DoA estimation via MT-BCS exploiting multiple-snapshots

M.Carlin, P. Rocca, G. Oliveri, and A. Massa

Abstract

In this report, an innovative strategy for the estimation of the directions of arrival of signals impinging on linear arrays of electromagnetic sensors has been assessed. Starting from a sparse representation of the problem solution, the DoA estimation problem has been addressed by means of a methodology based on the BCS paradigm. A customized implementation exploiting the measurements collected at multiple time instants (multiple-snapshots) providing robust and very accurate estimates when correlating the information from multiple snapshots has been validated.

MT-BCS DoA estimation

GOAL: The goal of this section is the analysis of the performances of the MT-BCS method for the DoA estimation with $W > 1$ snapshots. The performances of the method are compared with the standard single-task BCS (ST-BCS) and with the ROOT-MUSIC and ESPRIT algorithms.

$$\underline{\hat{x}}_h^{(ave)} = \frac{1}{W}\sum_{w=1}^{W} \left|\underline{\hat{x}}_h(t_w)\right| \quad (1)$$

being W the number of snapshots and $h \in \{ST - BCS, MT - BCS\}$. The main difference between the ST and $MT\ BCS$ furmulations is that in the second case the non-zero elements of the estimated vectors $\underline{\hat{x}}_h(t_w)$ are forced to be in the same locations.

Analysis vs number of snapshots W

Simulation Parameters

- Scenario
  - BPSK signals ($E_l^{inc} \in \{-1, 1\}$)
  - Number of incident signals: $L = 2$
  - Signal directions: $\underline{\theta} = \{0, 7\}$ $[deg]$
  - Signal to noise ratio: $SNR = 7\ dB$ (equivalent to a $SNR = 4\ dB$ if the literature's definition is taken into account)
- Array parameters
  - Elements spacing: $d = 0.5\lambda$
  - Number of elements: $M = 10$
- MT-BCS parameters
  - Number of angular locations: $K = 181$
  - $a = 3.162$
  - $b = 3.981 \times 10^1$
- BCS parameters
  - Number of angular locations: $K = 181$
  - $\sigma_0^2 = 4.642 \times 10^{-1}$
  - **Number of snapshots**: $W \in [1, 25]$

- Simulation
  - Number of independent realizations $Q = 150$ (the noise and the signal amplitudes are random, while the DoAs are fixed)

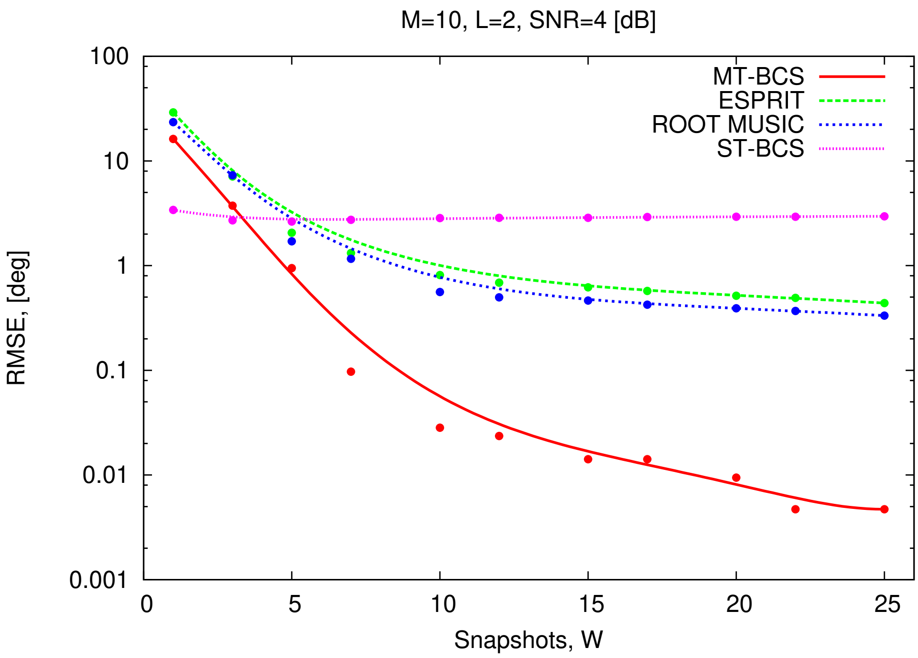

Figure 1: $RMSE$ vs the number of snapshots W.

Analysis vs SNR

Simulation Parameters

- Scenario
  - BPSK signals ($E_l^{inc} \in \{-1, 1\}$)
  - Number of incident signals: $L = 2$
  - Signal directions: $\underline{\theta} = \{0, 7\}$ $[deg]$
  - **Signal to noise ratio:** $SNR \in [-5, 20]$ dB ($SNR \in [-8, 17]$ dB if the literature's definition is taken into account)
- Array parameters
  - Elements spacing: $d = 0.5\lambda$
  - Number of elements: $M = 10$
- MT-BCS parameters
  - Number of angular locations: $K = 181$
  - $a = 3.162$
  - $b = 3.981 \times 10^1$
- BCS parameters
  - Number of angular locations: $K = 181$
  - $\sigma_0^2 = 4.642 \times 10^{-1}$
  - Number of snapshots**:** $W = 20$
- Simulation
  - Number of independent realizations $Q = 150$ (the noise and the signal amplitudes are random, while the DoAs are fixed)

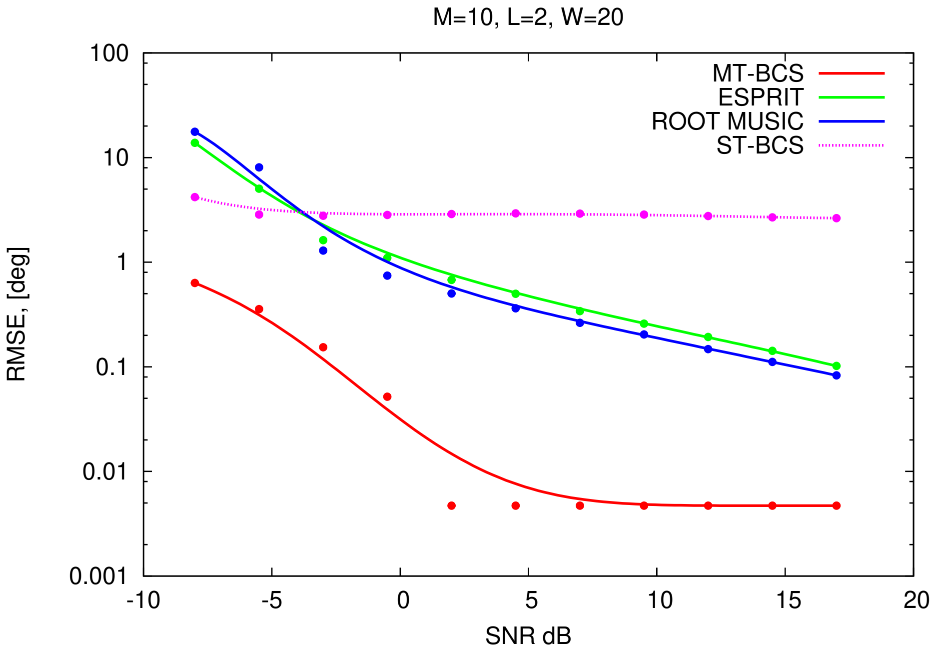

Figure 2: $RMSE$ vs the SNR.

Analysis vs $\Delta\theta^{l(l+1)}$

Simulation Parameters

- Scenario
  - BPSK signals ($E_l^{inc} \in \{-1, 1\}$)
  - Number of incident signals: $L = 2$
  - **Signals spacing**: $\Delta\theta^{l(l+1)} \in [2, 20]\ deg$
  - **Signals directions**: $\underline{\theta} = \left\{-\frac{\Delta\theta^{l(l+1)}}{2}, \frac{\Delta\theta^{l(l+1)}}{2}\right\}\ [deg]$
  - Signal to noise ratio: $SNR = 7\ dB$ (equivalent to a $SNR = 4\ dB$ if the literature's definition is taken into account)
- Array parameters
  - Elements spacing:
  - Number of elements: $M = 10$
- MT-BCS parameters
  - Number of angular locations: $K = 181$
  - $a = 3.162$
  - $b = 3.981 \times 10^1$
- BCS parameters
  - Number of angular locations: $K = 181$
  - $\sigma_0^2 = 4.642 \times 10^{-1}$
  - Number of snapshots**:** $W = 20$
- Simulation
  - Number of independent realizations $Q = 150$ (the noise and the signal amplitudes are random, while the DoAs are fixed)

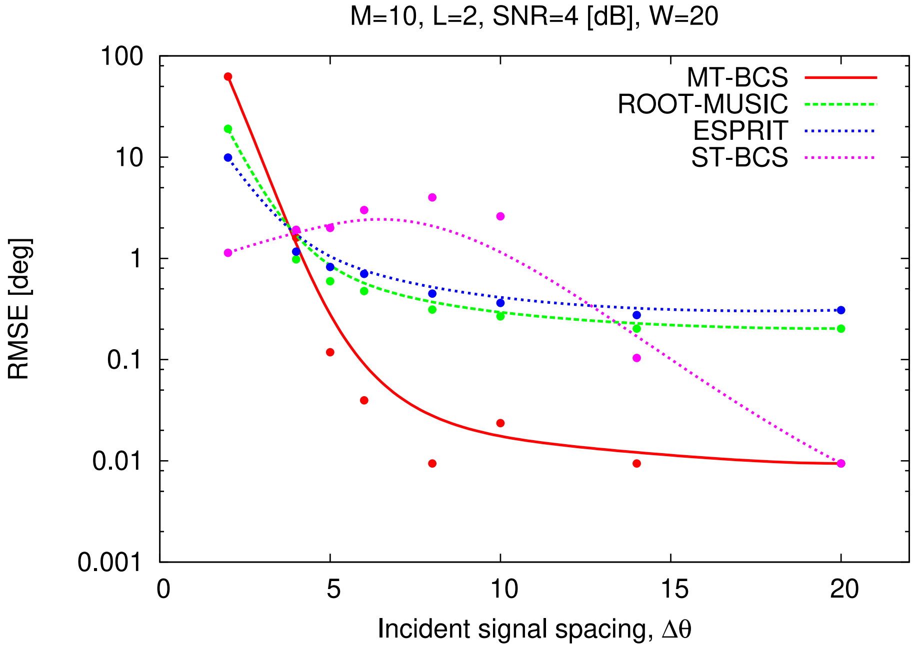

Figure 3: $RMSE$ vs the signal spacing $\Delta\theta^{l(l+1)}$.

MT-BCS vs ST-BCS comparison: estimation examples

Simulation Parameters

- Scenario
  - BPSK signals ($E_l^{inc} \in \{-1, 1\}$)
  - **Number of incident signals:** $L \in [1, 9]$
  - **Signal directions:**

L	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9
1	0	-	-	-	-	-	-	-	-
2	0	7	-	-	-	-	-	-	-
4	0	7	35		-	-	-	-	-
6	0	7	35	-20	22	-37	-	-	-
8	0	7	35	-20	22	-37	-9	-67	-
9	0	7	35	-20	22	-37	-9	-67	54

Table 1: Signal directions for different numbers of signals.

  - Signal to noise ratio**:** $SNR = 7\ dB$
- Array parameters
  - Elements spacing: $d = 0.5\lambda$
  - Number of elements: $M = 10$
- $ST - BCS$ and $MT - BCS$ parameters
  - Number of angular locations: $K = 181$
  - Number of snapshots**:** $W = 20$

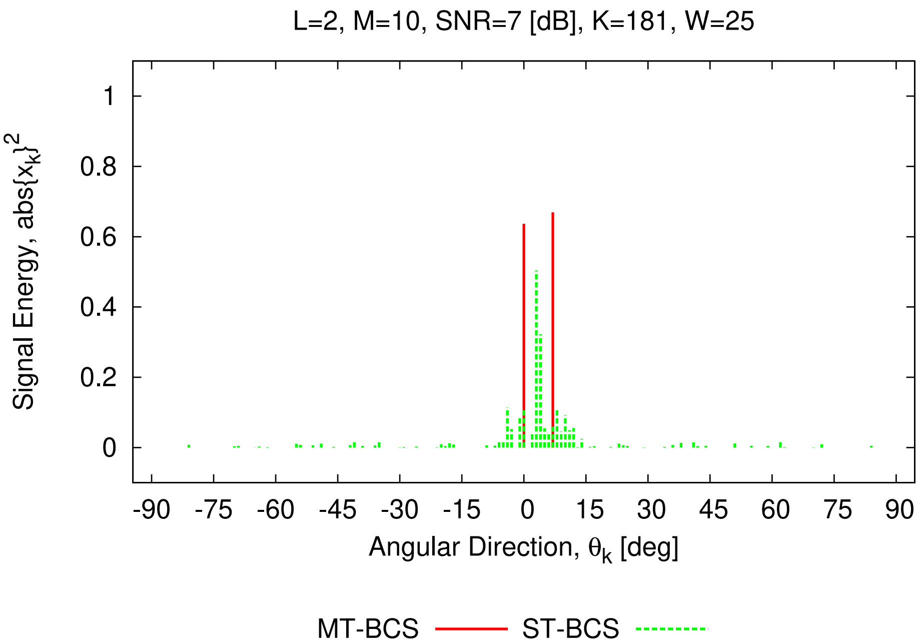

Figure 4: $MT-BCS$ vs $ST-BCS$: esstimated signal amplitudes when $L=2$ signals impinging on the array. The number of snapshots is $W=25$.

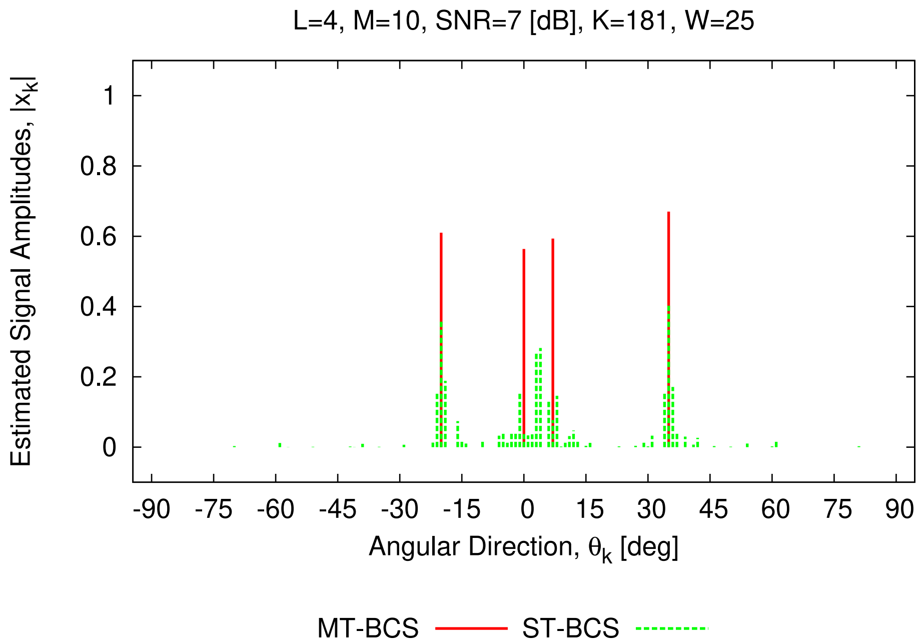

Figure 5: $MT-BCS$ vs $ST-BCS$: esstimated signal amplitudes when $L=4$ signals impinging on the array. The number of snapshots is $W=25$.

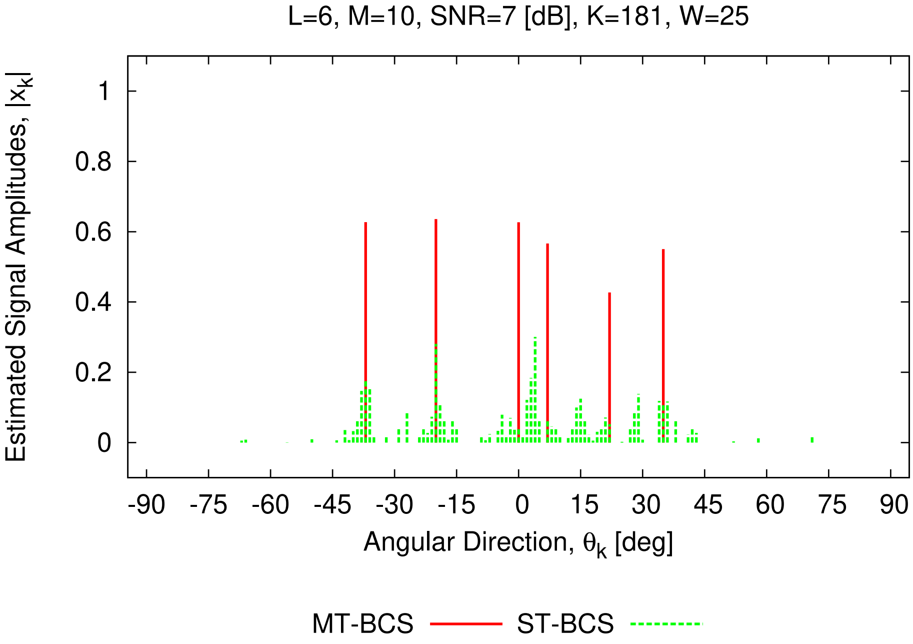

Figure 6: $MT - BCS$ vs $ST - BCS$: esstimated signal amplitudes when $L = 6$ signals impinging on the array. The number of snapshots is $W = 25$.

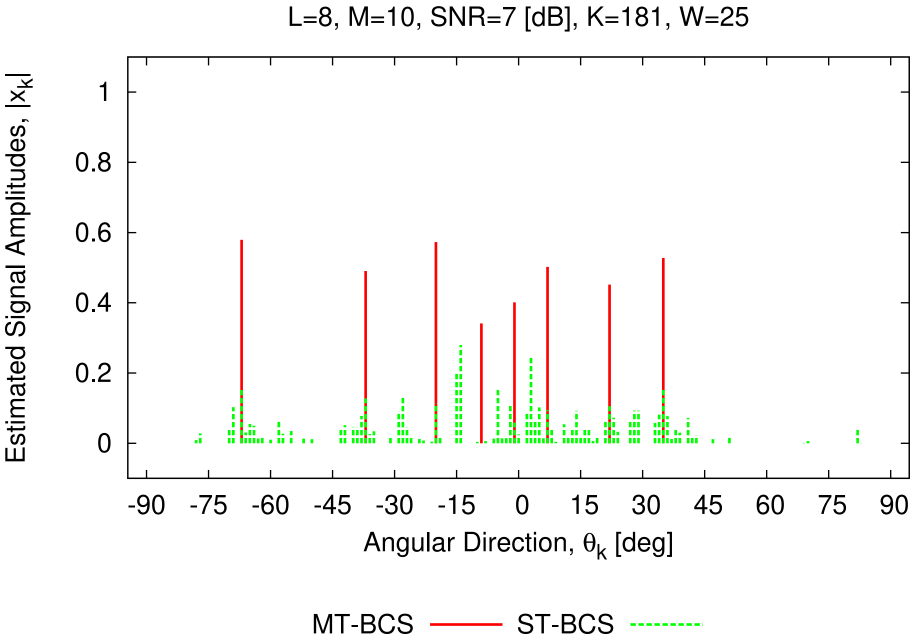

Figure 7: $MT - BCS$ vs $ST - BCS$: esstimated signal amplitudes when $L = 8$ signals impinging on the array. The number of snapshots is $W = 25$.

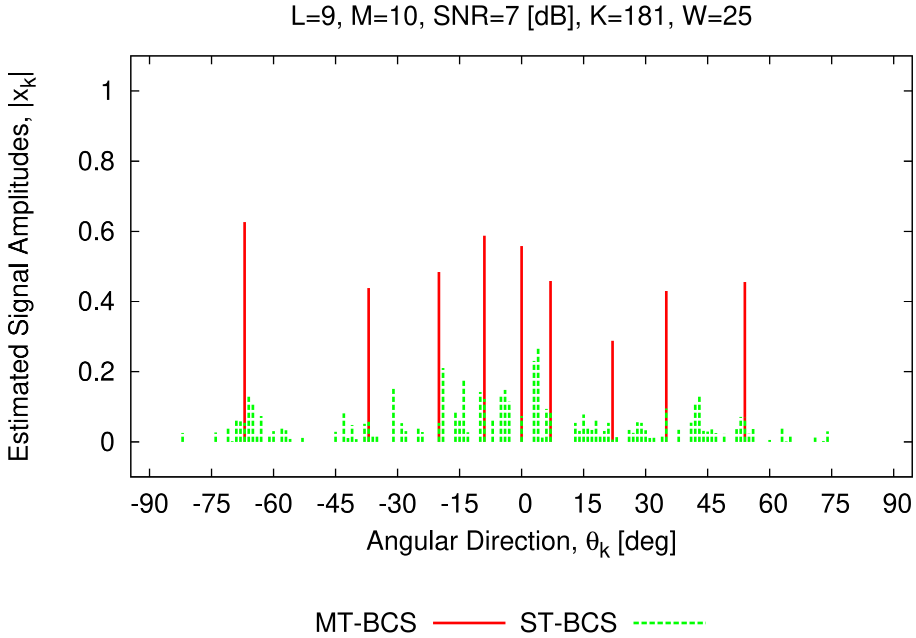

Figure 8: $MT-BCS$ vs $ST-BCS$: esstimated signal amplitudes when $L=9$ signals impinging on the array. The number of snapshots is $W=25$.

More information on the topics of this document can be found in the following list of references.

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