.50 & 1.50 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline Q^{(S)} & 100 & 144 & 25 & 25 \\ \hline SNR = 10dB \\ \hline S = 1 & S = 2 & S = 3 & S = 4 \\ \hline L^{(S)} & 6.00 & 1.50 & 1.50 & 1.50 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline Q^{(S)} & 100 & 144 & 36 & 25 \\ \hline SNR = 5dB \\ \hline S = 1 & S = 2 & S = 3 & S = 4 \\ \hline L^{(S)} & 6.00 & 1.80 & 1.80 & 1.80 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline S = 1 & S = 2 & S = 3 & S = 4 \\ \hline L^{(S)} & 6.00 & 1.80 & 1.80 & 1.80 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline \end{array}$	$Q^{(S)}$	100	144	36	25	
$\begin{array}{ c c c c c c } S = 1 & S = 2 & S = 3 & S = 4 \\ \hline L^{(S)} & 6.00 & 1.50 & 1.50 & 1.50 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline Q^{(S)} & 100 & 144 & 25 & 25 \\ \hline & S = 1 & S = 2 & S = 3 & S = 4 \\ \hline L^{(S)} & 6.00 & 1.50 & 1.50 & 1.50 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline Q^{(S)} & 100 & 144 & 36 & 25 \\ \hline & S = 1 & S = 2 & S = 3 & S = 4 \\ \hline L^{(S)} & 6.00 & 1.80 & 1.80 & 1.80 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline N^{(S)} & 100 & 208 & 1.80 & 1.80 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline \end{array}$			SNR =	= 20 dB		
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		S = 1	S=2	S=3	S = 4	
$\begin{array}{c c c c c c c } \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline Q^{(S)} & 100 & 144 & 25 & 25 \\ \hline Q^{(S)} & 100 & 144 & 25 & 5-4 \\ \hline S=1 & S=2 & S=3 & S=4 \\ \hline L^{(S)} & 6.00 & 1.50 & 1.50 & 1.50 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline Q^{(S)} & 100 & 144 & 36 & 25 \\ \hline S=1 & S=2 & S=3 & S=4 \\ \hline L^{(S)} & 6.00 & 1.80 & 1.80 & 1.80 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline N^{(S)} & 100 & 144 & 26 & 26 \\ \hline \end{array}$	$L^{(S)}$	6.00	1.50	1.50	1.50	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$N^{(S)}$	100	208	208	208	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$Q^{(S)}$	100	144	25	25	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $			SNR =	= 10 dB		
$\begin{array}{c ccccc} L^{(S)} & 6.00 & 1.50 & 1.50 & 1.50 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline Q^{(S)} & 100 & 144 & 36 & 25 \\ \hline & SSR = 5 dB \\ \hline S = 1 & S = 2 & S = 3 & S = 4 \\ \hline L^{(S)} & 6.00 & 1.80 & 1.80 & 1.80 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline Q^{(S)} & 100 & 144 & 26 & 26 \\ \hline \end{array}$		S = 1	S=2	S=3	S = 4	
$\begin{array}{c cccc} N^{(S)} & 100 & 208 & 208 & 208 \\ \hline Q^{(S)} & 100 & 144 & 36 & 25 \\ \hline & SNR = 5dB \\ \hline & S = 1 & S = 2 & S = 3 & S = 4 \\ \hline L^{(S)} & 6.00 & 1.80 & 1.80 & 1.80 \\ \hline N^{(S)} & 100 & 208 & 208 & 208 \\ \hline O^{(S)} & 100 & 144 & 26 & 26 \end{array}$	$L^{(S)}$	6.00	1.50	1.50	1.50	
$Q^{(S)}$ 100 144 36 25 SNR = 5dB $S = 1$ $S = 2$ $S = 3$ $S = 4$ $L^{(S)}$ 6.00 1.80 1.80 1.80 $N^{(S)}$ 100 208 208 208	$N^{(S)}$	100	208	208	208	
$SNR = 5dB$ $S = 1 S = 2 S = 3 S = 4$ $L^{(S)} 6.00 1.80 1.80 1.80$ $N^{(S)} 100 208 208 208$ $O^{(S)} 100 144 26 26$	$Q^{(S)}$	100	144	36	25	
$S=1$ $S=2$ $S=3$ $S=4$ $L^{(S)}$ 6.00 1.80 1.80 1.80 $N^{(S)}$ 100 208 208 208 $O^{(S)}$ 100 144 26 26		SNR = 5dB				
$L^{(S)}$ 6.00 1.80 1.80 1.80 $N^{(S)}$ 100 208 208 208 $O^{(S)}$ 100 144 26 26		S = 1	S=2	S=3	S = 4	
$N^{(S)}$ 100 208 208 208 208 $O^{(S)}$ 100 144 26 26	$L^{(S)}$	6.00	1.80	1.80	1.80	
O(S) 100 144 26 26	$N^{(S)}$	100	208	208	208	
$Q^{(-)}$ 100 144 30 30	$Q^{(S)}$	100	144	36	36	

Table XIV: Porous Object, $\tau = 0.05$ - Investigation domain parameters: restricted investigation domain size $L^{(S)}$, total number of cells $N^{(S)}$ and number of cells within the restricted domain size $Q^{(S)}$.



Figure 13: Porous Object, $\tau = 0.10$ - (a) Actual profile and (b)-(o) IMSA-BCS reconstructed profiles for (b)(e)(h) SNR = 20 [dB], (c)(f)(i) SNR = 10 [dB] and (d)(g)(l) SNR = 5 [dB] at the step (b)-(d) S = 1, (e)-(g) S = 2, and (h)-(l) S = 3.

	SNR = 50dB				
	S = 1	S=2	S=3	S = 4	
ξ_{tot}	8.27×10^{-3}	4.09×10^{-3}	3.13×10^{-3}	3.13×10^{-3}	
ξ_{int}	8.08×10^{-2}	5.31×10^{-2}	4.29×10^{-2}	4.29×10^{-2}	
ξ_{ext}	$5.61 imes 10^{-3}$	2.29×10^{-3}	1.69×10^{-3}	1.69×10^{-3}	
		SNR =	= 20 <i>dB</i>		
	S = 1	S=2	S = 3	S = 4	
ξ_{tot}	8.41×10^{-3}	4.13×10^{-3}	2.98×10^{-3}	2.98×10^{-3}	
ξ_{int}	7.98×10^{-2}	5.26×10^{-2}	3.83×10^{-2}	3.83×10^{-2}	
ξ_{ext}	5.62×10^{-3}	2.35×10^{-3}	$1.65 imes 10^{-3}$	1.65×10^{-3}	
-					
		SNR =	= 10 <i>dB</i>		
	S = 1	SNR = S = 2	= 10dB $S = 3$	S = 4	
ξtot	S = 1 8.91 × 10 ⁻³	$SNR = $ $S = 2$ 4.32×10^{-3}	= $10dB$ S = 3 3.06×10^{-3}	$S = 4$ 2.43×10^{-3}	
ξ_{tot} ξ_{int}	S = 1 8.91 × 10 ⁻³ 8.35 × 10 ⁻²	SNR = 0 S = 2 4.32×10^{-3} 5.38×10^{-2}	= $10dB$ S = 3 3.06×10^{-3} 4.04×10^{-2}	S = 4 2.43 × 10 ⁻³ 2.90 × 10 ⁻²	
$\frac{\xi_{tot}}{\xi_{int}}$	S = 1 8.91 × 10 ⁻³ 8.35 × 10 ⁻² 6.02 × 10 ⁻³	SNR = S = 2 4.32×10^{-3} 5.38×10^{-2} 2.40×10^{-3}	= $10dB$ S = 3 3.06×10^{-3} 4.04×10^{-2} 1.67×10^{-3}	S = 4 2.43 × 10 ⁻³ 2.90 × 10 ⁻² 1.25 × 10 ⁻³	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{ext}}$	S = 1 8.91 × 10 ⁻³ 8.35 × 10 ⁻² 6.02 × 10 ⁻³	SNR = S = 2 4.32×10^{-3} 5.38×10^{-2} 2.40×10^{-3} SNR =	= $10dB$ S = 3 3.06×10^{-3} 4.04×10^{-2} 1.67×10^{-3} = $5dB$	S = 4 2.43 × 10 ⁻³ 2.90 × 10 ⁻² 1.25 × 10 ⁻³	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{ext}}$	S = 1 8.91×10^{-3} 8.35×10^{-2} 6.02×10^{-3} S = 1	SNR = S = 2 4.32×10^{-3} 5.38×10^{-2} 2.40×10^{-3} SNR = S = 2	= $10dB$ S = 3 3.06×10^{-3} 4.04×10^{-2} 1.67×10^{-3} = $5dB$ S = 3	S = 4 2.43 × 10 ⁻³ 2.90 × 10 ⁻² 1.25 × 10 ⁻³ S = 4	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{tot}}$	S = 1 8.91 × 10 ⁻³ 8.35 × 10 ⁻² 6.02 × 10 ⁻³ $S = 1$ 9.61 × 10 ⁻³	SNR = S = 2 4.32×10^{-3} 5.38×10^{-2} 2.40×10^{-3} SNR = S = 2 5.32×10^{-3}	= $10dB$ S = 3 3.06×10^{-3} 4.04×10^{-2} 1.67×10^{-3} = $5dB$ S = 3 4.01×10^{-3}	S = 4 2.43 × 10 ⁻³ 2.90 × 10 ⁻² 1.25 × 10 ⁻³ $S = 4$ 3.14 × 10 ⁻³	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{tot}}$	S = 1 8.91×10^{-3} 8.35×10^{-2} 6.02×10^{-3} S = 1 9.61×10^{-3} 8.48×10^{-2}	SNR = S = 2 4.32×10^{-3} 5.38×10^{-2} 2.40×10^{-3} SNR = S = 2 5.32×10^{-3} 6.20×10^{-2}	= $10dB$ S = 3 3.06×10^{-3} 4.04×10^{-2} 1.67×10^{-3} = $5dB$ S = 3 4.01×10^{-3} 4.96×10^{-2}	S = 4 2.43 × 10 ⁻³ 2.90 × 10 ⁻² 1.25 × 10 ⁻³ $S = 4$ 3.14 × 10 ⁻³ 3.43 × 10 ⁻²	

Table XV: Porous Object, $\tau = 0.15$ - Reconstruction errors: total (ξ_{tot}) , internal (ξ_{int}) and external (ξ_{ext}) errors.

	SNR = 50dB			
	S = 1	S=2	S=3	S = 4
$L^{(S)}$	6.00	1.80	1.80	1.80
$N^{(S)}$	100	208	208	208
$Q^{(S)}$	100	144	36	36
		SNR =	= 20 dB	
	S = 1	S=2	S=3	S = 4
$L^{(S)}$	6.00	1.80	1.80	1.80
$N^{(S)}$	100	208	208	208
$Q^{(S)}$	100	144	36	36
		SNR =	= 10 dB	
	S = 1	S=2	S=3	S = 4
$L^{(S)}$	6.00	1.50	1.50	1.50
$N^{(S)}$	100	208	208	208
$Q^{(S)}$	100	144	36	25
	SNR = 5dB			
	S = 1	S=2	S=3	S = 4
$L^{(S)}$	6.00	1.80	1.80	1.80
$N^{(S)}$	100	208	208	208
$Q^{(S)}$	100	144	49	36

Table XVI: Porous Object, $\tau = 0.05$ - Investigation domain parameters: restricted investigation domain size $L^{(S)}$, total number of cells $N^{(S)}$ and number of cells within the restricted domain size $Q^{(S)}$.



Figure 14: Porous Object, $\tau = 0.20$ - (a) Actual profile and (b)-(o) IMSA-BCS reconstructed profiles for (b)(e)(h) SNR = 20 [dB], (c)(f)(i) SNR = 10 [dB] and (d)(g)(l) SNR = 5 [dB] at the step (b)-(d) S = 1, (e)-(g) S = 2, and (h)-(l) S = 3.

	SNR = 50dB				
	S = 1	S=2	S=3	S = 4	
ξ_{tot}	1.10×10^{-2}	6.19×10^{-3}	4.63×10^{-3}	4.63×10^{-3}	
ξ_{int}	1.02×10^{-1}	7.96×10^{-2}	6.28×10^{-2}	6.28×10^{-2}	
ξ_{ext}	7.43×10^{-3}	3.44×10^{-3}	2.39×10^{-3}	2.39×10^{-3}	
		SNR =	= 20 <i>dB</i>		
	S = 1	S=2	S=3	S = 4	
ξ_{tot}	1.11×10^{-2}	5.86×10^{-3}	4.51×10^{-3}	4.51×10^{-3}	
ξ_{int}	1.05×10^{-1}	7.45×10^{-2}	6.11×10^{-2}	6.11×10^{-2}	
ξ_{ext}	$7.50 imes 10^{-3}$	3.26×10^{-3}	$2.29 imes 10^{-3}$	2.29×10^{-3}	
-					
		SNR =	= 10 <i>dB</i>		
	S = 1	SNR = S = 2	= 10dB $S = 3$	S = 4	
ξtot	S = 1 1.20×10^{-2}	$SNR = $ $S = 2$ 6.26×10^{-3}	= $10dB$ S = 3 4.23×10^{-3}	$S = 4$ 4.23×10^{-3}	
ξ_{tot} ξ_{int}	S = 1 1.20×10^{-2} 1.09×10^{-1}	SNR = 0 S = 2 6.26×10^{-3} 7.70×10^{-2}	= $10dB$ S = 3 4.23×10^{-3} 5.01×10^{-2}	S = 4 4.23×10^{-3} 5.01×10^{-2}	
$\frac{\xi_{tot}}{\xi_{int}}$	S = 1 1.20×10^{-2} 1.09×10^{-1} 7.99×10^{-3}	SNR = S = 2 6.26×10^{-3} 7.70×10^{-2} 3.52×10^{-3}	= $10dB$ S = 3 4.23×10^{-3} 5.01×10^{-2} 2.22×10^{-3}	S = 4 4.23 × 10 ⁻³ 5.01 × 10 ⁻² 2.22 × 10 ⁻³	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{ext}}$	$S = 1$ 1.20×10^{-2} 1.09×10^{-1} 7.99×10^{-3}	SNR = S = 2 6.26×10^{-3} 7.70×10^{-2} 3.52×10^{-3} SNR =	= $10dB$ S = 3 4.23×10^{-3} 5.01×10^{-2} 2.22×10^{-3} = $5dB$	$S = 4$ 4.23×10^{-3} 5.01×10^{-2} 2.22×10^{-3}	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{ext}}$	S = 1 1.20 × 10 ⁻² 1.09 × 10 ⁻¹ 7.99 × 10 ⁻³ S = 1	SNR = S = 2 6.26×10^{-3} 7.70×10^{-2} 3.52×10^{-3} SNR = S = 2	= $10dB$ S = 3 4.23×10^{-3} 5.01×10^{-2} 2.22×10^{-3} = $5dB$ S = 3	S = 4 4.23×10^{-3} 5.01×10^{-2} 2.22×10^{-3} S = 4	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{tot}}$	$S = 1$ 1.20×10^{-2} 1.09×10^{-1} 7.99×10^{-3} $S = 1$ 1.39×10^{-2}	SNR = S = 2 6.26×10^{-3} 7.70×10^{-2} 3.52×10^{-3} SNR = S = 2 8.49×10^{-3}	= $10dB$ S = 3 4.23×10^{-3} 5.01×10^{-2} 2.22×10^{-3} = $5dB$ S = 3 6.74×10^{-3}	$S = 4$ 4.23×10^{-3} 5.01×10^{-2} 2.22×10^{-3} $S = 4$ 5.19×10^{-3}	
$\frac{\xi_{tot}}{\xi_{int}}$ $\frac{\xi_{ext}}{\xi_{tot}}$	$S = 1$ 1.20×10^{-2} 1.09×10^{-1} 7.99×10^{-3} $S = 1$ 1.39×10^{-2} 1.12×10^{-1}	SNR = S = 2 6.26×10^{-3} 7.70×10^{-2} 3.52×10^{-3} SNR = S = 2 8.49×10^{-3} 8.62×10^{-2}	= $10dB$ S = 3 4.23×10^{-3} 5.01×10^{-2} 2.22×10^{-3} = $5dB$ S = 3 6.74×10^{-3} 8.10×10^{-2}	$S = 4$ 4.23×10^{-3} 5.01×10^{-2} 2.22×10^{-3} $S = 4$ 5.19×10^{-3} 5.12×10^{-2}	

Table XVII: Porous Object, $\tau = 0.20$ - Reconstruction errors: total (ξ_{tot}) , internal (ξ_{int}) and external (ξ_{ext}) errors.

	SNR = 50 dB				
	S = 1	S = 2	S=3	S = 4	
$L^{(S)}$	6.00	1.80	1.80	1.80	
$N^{(S)}$	100	208	208	208	
$Q^{(S)}$	100	144	36	36	
		SNR =	= 20 dB		
	S = 1	S=2	S=3	S = 4	
$L^{(S)}$	6.00	1.80	1.80	1.80	
$N^{(S)}$	100	208	208	208	
$Q^{(S)}$	100	144	36	36	
		SNR =	= 10 dB		
	S = 1	S=2	S=3	S = 4	
$L^{(S)}$	6.00	1.80	1.80	1.80	
$N^{(S)}$	100	208	208	208	
$Q^{(S)}$	100	144	36	36	
	SNR = 5dB				
	S = 1	S=2	S=3	S = 4	
$L^{(S)}$	6.00	1.80	1.80	1.80	
$N^{(S)}$	100	247	247	247	
$Q^{(S)}$	100	196	64	36	

Table XVIII: Porous Object, $\tau = 0.05$ - Investigation domain parameters: restricted investigation domain size $L^{(S)}$, total number of cells $N^{(S)}$ and number of cells within the restricted domain size $Q^{(S)}$.

1.2.5 Porous Object, $\ell = 1.5\lambda$, $\tau = 0.20$ - IMSA-BCS multi-resolution grids



Figure 15: Porous Object, $\tau = 0.20$ - Example of IMSA-BCS multi-resolution grids for (a)(d) SNR = 20 [dB], (b)(e) SNR = 10 [dB] and (c)(f) SNR = 5 [dB] at the step (a)-(c) S = 1 and (d)-(f) S = 2, 3 4.



Figure 16: Porous Object - Reconstruction errors vs. IMSA step, S: (a)(b) total error, (c)(d) internal error and (e)(f) external error for $(a)(c)(e) \tau = 0.1$ and $(b)(d)(f) \tau = 0.2$.



Figure 17: Porous Object - Reconstruction errors vs. IMSA step, S: (a)(b) total error, (c)(d) internal error and (e)(f) external error for (a)(c)(e) SNR = 10dB and (b)(d)(f) SNR = 5dB.

More information on the topics of this document can be found in the following list of references.

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