A Multi-Resolution Approach for *BCS*-Based Imaging of Sparse Scatterers

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Abstract

In this work, a novel Bayesian compressive sensing (*BCS*)-based microwave imaging method is proposed. The developed technique suitably combines the regularization properties of *CS* techniques with those of the iterative multi-scale approach (*IMSA*), in order to exploit the progressively acquired information on the scatterer location and size and improve the overall accuracy of the retrieved images. Toward this end, an innovative information-driven relevance vector machine (*RVM*) has been developed. Some preliminary results are shown to verify the effectiveness of the proposed *IMSA-BCS* strategy.

1 Mathematical Formulation

Let us consider an inaccessible investigation domain Λ irradiated by a set of incident transverse-magnetic planes $E_{inc}^{v}\left(\mathbf{r}^{v}\right)$, $v=1,...,V$, impinging from the angular directions $\theta^{v}=\frac{2\pi}{V}\left(v-1\right)$, being V the number of views. In this working scenario, the scattered field $E_{scatt}^{v}\left(\mathbf{r}_{s}^{v}\right)$, $s=1,...,S$, is supposed to be measured through a set of S sensors equally displaced on a circular observation domain Θ, external to the investigation domain ($\Lambda\cap\Theta=0$), having radius ρ. The exact location of the sensors are identified by the position vector $\mathbf{r}_{s}^{v}=\left(\rho\cos\theta_{s}^{v}\sin\theta_{s}^{v}\right)$, being $\theta_{s}^{v}=\theta^{v}+\frac{2\pi}{S}\left(s-1\right)$.

This scattered field is known to be dependent on the equivalent currents $J_{eq}^{v}\left(\mathbf{r}\right)$ generated in the support of the unknown scatterers placed into the domain Λ, according to the *data equation*

$$E_{scatt}^{v}\left(\mathbf{r}_{s}^{v}\right)=-k_{0}^{2}\int_{\Lambda}J_{eq}^{v}\left(\mathbf{r}'\right)G\left(\mathbf{r}_{s}^{v}/\mathbf{r}'\right)\tag{1}$$

where $G\left(\mathbf{r}_{s}^{v}/\mathbf{r}'\right)$ is the Green's function in the free space and $k_{0}=\omega\sqrt{\varepsilon_{0}\mu_{0}}$. The material properties of the investigation domain Λ in terms of relative dielectric permittivity $\varepsilon_{r}\left(\mathbf{r}\right)$ and electric conductivity $\sigma\left(\mathbf{r}\right)$ are described by means of the object function

$$\tau\left(\mathbf{r}\right)=\varepsilon_{r}\left(\mathbf{r}\right)-\varepsilon_{0}-\frac{\sigma\left(\mathbf{r}\right)}{2\pi f\varepsilon_{0}}\tag{2}$$

f being the frequency of the TM plane wave.

In order to numerically deal with (1), the investigation domain is discretized into N sub-domains (cells), providing the matrix form of

$$\mathbf{E}^{v}=\mathbf{G}\mathbf{J}_{eq}^{v}+\mathbf{N}^{v}\tag{3}$$

$\mathbf{G}$ being the Green's matrix and $\mathbf{N}^{v}$ a zero mean additive Gaussian noise vector of variance σ^{2}. The dielectric features of the N sub-domains described through the discretized form of the object function τ are then retrieved through the following iterative strategy which combines a multi-resolution approach and the BCS method, aimed to maximize the a-posteriori probability of the equivalent sources given the scattered field as:

$$\widehat{\mathbf{J}}_{eq}^{v}=\arg\left\{\max\left[\mathcal{P}\left(\mathbf{J}_{eq}^{v}\middle|\,\mathbf{E}_{scatt}^{v}\right)\right]\right\},\qquad v=1,...,V\tag{4}$$

More in detail, the algorithms works as follows:

1. *Initialization*: Definition of input parameters of the BCS problem, namely the initial estimation of the noise on the scattered data, σ_{init}^{2}, the convergence parameter, γ, and the parameter related to the stopping criterion of the $IMSA$, χ. Set the region of interest equal to the whole domain $\mathcal{D}^{(1)}=\Lambda$;

2. *BCS inversion via "Constrained-RVM"*:

   (a) increase of the iteration index: $i=i+1$;

   (b) solution of the BCS problem within the Region of Interest (*RoI*) $\mathcal{D}^{(i-1)}$ defined at the $(i-1)$-th step, by maximizing the following cost function:

$$\ell\left(\mathbf{a}^{v}\right)=-0.5\left[2S\log\left(2\pi\right)+\log\left(\mathbf{C}\right)+\left(\mathbf{E}_{scatt}^{v}\right)^{T}\mathbf{C}^{-1}\left(\mathbf{E}_{scatt}^{v}\right)\right],\qquad v=1,...,V \tag{5}$$

where $\mathbf{C}=\sigma^{2}\mathbf{I}+\mathbf{G}\left[diag\left(\mathbf{a}^{v}\right)\right]^{-1}\mathbf{G}^{T}$ and being $\mathbf{a}^{v}$ the hyperparameter vector whose entries corresponding to the cells out of the *RoI* $\mathcal{D}^{(i-1)}$ are forced to ∞;

3. *Equivalent Current Retrieval*:

   Computation of the equivalent currents starting from the hyperparameter vector $\mathbf{a}^{v}$ according to:

$$\mathbf{J}_{eq}^{v}=\frac{1}{\sigma^{2}}\left[\frac{\mathbf{G}^{T}\mathbf{G}}{\sigma^{2}}diag\left(\mathbf{a}^{v}\right)\right]^{-1}\mathbf{G}^{T}\mathbf{E}_{scatt}^{v},\qquad v=1,...,V \tag{6}$$

4. *Features' Retrieval*:

   Reconstruction of the material properties of the investigation domain taking advantage from the first order Born approximation through

$$\tau\left(\mathbf{r}_{n}^{(i)}\right)=\frac{1}{V}\sum\frac{\mathbf{J}_{eq}^{v}\left(\mathbf{r}_{n}^{(i)}\right)}{\mathbf{E}_{inc}^{v}\left(\mathbf{r}_{n}^{(i)}\right)},\qquad n=1,...,N \tag{7}$$

   being $\mathbf{r}_{n}^{(i)}$ the barycenter of the n-th cell within the *RoI* $\mathcal{D}^{(i-1)}$;

5. *Convergence Check*:

   Definition of the new *RoI* $\mathcal{D}^{(i)}$ according to the contrast function distribution and evaluation of the following termination condition:

$$\left(\frac{L^{(i-1)}-L^{(i)}}{L^{(i)}}\right)<\chi \tag{8}$$

   being $L^{(i)}$ the side of the *RoI* $\mathcal{D}^{(i)}$. If such a condition is met, then stop the iterative process, otherwise go to step 2.

2 Preliminary Numerical Assessment

2.1 L-shaped Object, $\ell = 1.5\lambda$

Test Case Description

Direct solver:

- Side of the investigation domain: $L = 6.0\lambda$
- Cubic domain divided in $\sqrt{D} \times \sqrt{D}$ cells
- Number of cells for the direct solver: $D = 1600$ (discretization $= \lambda/10$)

Investigation domain:

- Cubic domain divided in $\sqrt{N} \times \sqrt{N}$ cells
- Number of cells for the inversion:
  - First Step IMSA: $N^{(1)} = 100$ (discretization $= \lambda/10$)
  - Following Steps IMSA: $N^{(i)}$ not fixed, defined according to the estimated *RoI* $\mathcal{D}^{(i)}$

Measurement domain:

- Total number of measurements: $M = 60$
- Measurement points placed on circles of radius $\rho = 4.5\lambda$

Sources:

- Plane waves
- Number of views: $V = 60$; $\theta^{v}_{inc} = 0° + (v - 1) \times (360/V)$
- Amplitude: $A = 1.0$
- Frequency: $F = 300$ MHz ($\lambda = 1$)

Background:

- $\varepsilon_r = 1.0$
- $\sigma = 0$ [S/m]

Scatterer

- L-shaped object, $\ell = 1.5\lambda$
- $\varepsilon_r \in \{1.01, 1.02, 1.04, 1.05, 1.06, 1.08, 1.10, 1.15, 1.20\}$
- $\sigma = 0$ [S/m]

2.1.1 L-shaped Object, $\ell = 1.5\lambda$, $\tau = 0.02$ - IMSA-BCS reconstructed profiles

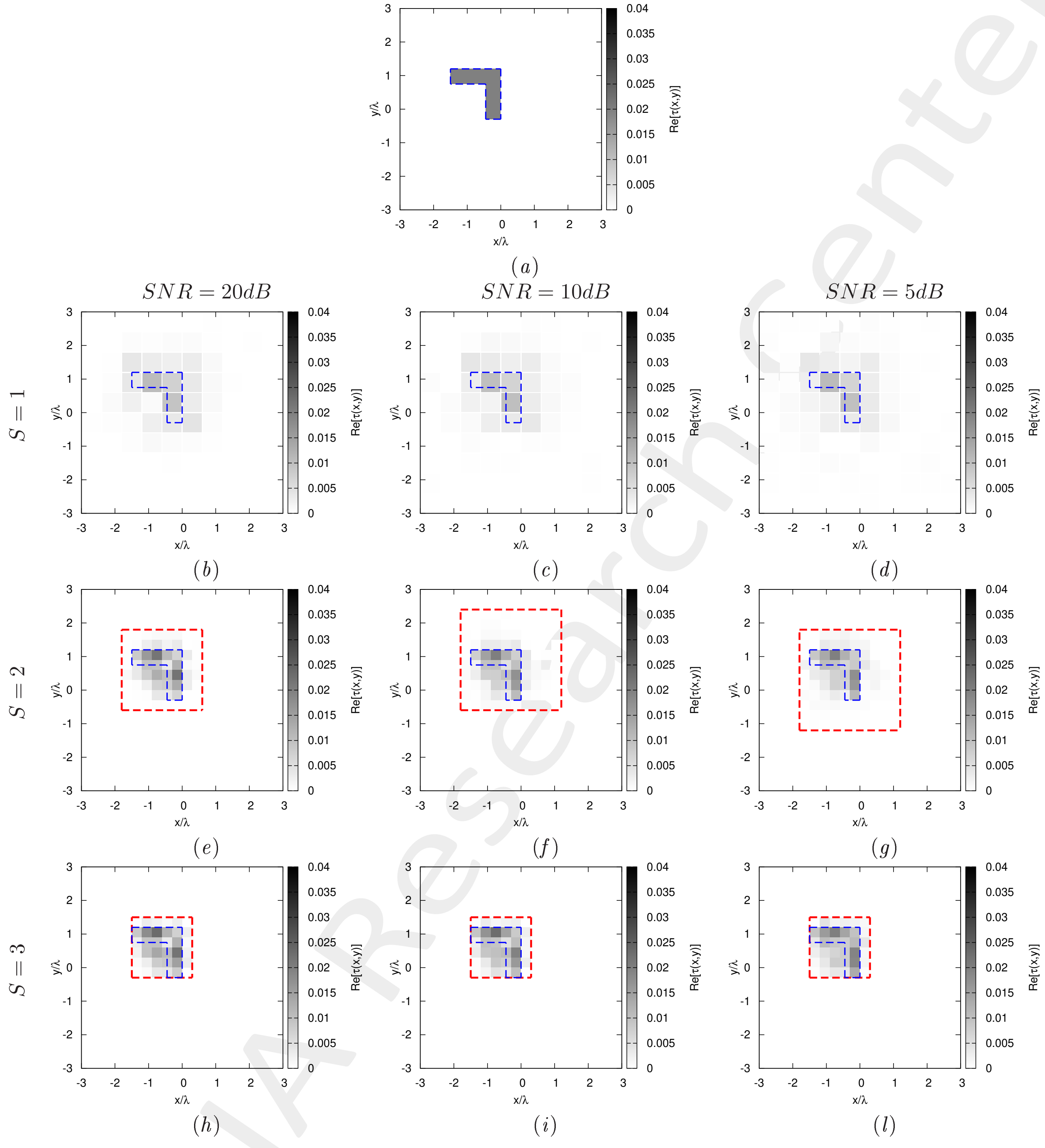


Figure 1: *L-shaped Object*, $\ell = 1.5\lambda$, $\tau = 0.02$ - (a) Actual profile and (b)-(o) IMSA-BCS reconstructed profiles for $(b)(e)(h)$ $SNR = 20$ [dB], $(c)(f)(i)$ $SNR = 10$ [dB] and $(d)(g)(l)$ $SNR = 5$ [dB] at the step (b)-(d) $S = 1$, (e)-(g) $S = 2$, and (h)-(l) $S = 3$.

	$SNR = 50dB$			
	$S = 1$	$S = 2$	$S = 3$	$S = 4$
ξ_{tot}	9.01×10^{-4}	5.09×10^{-4}	5.64×10^{-4}	5.64×10^{-4}
ξ_{int}	1.20×10^{-2}	8.97×10^{-3}	1.04×10^{-2}	1.04×10^{-2}
ξ_{ext}	5.30×10^{-4}	2.29×10^{-4}	2.39×10^{-4}	2.39×10^{-4}
	$SNR = 20dB$			
	$S = 1$	$S = 2$	$S = 3$	$S = 4$
ξ_{tot}	9.09×10^{-4}	5.21×10^{-4}	4.85×10^{-4}	5.72×10^{-4}
ξ_{int}	1.21×10^{-2}	9.18×10^{-3}	8.73×10^{-3}	1.04×10^{-2}
ξ_{ext}	5.34×10^{-4}	2.35×10^{-4}	2.13×10^{-4}	2.47×10^{-4}
	$SNR = 10dB$			
	$S = 1$	$S = 2$	$S = 3$	$S = 4$
ξ_{tot}	9.38×10^{-4}	5.18×10^{-4}	4.69×10^{-4}	5.42×10^{-4}
ξ_{int}	1.22×10^{-2}	8.85×10^{-3}	8.26×10^{-3}	1.01×10^{-2}
ξ_{ext}	5.56×10^{-4}	2.42×10^{-4}	2.12×10^{-4}	2.24×10^{-4}
	$SNR = 5dB$			
	$S = 1$	$S = 2$	$S = 3$	$S = 4$
ξ_{tot}	9.73×10^{-4}	5.31×10^{-4}	4.34×10^{-4}	4.34×10^{-4}
ξ_{int}	1.22×10^{-2}	8.84×10^{-3}	7.44×10^{-3}	7.44×10^{-3}
ξ_{ext}	5.85×10^{-4}	2.50×10^{-4}	2.01×10^{-4}	2.01×10^{-4}

Table I: *L-shaped Object*, $\ell = 1.5\lambda$, $\tau = 0.02$ - Reconstruction errors: total (ξ_{tot}), internal (ξ_{int}) and external (ξ_{ext}) errors.

	$SNR = 50dB$			
	$S = 1$	$S = 2$	$S = 3$	$S = 4$
$L^{(S)}$	6.00	1.50	1.50	1.50
$N^{(S)}$	100	148	148	148
$Q^{(S)}$	100	64	25	25
	$SNR = 20dB$			
	$S = 1$	$S = 2$	$S = 3$	$S = 4$
$L^{(S)}$	6.00	1.50	1.50	1.50
$N^{(S)}$	100	148	148	148
$Q^{(S)}$	100	64	36	25
	$SNR = 10dB$			
	$S = 1$	$S = 2$	$S = 3$	$S = 4$
$L^{(S)}$	6.00	1.50	1.50	1.50
$N^{(S)}$	100	175	175	175
$Q^{(S)}$	100	100	36	25
	$SNR = 5dB$			
	$S = 1$	$S = 2$	$S = 3$	$S = 4$
$L^{(S)}$	6.00	1.80	1.80	1.80
$N^{(S)}$	100	175	175	175
$Q^{(S)}$	100	100	36	36

Table II: *L-shaped Object*, $\ell = 1.5\lambda$, $\tau = 0.02$ - Investigation domain parameters: restricted investigation domain size $L^{(S)}$, total number of cells $N^{(S)}$ and number of cells within the restricted domain size $Q^{(S)}$.

2.1.2 L-shaped Object, $\ell = 1.5\lambda$, $\tau = 0.05$ - IMSA-BCS reconstructed profiles

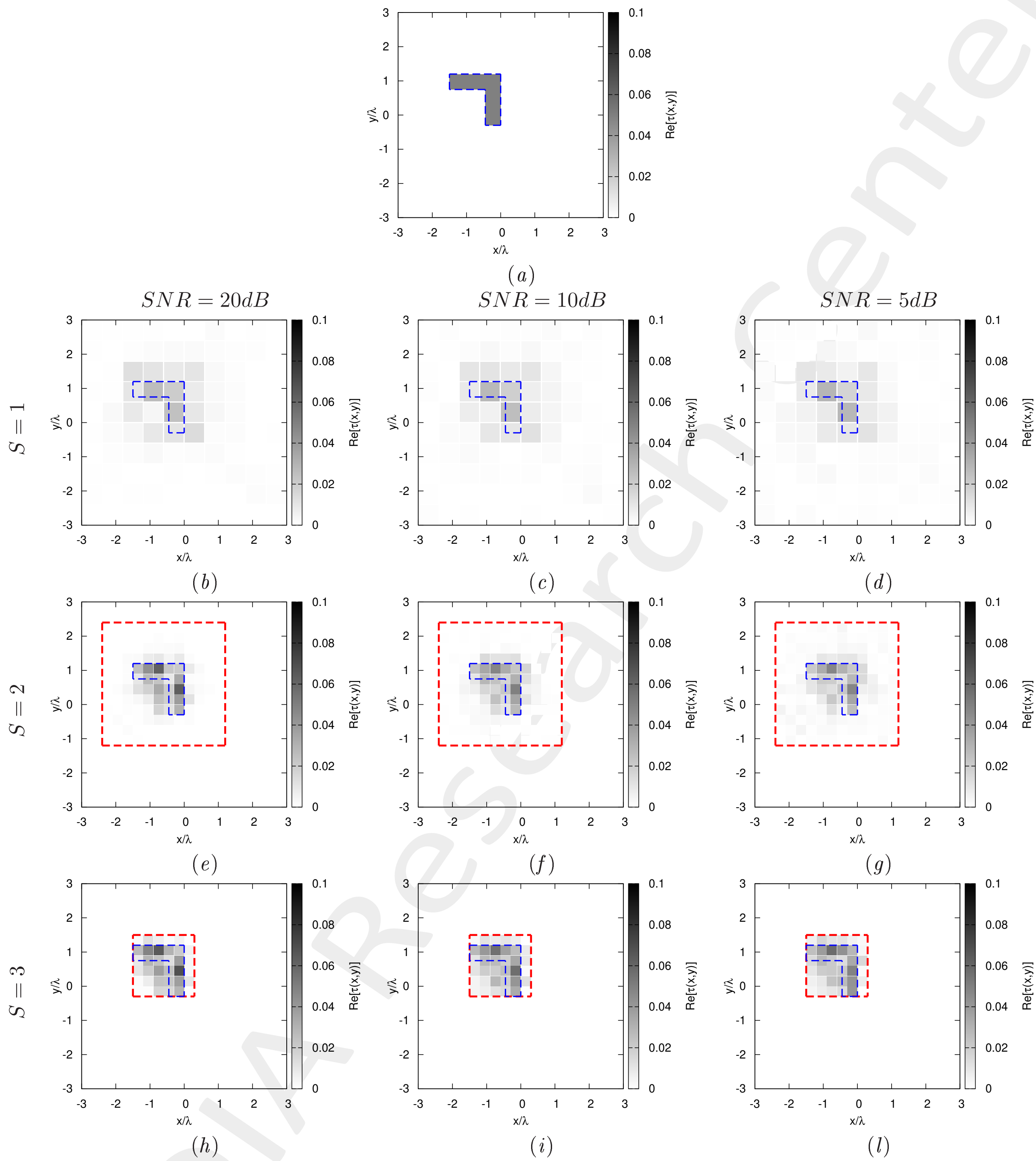


Figure 2: *L-shaped Object*, $\ell = 1.5\lambda$, $\tau = 0.05$ - (a) Actual profile and (b)-(o) IMSA-BCS reconstructed profiles for $(b)(e)(h)$ $SNR = 20$ [dB], $(c)(f)(i)$ $SNR = 10$ [dB] and $(d)(g)(l)$ $SNR = 5$ [dB] at the step (b)-(d) $S = 1$, (e)-(g) $S = 2$, and (h)-(l) $S = 3$.

	$SNR = 50dB$			
	$S=1$	$S=2$	$S=3$	$S=4$
ξ_{tot}	2.75×10^{-3}	1.31×10^{-3}	1.23×10^{-3}	1.23×10^{-3}
ξ_{int}	2.82×10^{-2}	2.00×10^{-2}	1.99×10^{-2}	1.99×10^{-2}
ξ_{ext}	1.87×10^{-3}	6.78×10^{-4}	6.12×10^{-4}	6.12×10^{-4}
	$SNR = 20dB$			
	$S=1$	$S=2$	$S=3$	$S=4$
ξ_{tot}	2.77×10^{-3}	1.39×10^{-3}	1.23×10^{-3}	1.23×10^{-3}
ξ_{int}	2.84×10^{-2}	2.14×10^{-2}	1.98×10^{-2}	1.98×10^{-2}
ξ_{ext}	1.86×10^{-3}	7.21×10^{-4}	6.17×10^{-4}	6.17×10^{-4}
	$SNR = 10dB$			
	$S=1$	$S=2$	$S=3$	$S=4$
ξ_{tot}	2.81×10^{-3}	1.47×10^{-3}	1.19×10^{-3}	1.19×10^{-3}
ξ_{int}	2.83×10^{-2}	1.98×10^{-2}	1.74×10^{-2}	1.74×10^{-2}
ξ_{ext}	1.87×10^{-3}	8.23×10^{-4}	6.50×10^{-4}	6.50×10^{-4}
	$SNR = 5dB$			
	$S=1$	$S=2$	$S=3$	$S=4$
ξ_{tot}	2.93×10^{-3}	1.67×10^{-3}	1.19×10^{-3}	1.19×10^{-3}
ξ_{int}	2.82×10^{-2}	2.20×10^{-2}	1.66×10^{-2}	1.66×10^{-2}
ξ_{ext}	1.98×10^{-3}	9.33×10^{-4}	6.73×10^{-4}	6.73×10^{-4}

Table III: *L-shaped Object*, $\ell = 1.5\lambda$, $\tau = 0.05$ - Reconstruction errors: total (ξ_{tot}), internal (ξ_{int}) and external (ξ_{ext}) errors.

	$SNR = 50dB$			
	$S=1$	$S=2$	$S=3$	$S=4$
$L^{(S)}$	6.00	1.80	1.80	1.80
$N^{(S)}$	100	208	208	208
$Q^{(S)}$	100	144	36	36
	$SNR = 20dB$			
	$S=1$	$S=2$	$S=3$	$S=4$
$L^{(S)}$	6.00	1.80	1.80	1.80
$N^{(S)}$	100	208	208	208
$Q^{(S)}$	100	144	36	36
	$SNR = 10dB$			
	$S=1$	$S=2$	$S=3$	$S=4$
$L^{(S)}$	6.00	1.80	1.80	1.80
$N^{(S)}$	100	208	208	208
$Q^{(S)}$	100	144	36	36
	$SNR = 5dB$			
	$S=1$	$S=2$	$S=3$	$S=4$
$L^{(S)}$	6.00	1.80	1.80	1.80
$N^{(S)}$	100	208	208	208
$Q^{(S)}$	100	144	36	36

Table IV: *L-shaped Object*, $\ell = 1.5\lambda$, $\tau = 0.05$ - Investigation domain parameters: restricted investigation domain size $L^{(S)}$, total number of cells $N^{(S)}$ and number of cells within the restricted domain size $Q^{(S)}$.

2.1.3 L-shaped Object, $\ell = 1.5\lambda$, $\tau = 0.10$ - IMSA-BCS reconstructed profiles

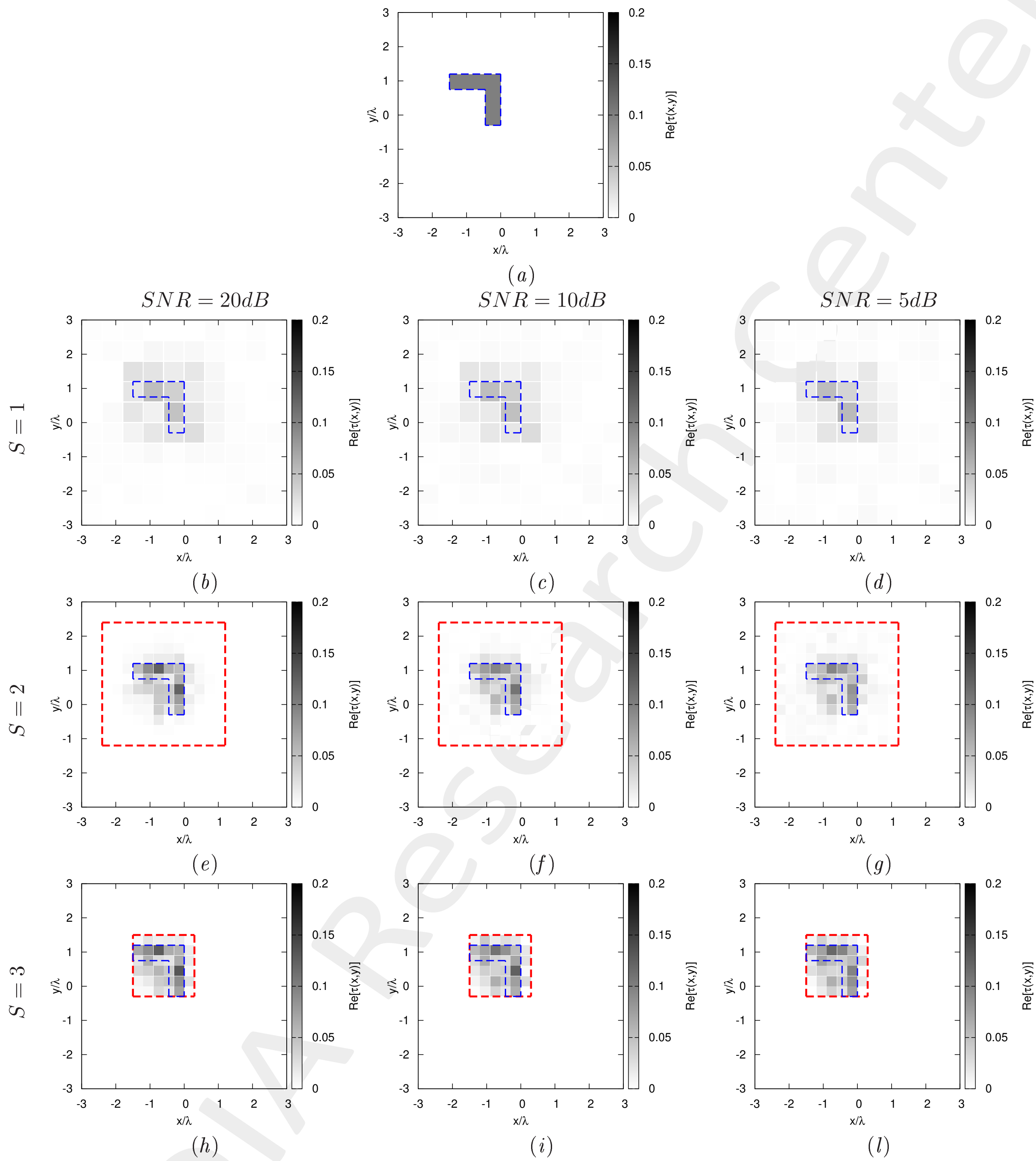


Figure 3: *L-shaped Object*, $\ell = 1.5\lambda$, $\tau = 0.10$ - (a) Actual profile and (b)-(o) IMSA-BCS reconstructed profiles for $(b)(e)(h)$ $SNR = 20$ [dB], $(c)(f)(i)$ $SNR = 10$ [dB] and $(d)(g)(l)$ $SNR = 5$ [dB] at the step (b)-(d) $S = 1$, (e)-(g) $S = 2$, and (h)-(l) $S = 3$.

	$SNR = 50dB$			
	$S=1$	$S=2$	$S=3$	$S=4$
ξ_{tot}	5.91×10^{-3}	2.63×10^{-3}	2.36×10^{-3}	2.36×10^{-3}
ξ_{int}	5.53×10^{-2}	3.67×10^{-2}	3.50×10^{-2}	3.50×10^{-2}
ξ_{ext}	4.05×10^{-3}	1.40×10^{-3}	1.21×10^{-3}	1.21×10^{-3}
	$SNR = 20dB$			
	$S=1$	$S=2$	$S=3$	$S=4$
ξ_{tot}	5.89×10^{-3}	2.85×10^{-3}	2.42×10^{-3}	2.42×10^{-3}
ξ_{int}	5.55×10^{-2}	3.99×10^{-2}	3.57×10^{-2}	3.57×10^{-2}
ξ_{ext}	4.03×10^{-3}	1.51×10^{-3}	1.25×10^{-3}	1.25×10^{-3}
	$SNR = 10dB$			
	$S=1$	$S=2$	$S=3$	$S=4$
ξ_{tot}	5.97×10^{-3}	2.91×10^{-3}	2.55×10^{-3}	2.55×10^{-3}
ξ_{int}	5.49×10^{-2}	3.67×10^{-2}	3.55×10^{-2}	3.55×10^{-2}
ξ_{ext}	4.06×10^{-3}	1.64×10^{-3}	1.39×10^{-3}	1.39×10^{-3}
	$SNR = 5dB$			
	$S=1$	$S=2$	$S=3$	$S=4$
ξ_{tot}	6.34×10^{-3}	3.91×10^{-3}	2.47×10^{-3}	2.47×10^{-3}
ξ_{int}	5.49×10^{-2}	$NaN\times10^{-a}$	3.22×10^{-2}	3.22×10^{-2}
ξ_{ext}	4.22×10^{-3}	3.66×10^{-3}	1.36×10^{-3}	1.36×10^{-3}

Table V: *L-shaped Object,* $\ell = 1.5\lambda$, $\tau = 0.10$ - Reconstruction errors: total (ξ_{tot}), internal (ξ_{int}) and external (ξ_{ext}) errors.

	$SNR = 50dB$			
	$S=1$	$S=2$	$S=3$	$S=4$
$L^{(S)}$	6.00	1.80	1.80	1.80
$N^{(S)}$	100	208	208	208
$Q^{(S)}$	100	144	36	36
	$SNR = 20dB$			
	$S=1$	$S=2$	$S=3$	$S=4$
$L^{(S)}$	6.00	1.80	1.80	1.80
$N^{(S)}$	100	208	208	208
$Q^{(S)}$	100	144	36	36
	$SNR = 10dB$			
	$S=1$	$S=2$	$S=3$	$S=4$
$L^{(S)}$	6.00	1.80	1.80	1.80
$N^{(S)}$	100	208	208	208
$Q^{(S)}$	100	144	36	36
	$SNR = 5dB$			
	$S=1$	$S=2$	$S=3$	$S=4$
$L^{(S)}$	6.00	1.80	1.80	1.80
$N^{(S)}$	100	208	208	208
$Q^{(S)}$	100	144	36	36

Table VI: *L-shaped Object,* $\ell = 1.5\lambda$, $\tau = 0.10$ - Investigation domain parameters: restricted investigation domain size $L^{(S)}$, total number of cells $N^{(S)}$ and number of cells within the restricted domain size $Q^{(S)}$.

2.1.4 L-shaped Object, $\ell = 1.5\lambda$, $\tau = 0.15$ - IMSA-BCS reconstructed profiles

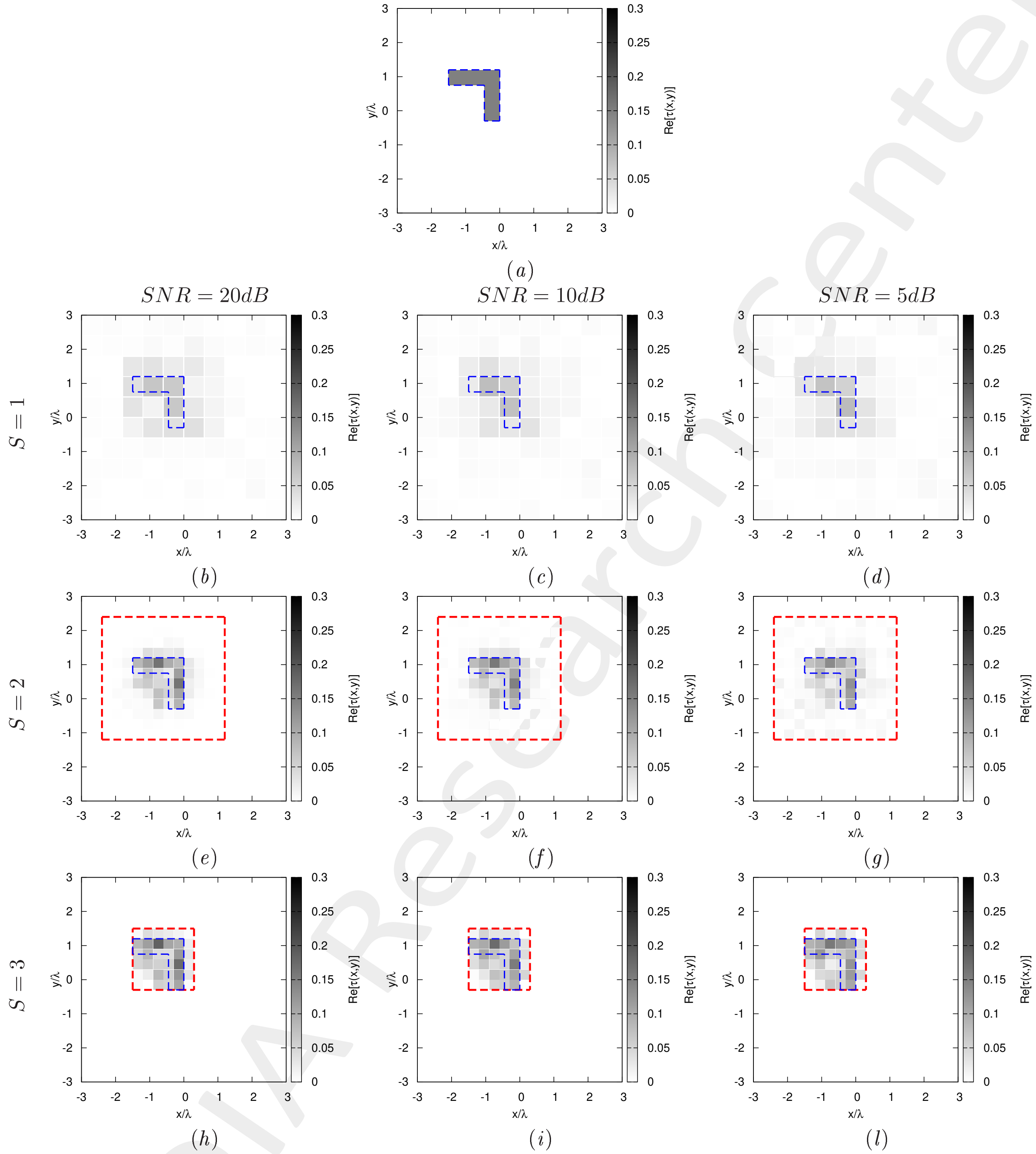


Figure 4: *L-shaped Object*, $\ell = 1.5\lambda$, $\tau = 0.15$ - (a) Actual profile and (b)-(o) IMSA-BCS reconstructed profiles for $(b)(e)(h)$ $SNR = 20$ [dB], $(c)(f)(i)$ $SNR = 10$ [dB] and $(d)(g)(l)$ $SNR = 5$ [dB] at the step (b)-(d) $S = 1$, (e)-(g) $S = 2$, and (h)-(l) $S = 3$.

	$SNR = 50dB$			
	$S = 1$	$S = 2$	$S = 3$	$S = 4$
ξ_{tot}	8.88×10^{-3}	4.04×10^{-3}	3.48×10^{-3}	3.48×10^{-3}
ξ_{int}	8.11×10^{-2}	5.20×10^{-2}	4.62×10^{-2}	4.62×10^{-2}
ξ_{ext}	6.00×10^{-3}	2.17×10^{-3}	1.79×10^{-3}	1.79×10^{-3}
	$SNR = 20dB$			
	$S = 1$	$S = 2$	$S = 3$	$S = 4$
ξ_{tot}	8.92×10^{-3}	4.26×10^{-3}	3.69×10^{-3}	3.69×10^{-3}
ξ_{int}	8.01×10^{-2}	5.46×10^{-2}	4.97×10^{-2}	4.97×10^{-2}
ξ_{ext}	5.97×10^{-3}	2.33×10^{-3}	1.92×10^{-3}	1.92×10^{-3}
	$SNR = 10dB$			
	$S = 1$	$S = 2$	$S = 3$	$S = 4$
ξ_{tot}	9.16×10^{-3}	4.68×10^{-3}	3.87×10^{-3}	3.87×10^{-3}
ξ_{int}	8.02×10^{-2}	5.41×10^{-2}	5.03×10^{-2}	5.03×10^{-2}
ξ_{ext}	6.17×10^{-3}	2.59×10^{-3}	2.10×10^{-3}	2.10×10^{-3}
	$SNR = 5dB$			
	$S = 1$	$S = 2$	$S = 3$	$S = 4$
ξ_{tot}	1.02×10^{-2}	5.50×10^{-3}	3.85×10^{-3}	3.85×10^{-3}
ξ_{int}	7.92×10^{-2}	6.05×10^{-2}	4.50×10^{-2}	4.50×10^{-2}
ξ_{ext}	6.81×10^{-3}	3.12×10^{-3}	2.06×10^{-3}	2.06×10^{-3}

Table VII: *L-shaped Object*, $\ell = 1.5\lambda$, $\tau = 0.10$ - Reconstruction errors: total (ξ_{tot}), internal (ξ_{int}) and external (ξ_{ext}) errors.

	$SNR = 50dB$			
	$S = 1$	$S = 2$	$S = 3$	$S = 4$
$L^{(S)}$	6.00	1.80	1.80	1.80
$N^{(S)}$	100	208	208	208
$Q^{(S)}$	100	144	36	36
	$SNR = 20dB$			
	$S = 1$	$S = 2$	$S = 3$	$S = 4$
$L^{(S)}$	6.00	1.80	1.80	1.80
$N^{(S)}$	100	208	208	208
$Q^{(S)}$	100	144	36	36
	$SNR = 10dB$			
	$S = 1$	$S = 2$	$S = 3$	$S = 4$
$L^{(S)}$	6.00	1.80	1.80	1.80
$N^{(S)}$	100	208	208	208
$Q^{(S)}$	100	144	36	36
	$SNR = 5dB$			
	$S = 1$	$S = 2$	$S = 3$	$S = 4$
$L^{(S)}$	6.00	1.80	1.80	1.80
$N^{(S)}$	100	208	208	208
$Q^{(S)}$	100	144	36	36

Table VIII: *L-shaped Object*, $\ell = 1.5\lambda$, $\tau = 0.15$ - Investigation domain parameters: restricted investigation domain size $L^{(S)}$, total number of cells $N^{(S)}$ and number of cells within the restricted domain size $Q^{(S)}$.

2.1.5 L-shaped Object, $\ell = 1.5\lambda$, $\tau = 0.20$ - IMSA-BCS reconstructed profiles

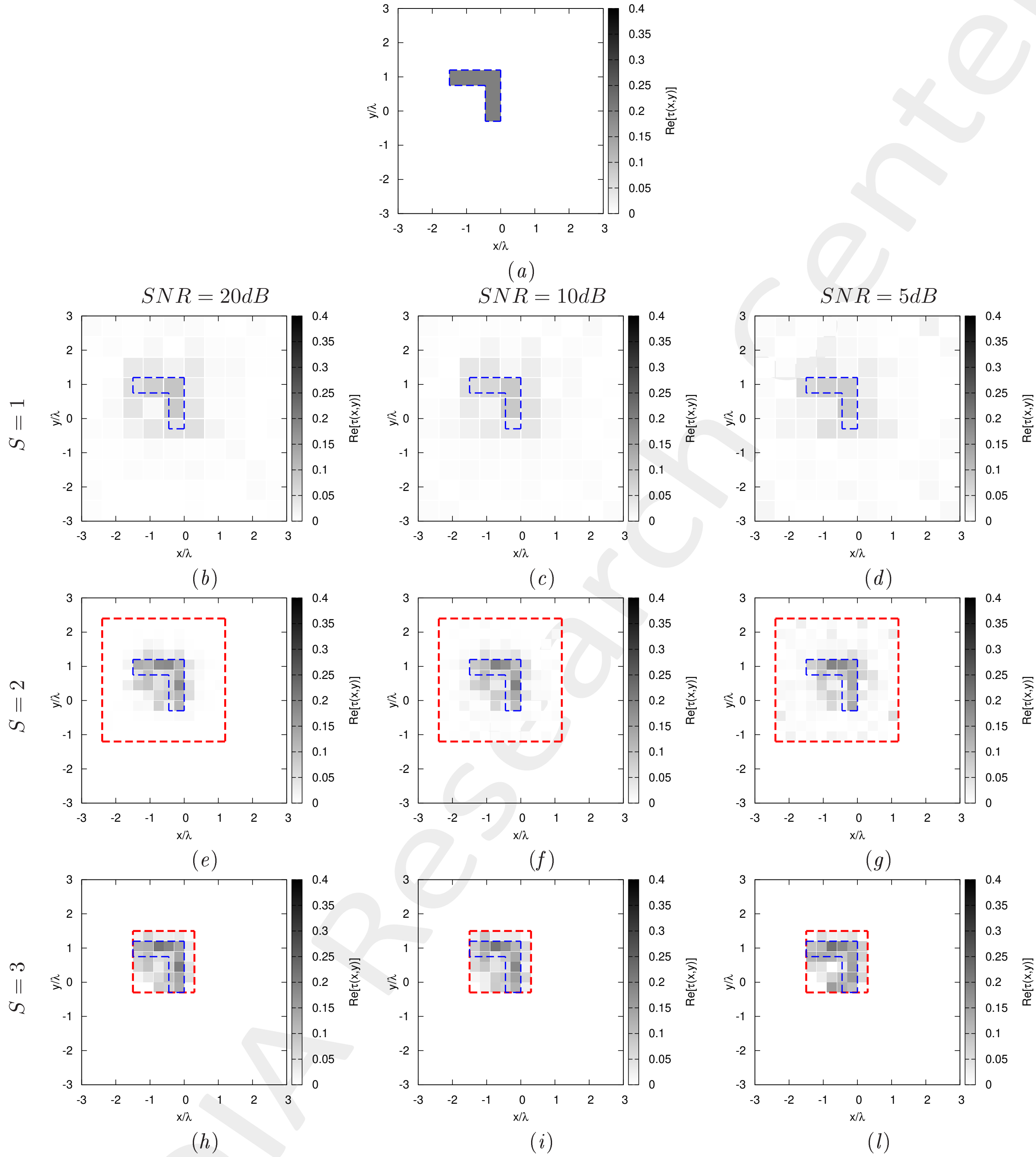


Figure 5: *L-shaped Object*, $\ell = 1.5\lambda$, $\tau = 0.20$ - (a) Actual profile and (b)-(o) IMSA-BCS reconstructed profiles for $(b)(e)(h)$ $SNR = 20$ [dB], $(c)(f)(i)$ $SNR = 10$ [dB] and $(d)(g)(l)$ $SNR = 5$ [dB] at the step (b)-(d) $S = 1$, (e)-(g) $S = 2$, and (h)-(l) $S = 3$.

	$SNR = 50dB$			
	$S = 1$	$S = 2$	$S = 3$	$S = 4$
ξ_{tot}	1.21×10^{-2}	6.02×10^{-3}	4.90×10^{-3}	4.90×10^{-3}
ξ_{int}	1.07×10^{-1}	7.30×10^{-2}	5.81×10^{-2}	5.81×10^{-2}
ξ_{ext}	8.04×10^{-3}	3.23×10^{-3}	2.58×10^{-3}	2.58×10^{-3}
	$SNR = 20dB$			
	$S = 1$	$S = 2$	$S = 3$	$S = 4$
ξ_{tot}	1.19×10^{-2}	5.81×10^{-3}	4.83×10^{-3}	4.83×10^{-3}
ξ_{int}	1.03×10^{-1}	6.95×10^{-2}	5.87×10^{-2}	5.87×10^{-2}
ξ_{ext}	7.98×10^{-3}	3.13×10^{-3}	2.51×10^{-3}	2.51×10^{-3}
	$SNR = 10dB$			
	$S = 1$	$S = 2$	$S = 3$	$S = 4$
ξ_{tot}	1.27×10^{-2}	6.40×10^{-3}	5.23×10^{-3}	5.23×10^{-3}
ξ_{int}	1.07×10^{-1}	7.19×10^{-2}	6.41×10^{-2}	6.41×10^{-2}
ξ_{ext}	8.49×10^{-3}	3.47×10^{-3}	2.75×10^{-3}	2.75×10^{-3}
	$SNR = 5dB$			
	$S = 1$	$S = 2$	$S = 3$	$S = 4$
ξ_{tot}	1.44×10^{-2}	8.26×10^{-3}	5.85×10^{-3}	5.85×10^{-3}
ξ_{int}	1.06×10^{-1}	7.72×10^{-2}	5.98×10^{-2}	5.98×10^{-2}
ξ_{ext}	9.55×10^{-3}	4.70×10^{-3}	3.09×10^{-3}	3.09×10^{-3}

Table IX: *L-shaped Object*, $\ell = 1.5\lambda$, $\tau = 0.20$ - Reconstruction errors: total (ξ_{tot}), internal (ξ_{int}) and external (ξ_{ext}) errors.

	$SNR = 50dB$			
	$S = 1$	$S = 2$	$S = 3$	$S = 4$
$L^{(S)}$	6.00	1.80	1.80	1.80
$N^{(S)}$	100	208	208	208
$Q^{(S)}$	100	144	36	36
	$SNR = 20dB$			
	$S = 1$	$S = 2$	$S = 3$	$S = 4$
$L^{(S)}$	6.00	1.80	1.80	1.80
$N^{(S)}$	100	208	208	208
$Q^{(S)}$	100	144	36	36
	$SNR = 10dB$			
	$S = 1$	$S = 2$	$S = 3$	$S = 4$
$L^{(S)}$	6.00	1.80	1.80	1.80
$N^{(S)}$	100	208	208	208
$Q^{(S)}$	100	144	36	36
	$SNR = 5dB$			
	$S = 1$	$S = 2$	$S = 3$	$S = 4$
$L^{(S)}$	6.00	1.80	1.80	1.80
$N^{(S)}$	100	208	208	208
$Q^{(S)}$	100	144	36	36

Table X: *L-shaped Object*, $\ell = 1.5\lambda$, $\tau = 0.20$ - Investigation domain parameters: restricted investigation domain size $L^{(S)}$, total number of cells $N^{(S)}$ and number of cells within the restricted domain size $Q^{(S)}$.

2.1.6 L-shaped Object, $\ell = 1.5\lambda$, $\tau = 0.20$ - IMSA-BCS multi-resolution grids

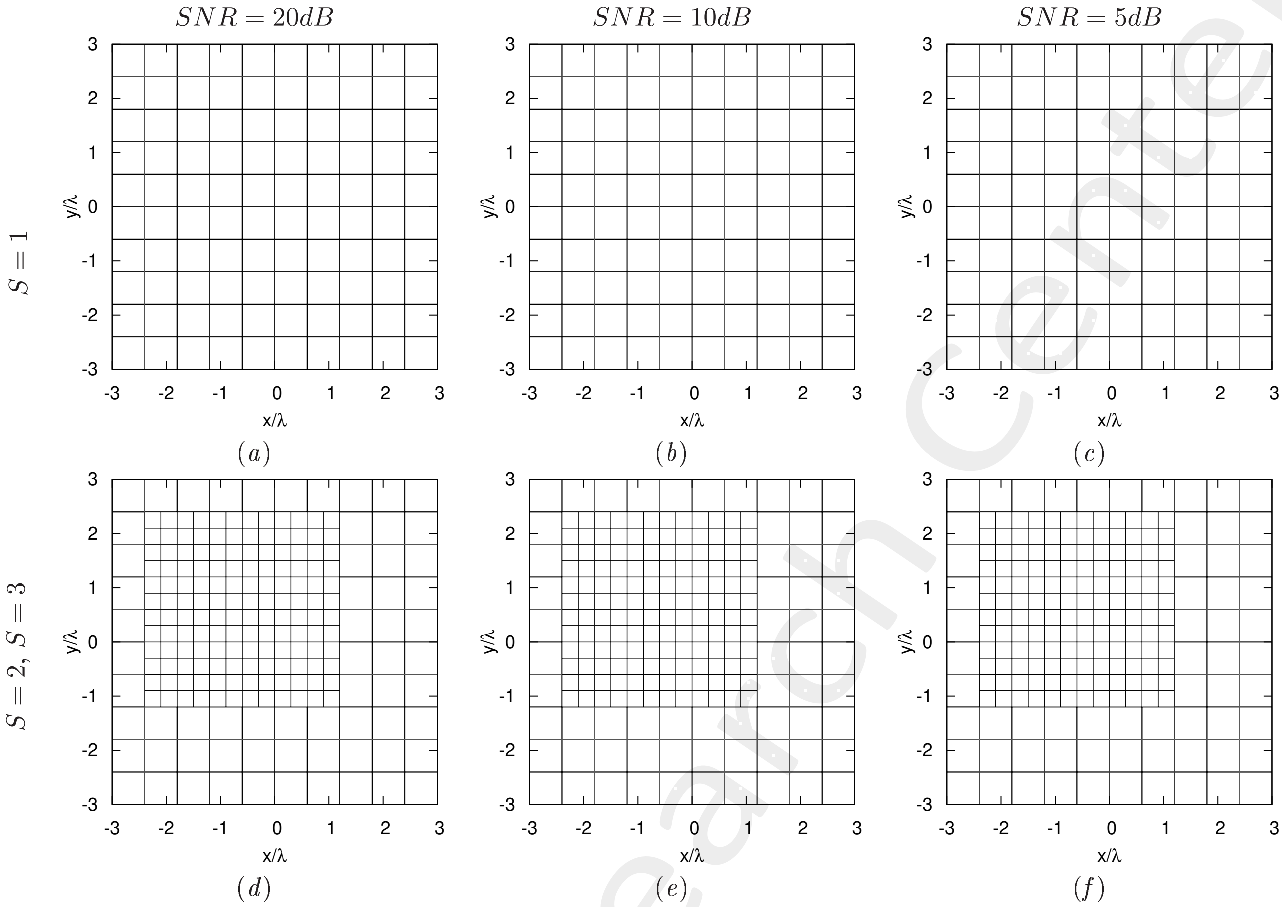
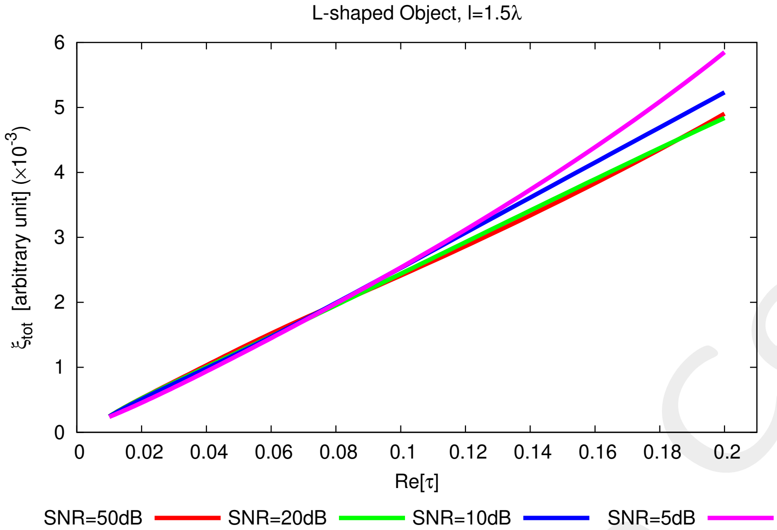
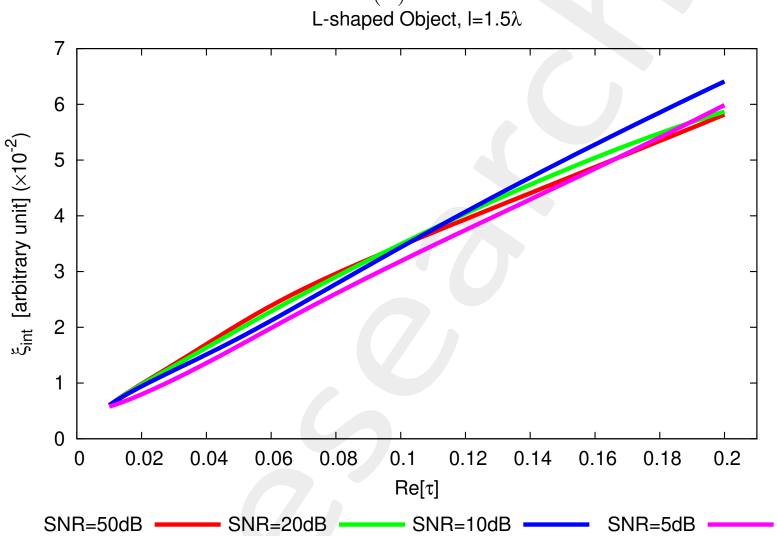


Figure 6: *L-shaped Object*, $\ell = 1.5\lambda$, $\tau = 0.20$ - Example of IMSA-BCS multi-resolution grids for $(a)(d)$ $SNR = 20$ [dB], $(b)(e)$ $SNR = 10$ [dB] and $(c)(f)$ $SNR = 5$ [dB] at the step (a)-(c) $S = 1$ and (d)-(f) $S = 2$, 3.

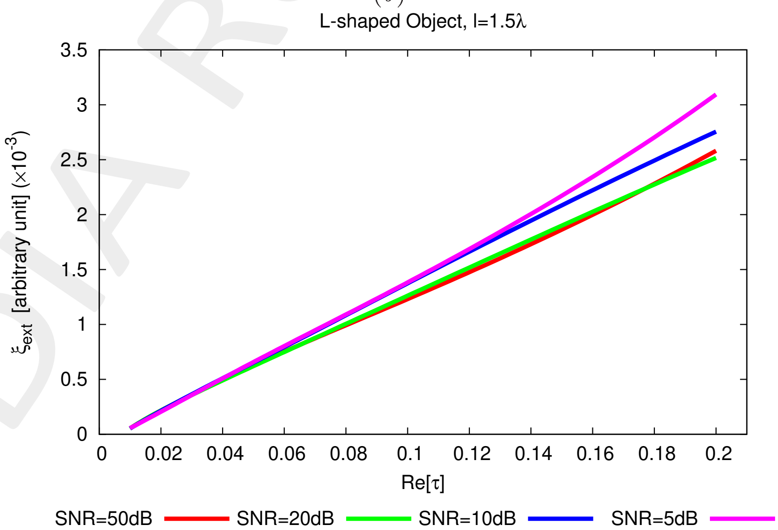
2.1.7 L-shaped Object, $\ell = 1.5\lambda$ - Resume: Errors vs. τ



(a)



(b)



(c)

Figure 7: *L-shaped Object*, $\ell = 1.5\lambda$ - Reconstruction errors vs. τ: (a) total error, (b) internal error and (c) external error.

2.1.8 L-shaped Object, $\ell = 1.5\lambda$ - Resume: Errors vs. SNR

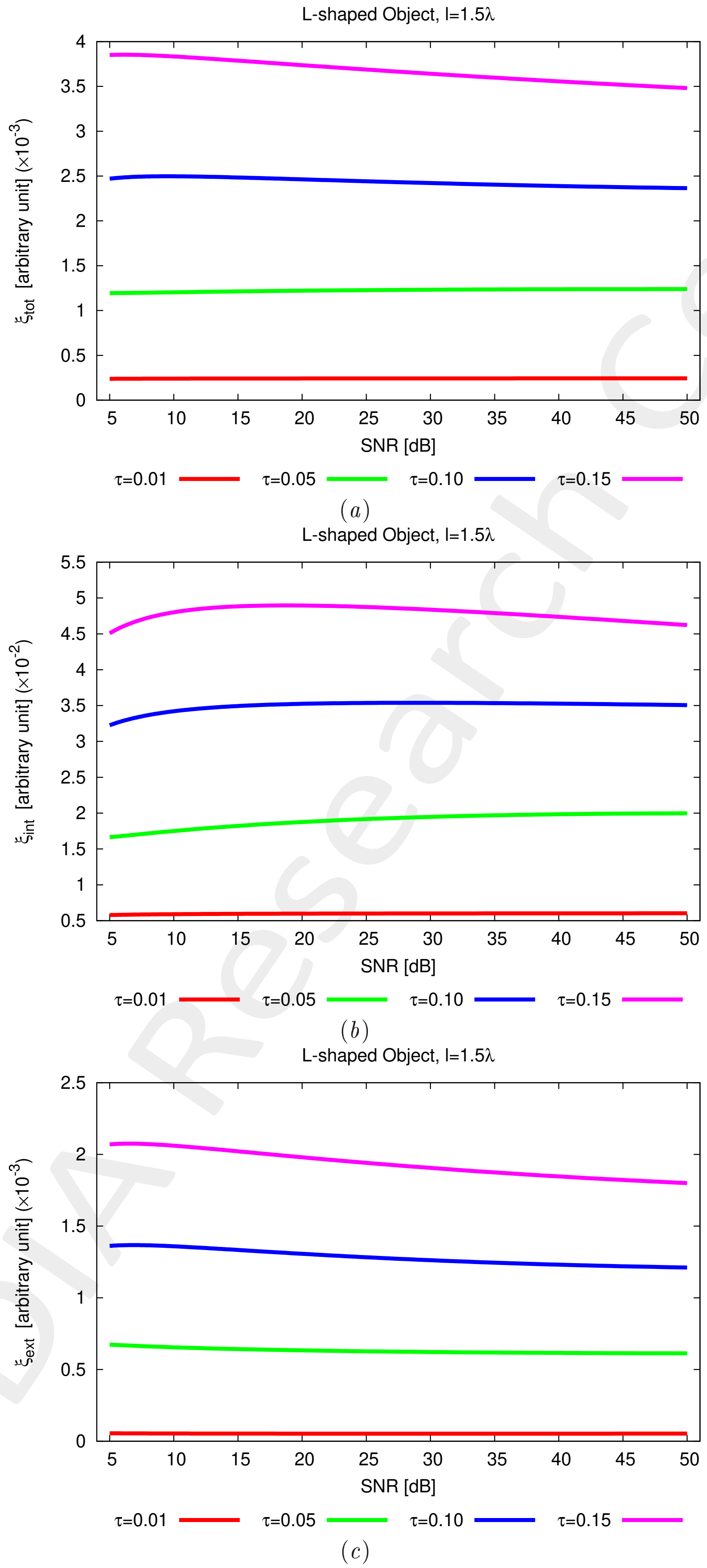


Figure 8: *L-shaped Object*, $\ell = 1.5\lambda$ - Reconstruction errors vs. SNR: (a) total error, (b) internal error and (c) external error.

2.1.9 L-shaped Object, $\ell = 1.5\lambda$ - Resume: Errors vs. *IMSA* step, S

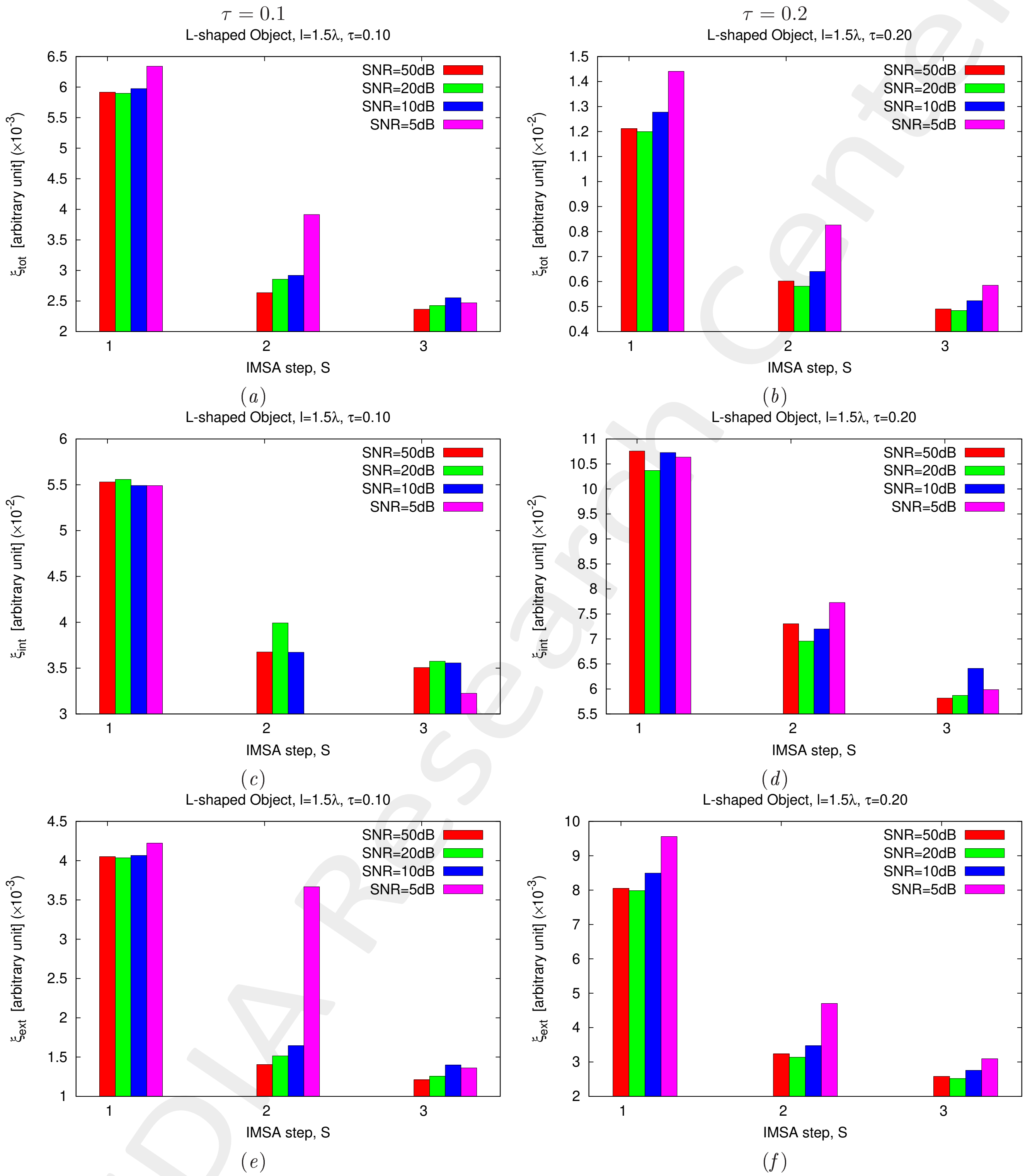


Figure 9: *L-shaped Object*, $\ell = 1.5\lambda$ - Reconstruction errors vs. *IMSA* step, S: $(a)(b)$ total error, $(c)(d)$ internal error and $(e)(f)$ external error for $(a)(c)(e)$ $\tau = 0.1$ and $(b)(d)(f)$ $\tau = 0.2$.

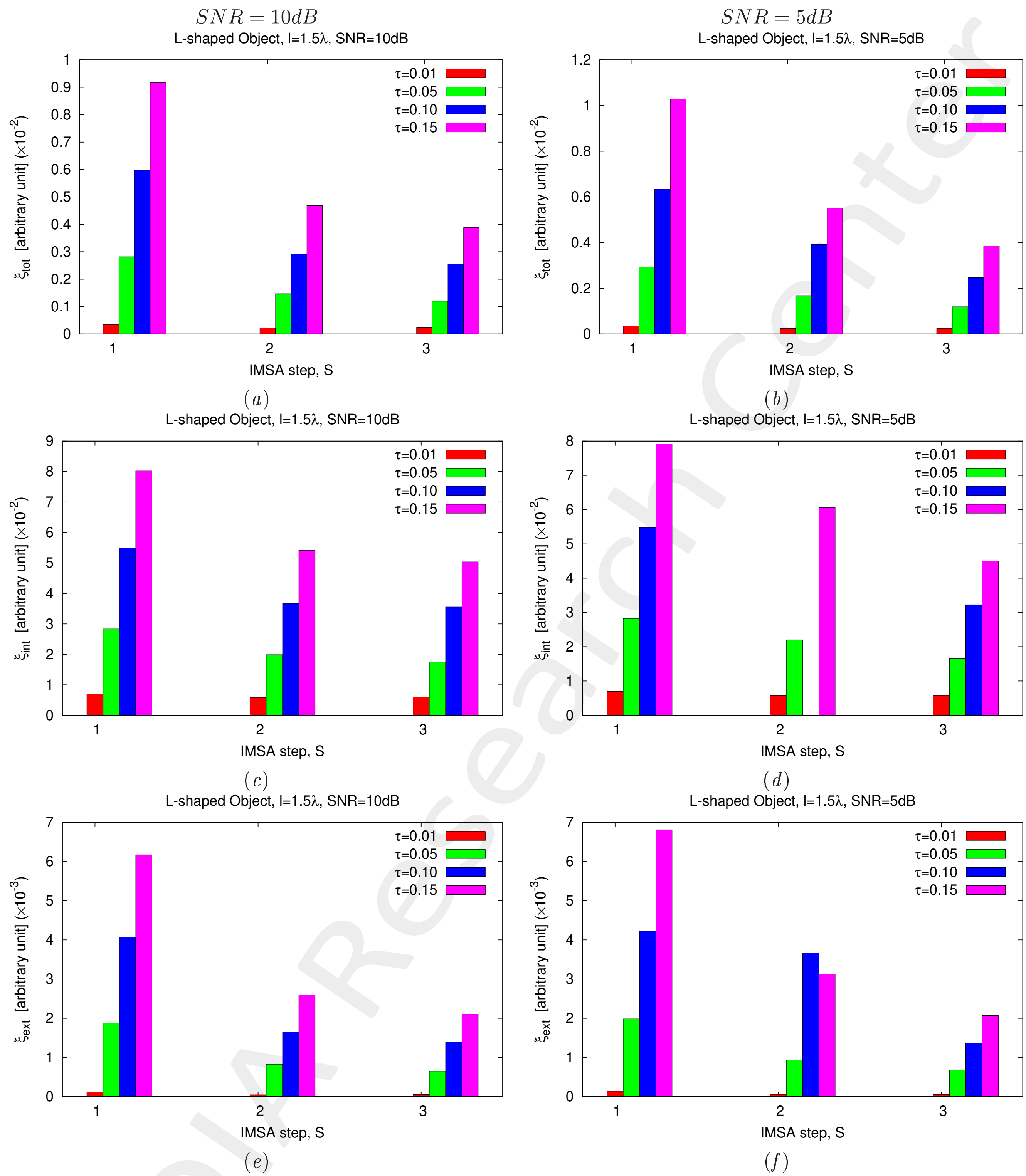


Figure 10: *L-shaped Object*, $\ell = 1.5\lambda$ - Reconstruction errors vs. $IMSA$ step, S: $(a)(b)$ total error, $(c)(d)$ internal error and $(e)(f)$ external error for $(a)(c)(e)$ $SNR = 10dB$ and $(b)(d)(f)$ $SNR = 5dB$.

More information on the topics of this document can be found in the following list of references.

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