Numerical Analysis of the DoA estimation through BCS

M.Carlin, P. Rocca, G. Oliveri, and A. Massa

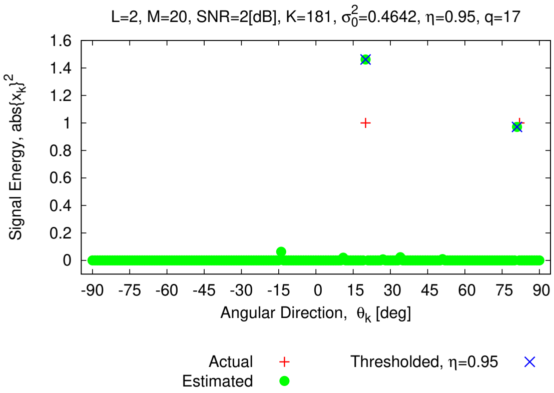
Abstract

This report deals with the Direction of arrival (DoA) estimation problem formulated within the Bayesian Compressive Sensing (BCS) framework. The performances of a novel method that directly works on the voltages measured at the output of the array elements without requiring the computation of the covariance matrix and that provide accurate and reliable DoAs estimation also without the a-priori knowledge on the number of incident signals have been investigated. The effectiveness of the proposed approach is assessed through an extensive numerical analysis addressing different scenarios, signal configurations, and noise conditions.

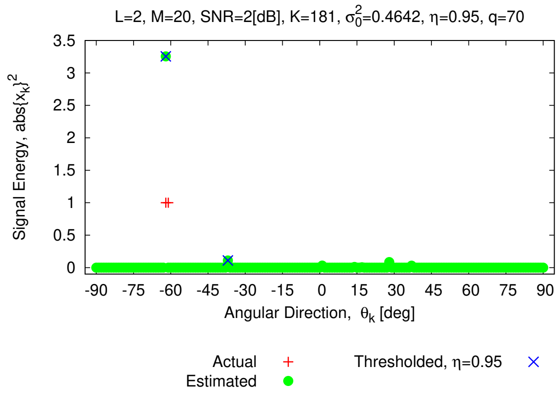
Estimation examples

$L = 2,\ SNR = 2\ dB$

$\widetilde{L} = L$

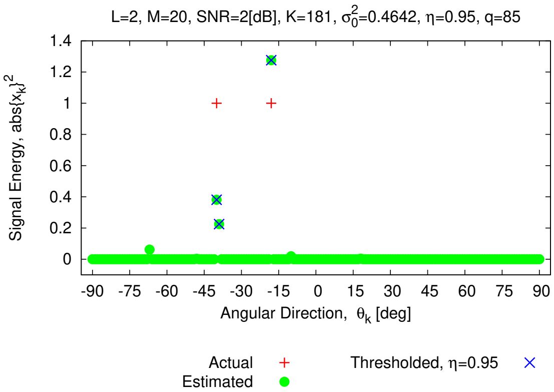


(a) $RMSE_{\theta} = 0.71\ \ [deg]$

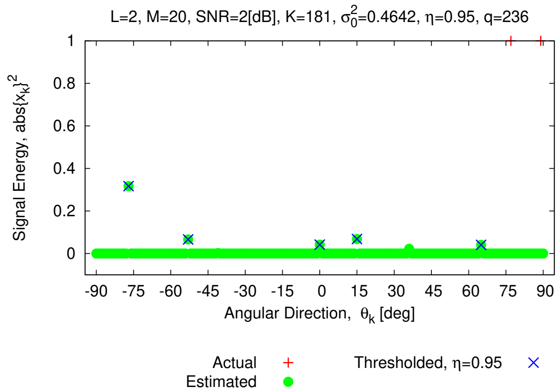


(b) $RMSE_{\theta} = 16.97\ \ [deg]$

$\widetilde{L} > L$

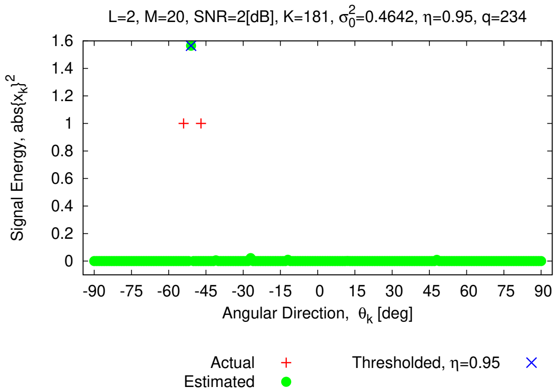


(c) $RMSE_{\theta} = 0.71\ \ [deg]$



(d) $RMSE_{\theta} = 161.50\ \ [deg]$

$\widetilde{L} < L$

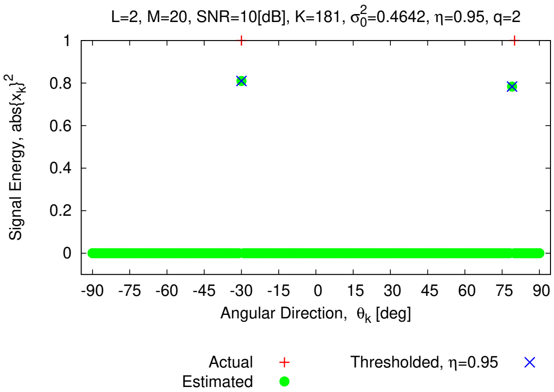


(e) $RMSE_{\theta} = 127.30\ \ [deg]$

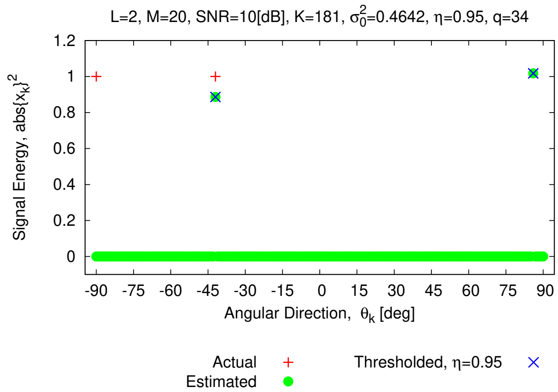
Figure 1: *BCS DoA estimation examples*: (a) $\widetilde{L} = L$ with low $RMSE_{\theta}$, (b) $\widetilde{L} = L$ with high $RMSE_{\theta}$, (c) $\widetilde{L} > L$ with low $RMSE_{\theta}$, (d) $\widetilde{L} > L$ with high $RMSE_{\theta}$ and (e) $\widetilde{L} < L$ with high $RMSE_{\theta}$ (in this case it is impossible to obtain a low $RMSE_{\theta}$).

$L = 2,\ SNR = 10\ dB$

$\widetilde{L} = L$

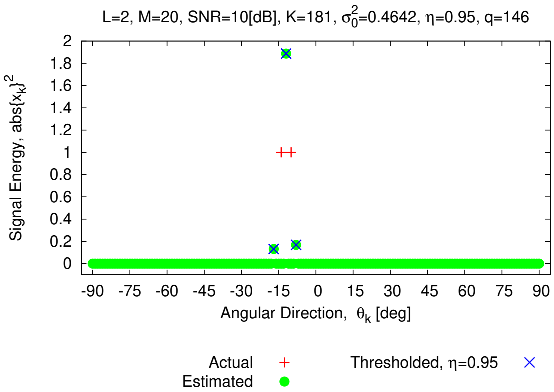


(a) $RMSE_\theta = 0.71\ \ [deg]$

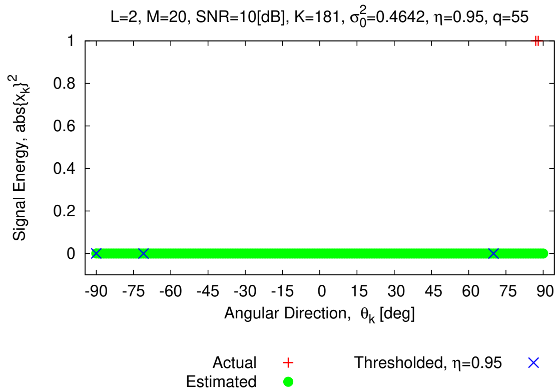


(b) $RMSE_\theta = 96.66\ \ [deg]$

$\widetilde{L} > L$

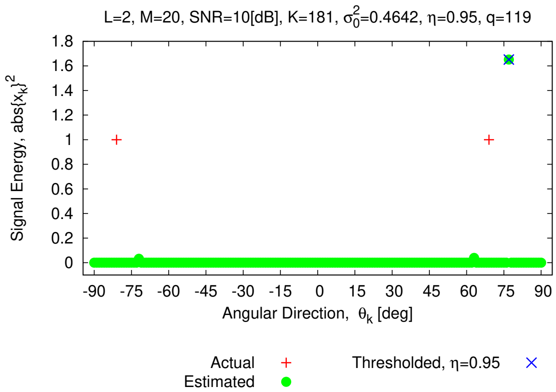


(c) $RMSE_\theta = 2.92\ \ [deg]$



(d) $RMSE_\theta = 168.67\ \ [deg]$

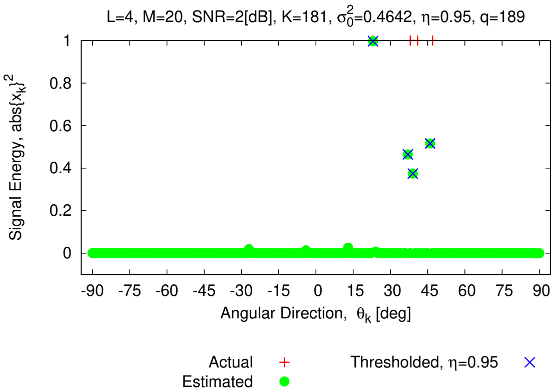
$\widetilde{L} < L$



(e) $RMSE_\theta = 127.41\ \ [deg]$

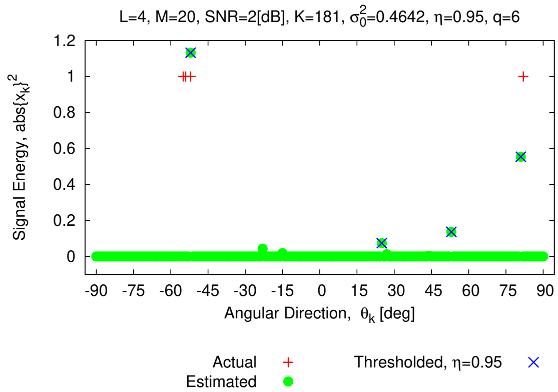
Figure 2: *BCS DoA estimation examples*: (a) $\widetilde{L} = L$ with low $RMSE_\theta$, (b) $\widetilde{L} = L$ with high $RMSE_\theta$, (c) $\widetilde{L} > L$ with low $RMSE_\theta$, (d) $\widetilde{L} > L$ with high $RMSE_\theta$ and (e) $\widetilde{L} < L$ with high $RMSE_\theta$ (in this case it is impossible to obtain a low $RMSE_\theta$).

$L = 4,\, SNR = 2\ dB$

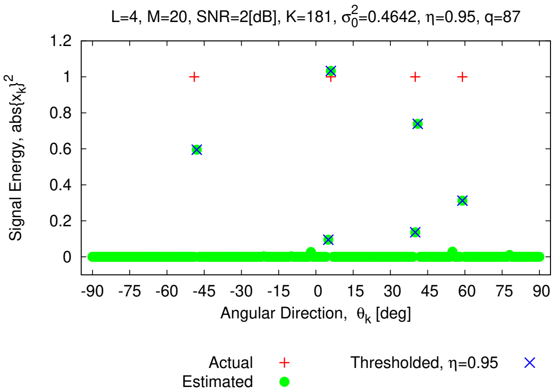


$\widetilde{L} = L$

(a) $RMSE_\theta = 1.22\,[deg]$

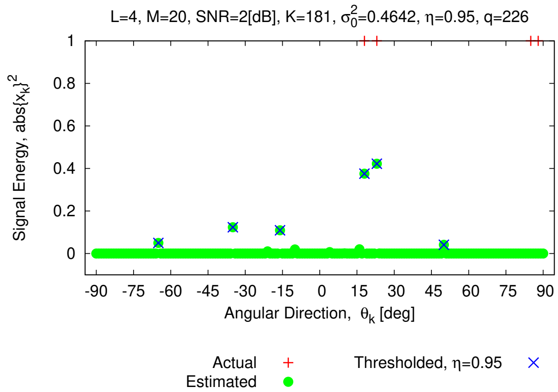


(b) $RMSE_\theta = 66.01\ \ [deg]$

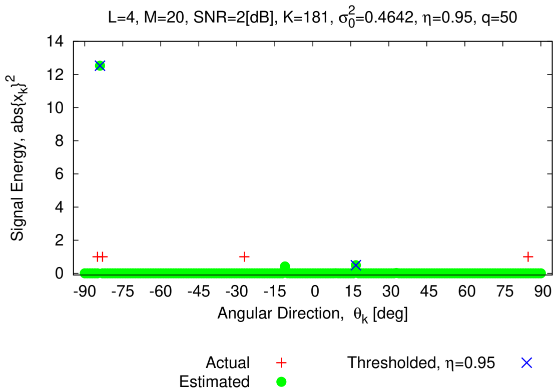


$\widetilde{L} > L$

(c) $RMSE_\theta = 0.87\ \ [deg]$



(d) $RMSE_\theta = 73.72\ \ [deg]$

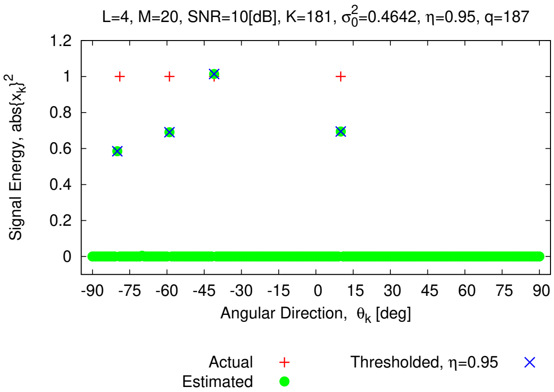


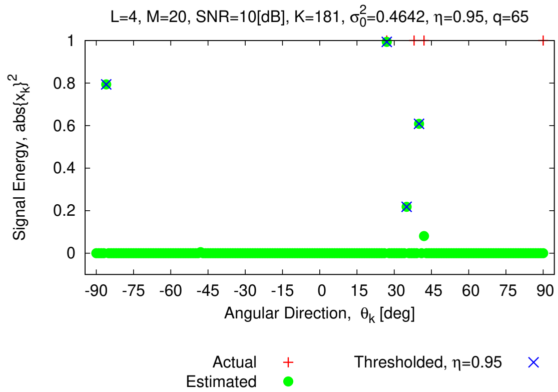
$\widetilde{L} < L$

(e) $RMSE_\theta = 129.17\ \ [deg]$

Figure 3: *BCS DoA estimation examples*: (a) $\widetilde{L} = L$ with low $RMSE_\theta$, (b) $\widetilde{L} = L$ with high $RMSE_\theta$, (c) $\widetilde{L} > L$ with low $RMSE_\theta$, (d) $\widetilde{L} > L$ with high $RMSE_\theta$ and (e) $\widetilde{L} < L$ with high $RMSE_\theta$ (in this case it is impossible to obtain a low $RMSE_\theta$).

$L = 4,\ SNR = 10\ dB$

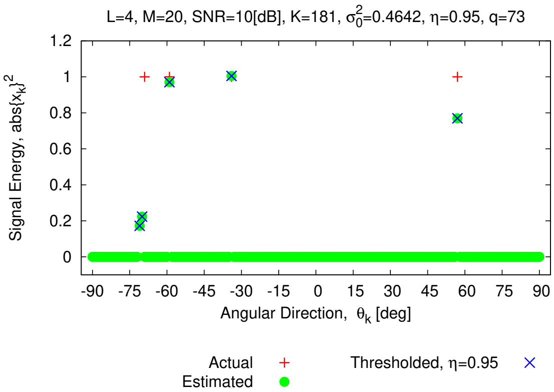


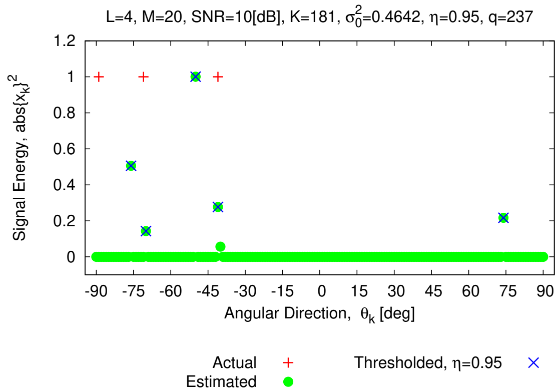


$\widetilde{L} = L$

(a) $RMSE_\theta = 0.50\ [deg]$

(b) $RMSE_\theta = 62.13\ [deg]$

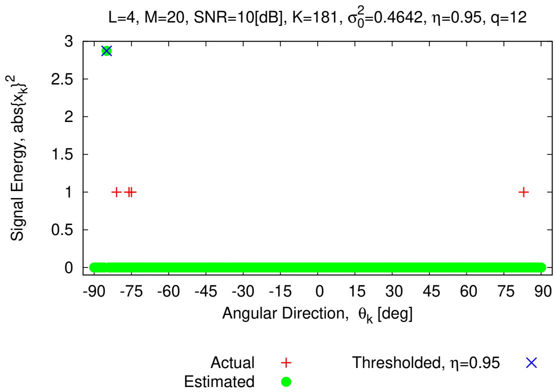




$\widetilde{L} > L$

(c) $RMSE_\theta = 1.12\ [deg]$

(d) $RMSE_\theta = 57.87\ [deg]$



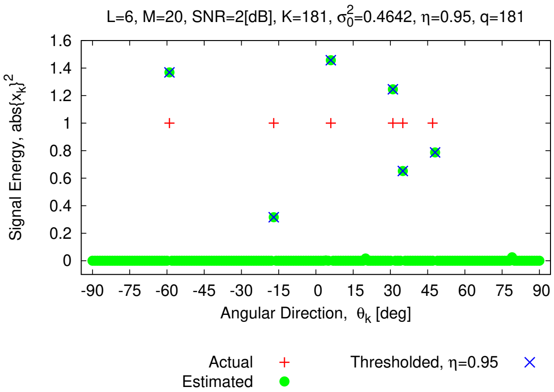
$\widetilde{L} < L$

(e) $RMSE_\theta = 155.90\ [deg]$

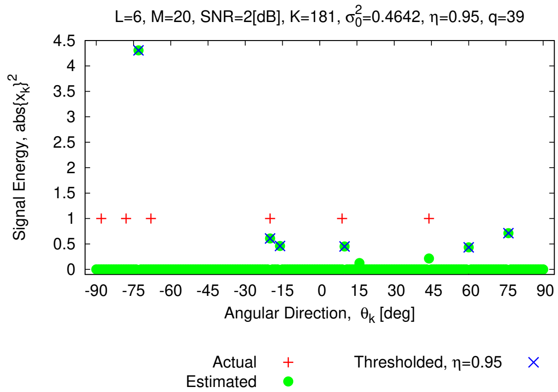
Figure 4: *BCS DoA estimation examples*: (a) $\widetilde{L} = L$ with low $RMSE_\theta$, (b) $\widetilde{L} = L$ with high $RMSE_\theta$, (c) $\widetilde{L} > L$ with low $RMSE_\theta$, (d) $\widetilde{L} > L$ with high $RMSE_\theta$ and (e) $\widetilde{L} < L$ with high $RMSE_\theta$ (in this case it is impossible to obtain a low $RMSE_\theta$).

$L = 6, SNR = 2\ dB$

$\widetilde{L} = L$

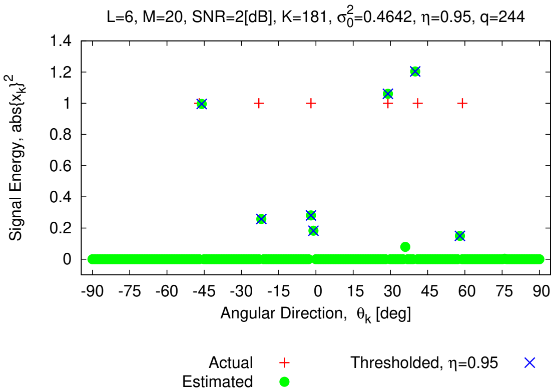


(a) $RMSE_\theta = 0.41\ \ [deg]$

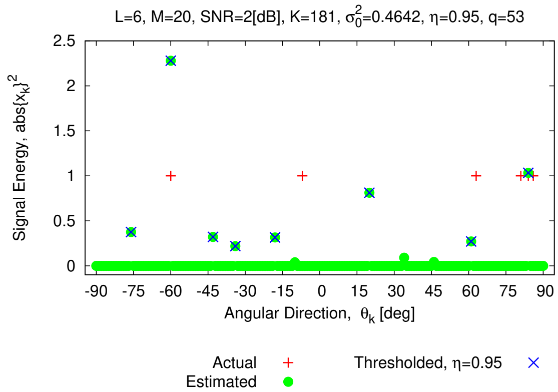


(b) $RMSE_\theta = 46.58\ \ [deg]$

$\widetilde{L} > L$

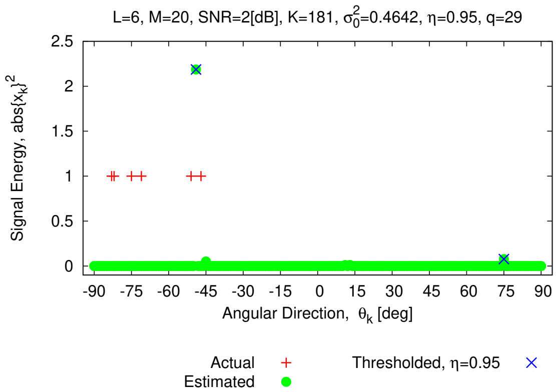


(c) $RMSE_\theta = 0.91\ \ [deg]$



(d) $RMSE_\theta = 50.58\ \ [deg]$

$\widetilde{L} < L$



(e) $RMSE_\theta = 155.18\ \ [deg]$

Figure 5: *BCS DoA estimation examples*: (a) $\widetilde{L} = L$ with low $RMSE_\theta$, (b) $\widetilde{L} = L$ with high $RMSE_\theta$, (c) $\widetilde{L} > L$ with low $RMSE_\theta$, (d) $\widetilde{L} > L$ with high $RMSE_\theta$ and (e) $\widetilde{L} < L$ with high $RMSE_\theta$ (in this case it is impossible to obtain a low $RMSE_\theta$).

Performances analysis

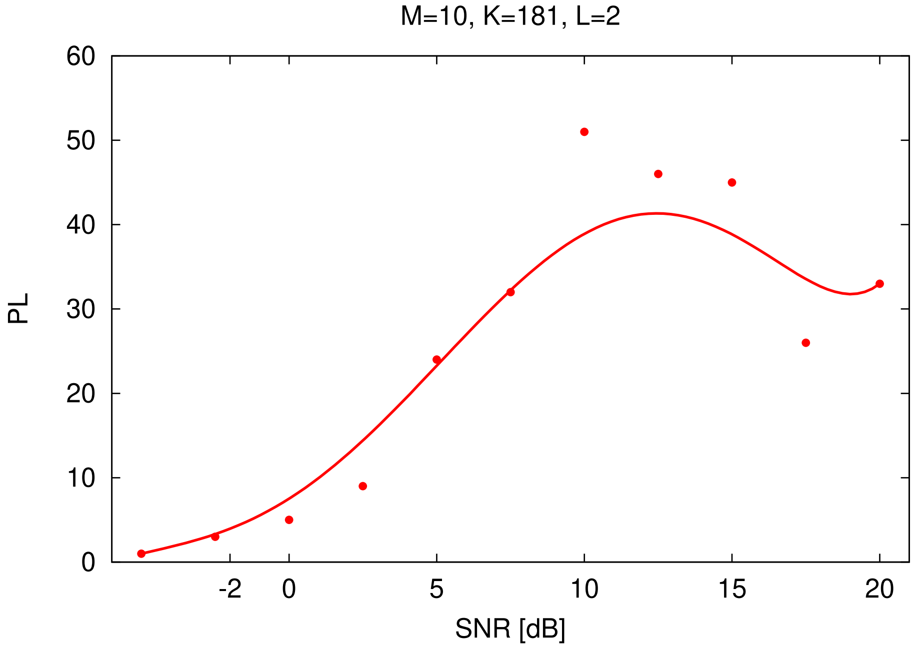
GOAL: this section is aimed to analyzing the performances of the BCS method in terms of $RMSE$ and PL. The $RMSE$ has been computed considering two cases:

- the number of signals is not known
- the number of signals in known

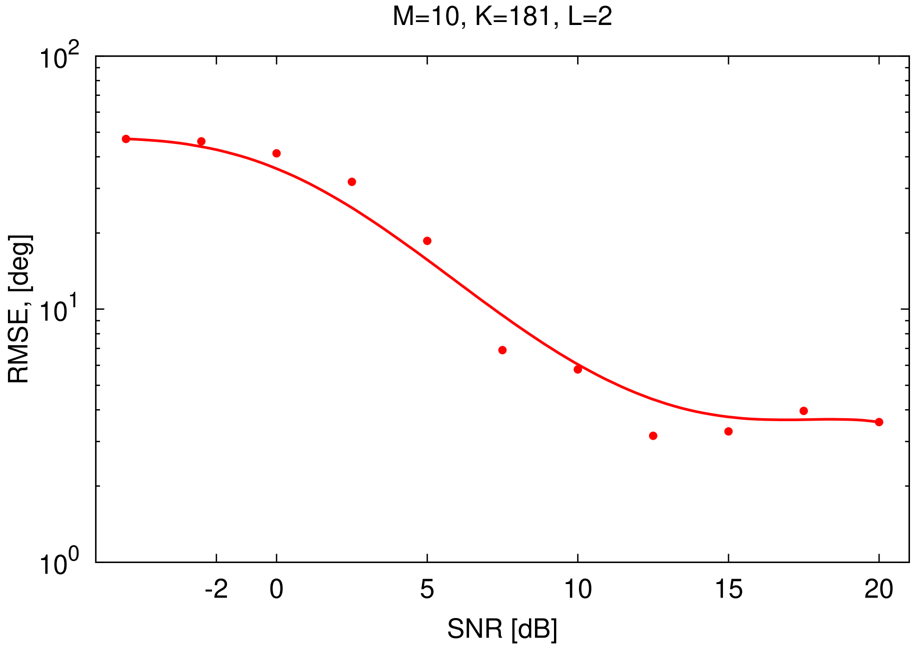
0.0.1 Performances vs the SNR

Simulation Parameters

- Scenario
  - BPSK signals ($E_l^{inc} \in \{-1, 1\}$)
  - Number of incident signals: $L = 2$
  - Signal directions: $\underline{\theta} = \{0, 7\}$ $[deg]$
  - **Signal to noise ratio:** $SNR = [-5, 20]$ dB
- Array parameters
  - Elements spacing: $d = 0.5\lambda$
  - Number of elements: $M = 10$
- BCS parameters
  - Number of angular locations: $K = 181$
  - $\sigma_0^2 = 4.642 \times 10^{-1}$
- Simulation
  - Number of independent realizations $Q = 100$ (the noise and the signal amplitudes are random, while the DoAs are fixed)



(a)



(b)

Figure 6: *BCS performances analysis:* (a) P_L and (b) $RMSE$ vs the SNR value (L is unknown).

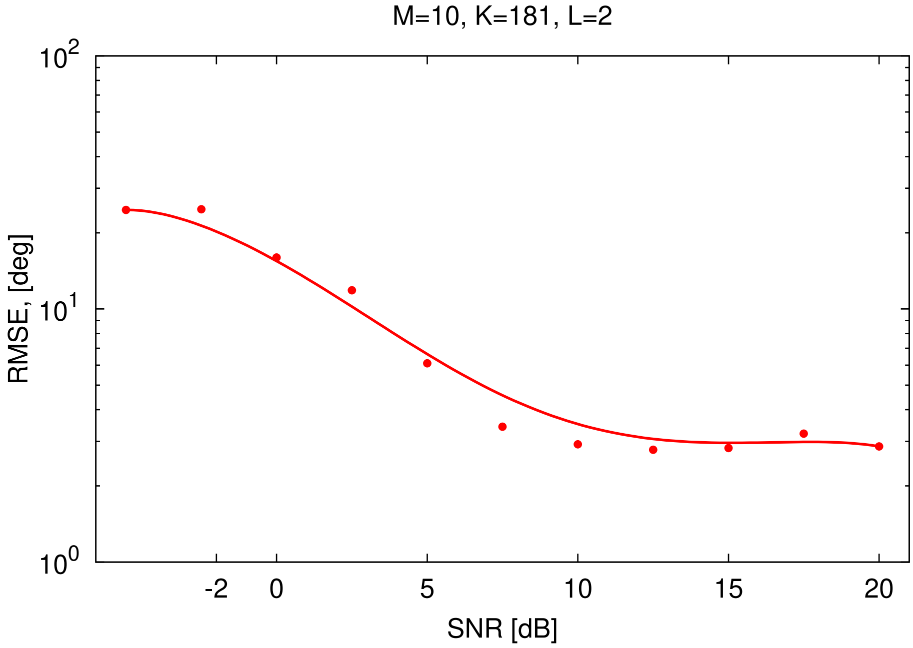
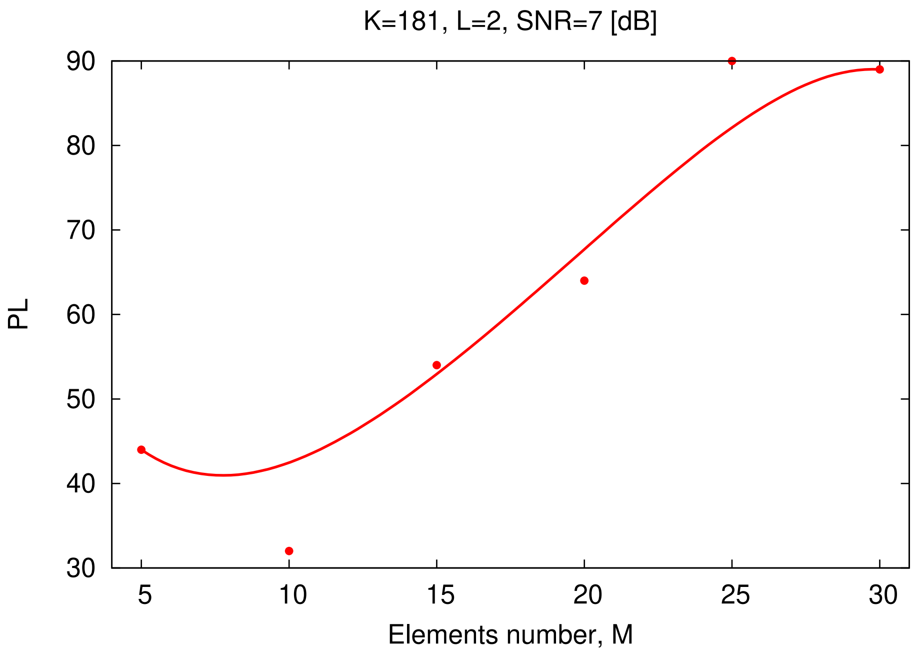


Figure 7: *BCS performances analysis:* $RMSE$ vs the SNR value (L is known).

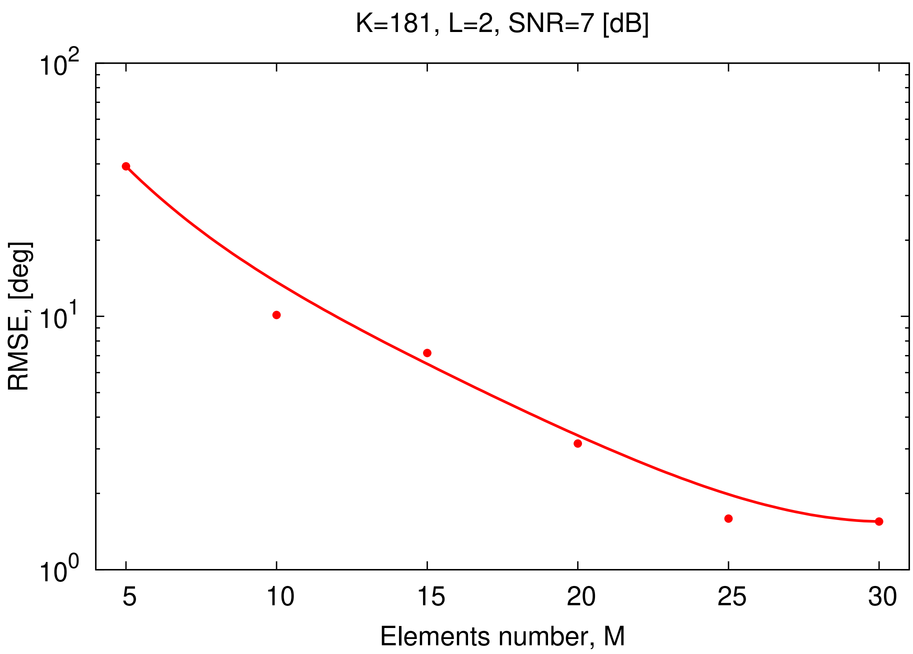
Performances vs the number of array elements M

Simulation Parameters

- Scenario
  - BPSK signals ($E_l^{inc} \in \{-1, 1\}$)
  - Number of incident signals: $L = 2$
  - Signal directions: $\underline{\theta} = \{0, 7\}$ $[deg]$
  - Signal to noise ratio: $SNR = 7$ dB
- Array parameters
  - Elements spacing: $d = 0.5\lambda$
  - **Number of elements:** $M \in [5, 30]$
- BCS parameters
  - Number of angular locations: $K = 181$
  - $\sigma_0^2 = 4.642 \times 10^{-1}$
- Simulation
  - Number of independent realizations $Q = 100$ (the noise and the signal amplitudes are random, while the DoAs are fixed)



(a)



(b)

Figure 8: *BCS performances analysis:* (a) P_L and (b) $RMSE$ vs the number of array elements M (L is unknown).

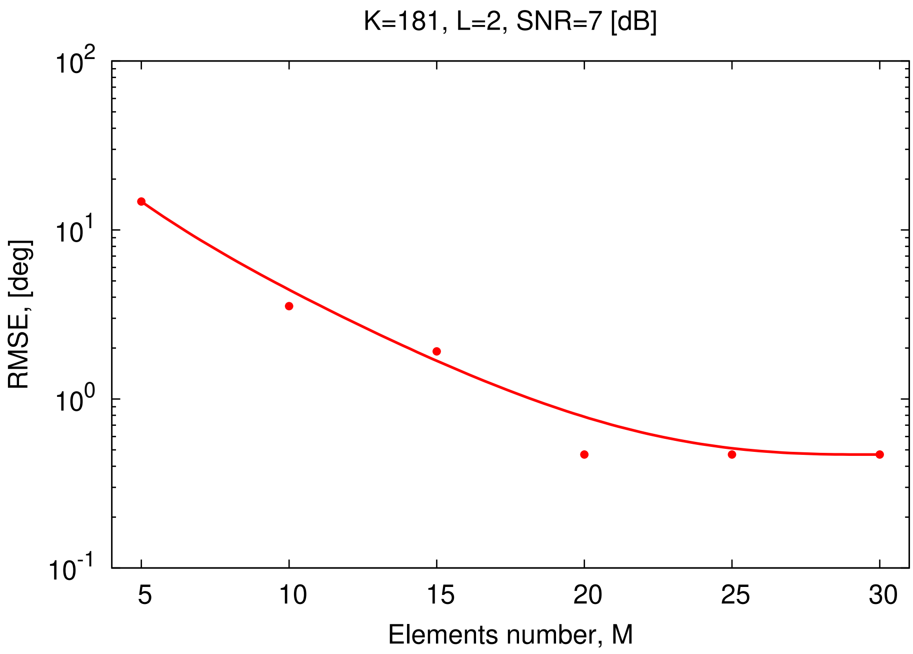


Figure 9: *BCS performances analysis:* $RMSE$ vs vs the number of array elements M (L is known).

Performances vs the number of incident signals L

Simulation Parameters

- Scenario
  - BPSK signals ($E_l^{inc} \in \{-1, 1\}$)
  - **Number of incident signals:** $L \in [1, 7]$
  - **Signal directions:**

L	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7
1	0	-	-	-	-	-	-
2	0	7	-	-	-	-	-
3	0	7	-12	-	-	-	-
4	0	7	-12	-24	-	-	-
5	0	7	-12	-24	45	-	-
6	0	7	-12	-24	45	75	-
7	0	7	-12	-24	45	75	-33

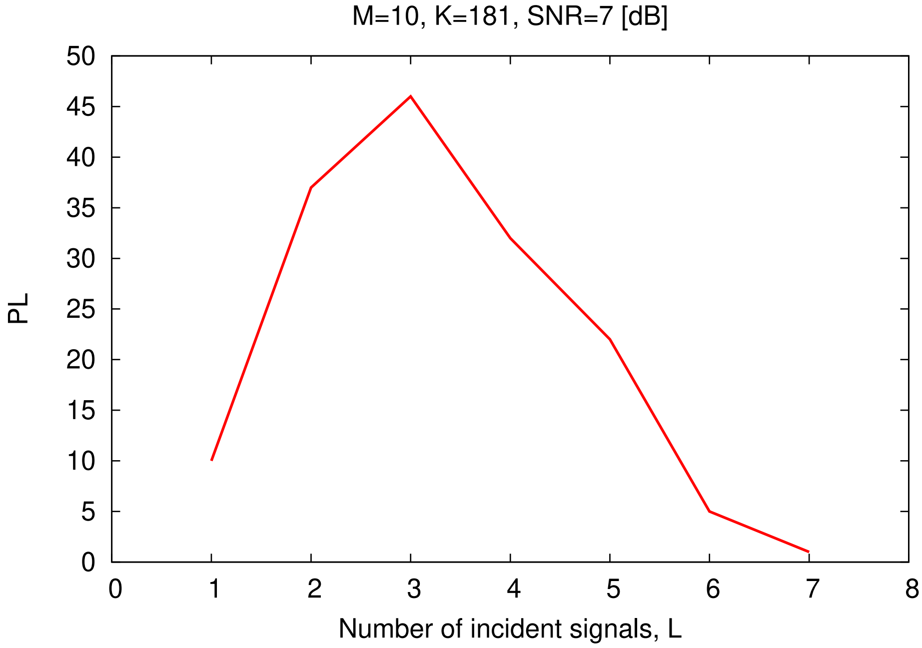
Table 1: Signal directions for different numbers of signals.

  - Signal to noise ratio**:** $SNR = 7\ dB$

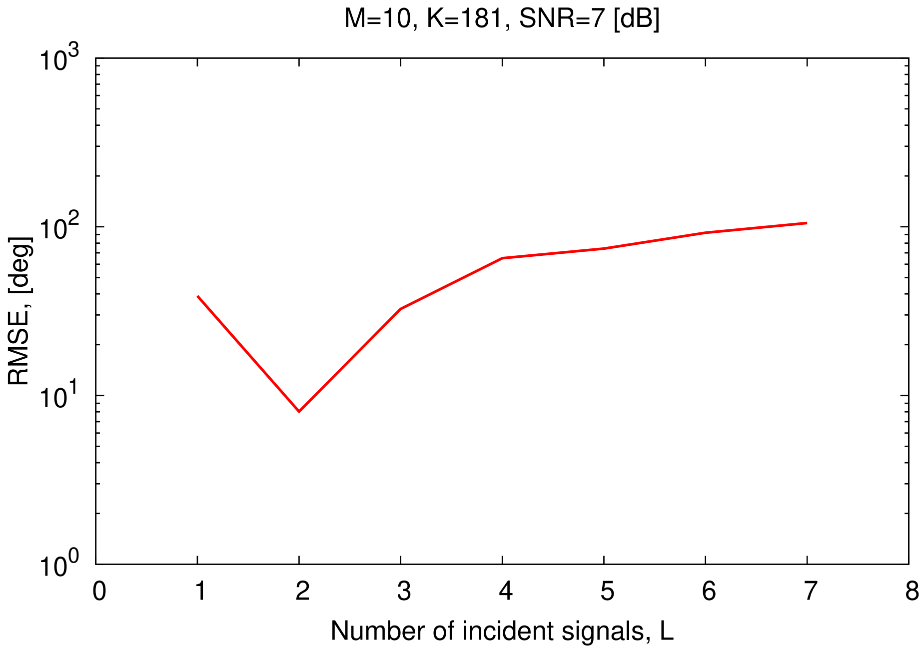
- Array parameters
  - Elements spacing: $d = 0.5\lambda$
  - Number of elements: $M = 10$

- BCS parameters
  - Number of angular locations: $K = 181$
  - $\sigma_0^2 = 4.642 \times 10^{-1}$

- Simulation
  - Number of independent realizations $Q = 100$ (the noise and the signal amplitudes are random, while the DoAs are fixed)



(a)



(b)

Figure 10: *BCS performances analysis:* (a) P_L and (b) $RMSE$ vs the number of incident signals L (L is unknown).

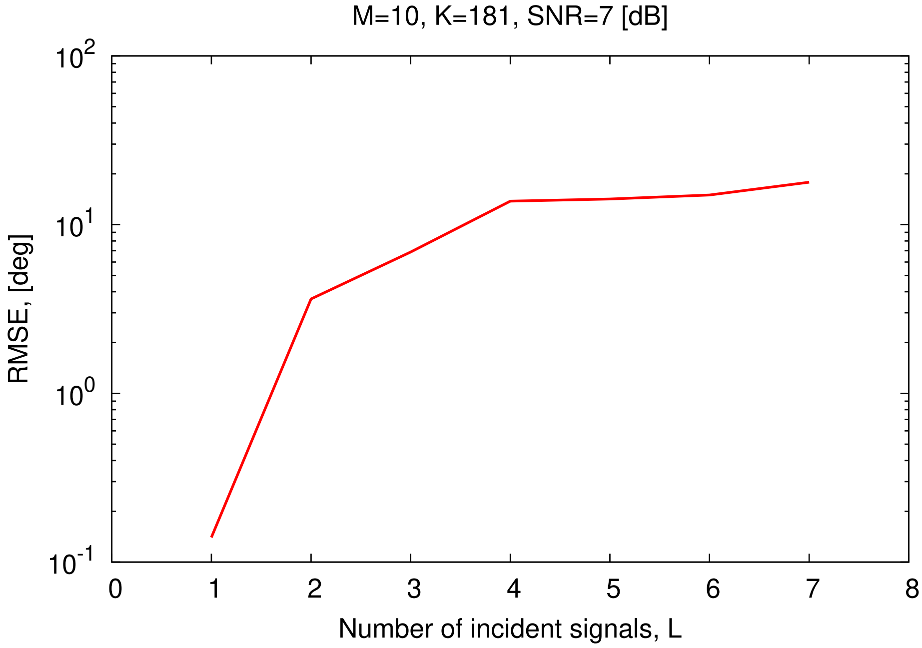
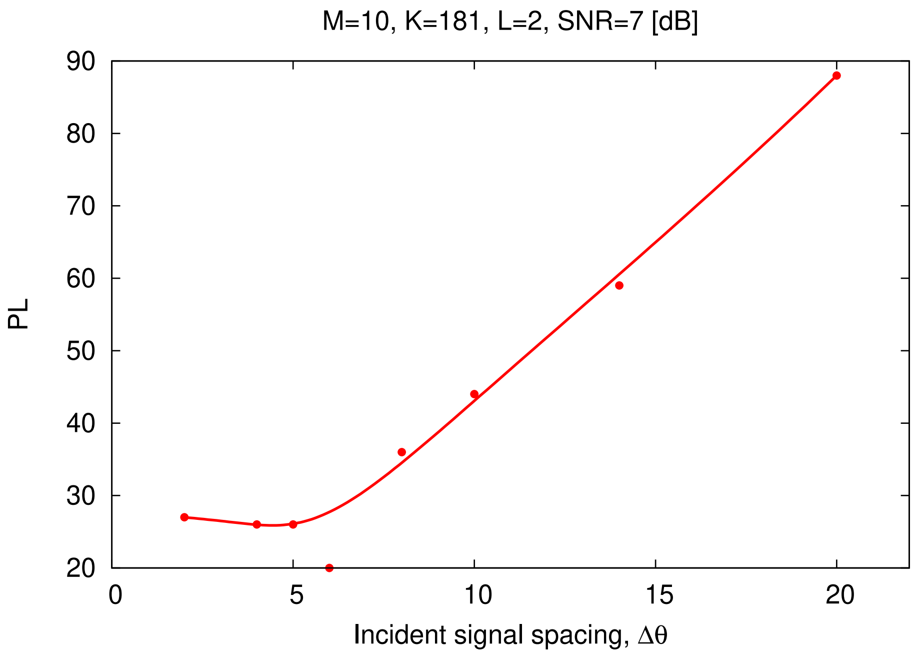


Figure 11: *BCS performances analysis:* $RMSE$ vs the number of incident signals L (L is known).

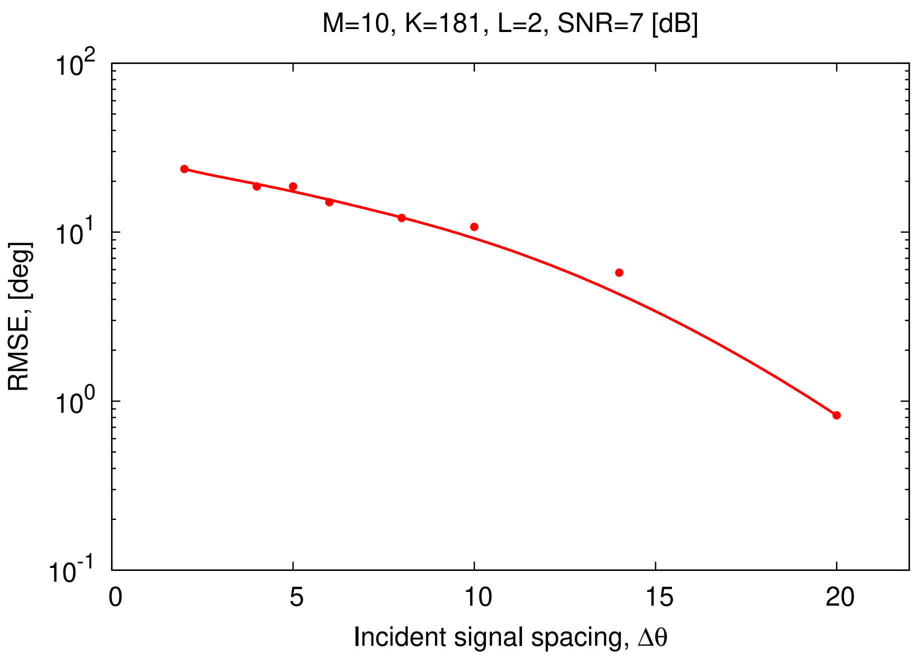
Performances vs the signal spacing $\Delta\theta$

Simulation Parameters

- Scenario
  - BPSK signals ($E_l^{inc} \in \{-1, 1\}$)
  - Number of incident signals: $L = 2$
  - **Signals spacing**: $\Delta\theta^{l(l+1)} \in [2, 20]$ deg
  - **Signals directions**: $\underline{\theta} = \left\{-\frac{\Delta\theta^{l(l+1)}}{2}, \frac{\Delta\theta^{l(l+1)}}{2}\right\}$ $[deg]$
  - Signal to noise ratio**:** $SNR = 7\ dB$
- Array parameters
  - Elements spacing: $d = 0.5\lambda$
  - Number of elements: $M = 10$
- BCS parameters
  - Number of angular locations: $K = 181$
  - $\sigma_0^2 = 4.642 \times 10^{-1}$
- Simulation
  - Number of independent realizations $Q = 100$ (the noise and the signal amplitudes are random, while the DoAs are fixed)



(a)



(b)

Figure 12: *BCS performances analysis:* (a) P_L and (b) $RMSE$ vs the distance between incident signals (L is unknown).

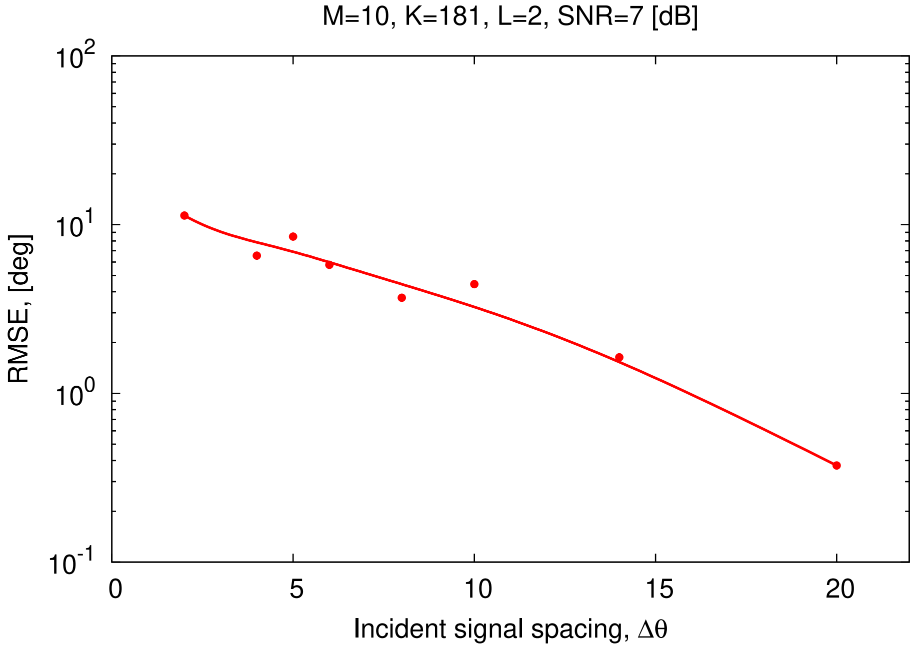


Figure 13: *BCS performances analysis:* $RMSE$ vs the distance between incident signals (L is known).

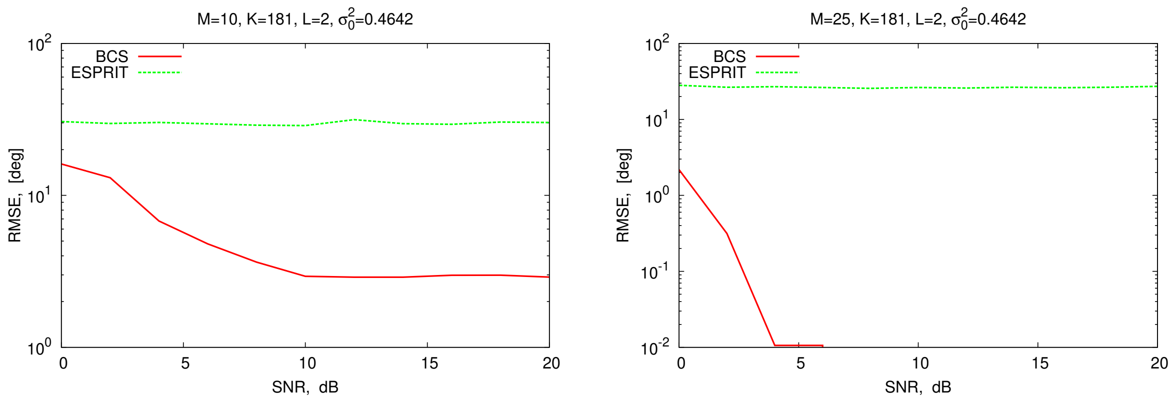
Single Snapshot Comparison with ESPRIT

Analysis of the performances of the algorithm when only $W = 1$ snapshot is considered

The objective of this section is to compare the performance of the proposed method with the performances of the ESPRIT method. In order to compare the two methods under the same conditions, the DoA estimation is performed only from one snapshot. As it can be noticed, the ESPRIT method does not work with just one snapshot.

Case 01: signals DoAs locked to the BCS grid In the following example the directions of arrival of the impinging signals are locked to the user-defined grid used by the BCS solver.

- Scenario
  - $L = 2$
  - $\underline{\theta} = \{0, 7\}$ $[deg]$
  - BPSK signals
  - $SNR \in [0, 20]$ dB (def. Imaging)
- Array
  - $M = \{10, 25\}$
  - $d = 0.5\lambda$
  - $W = 1$
- Method
  - $\sigma_0^2 = 4.642 \times 10^{-1}$
  - $K = 181$
- Simulation
  - $Q = 200$ independent realizations



(a) (b)

Figure 14: $RMSE$ vs SNR when only $W = 1$ snapshots are acquired. As it can be observed, compared to BCS, the performances of $ESPRIT$ are very poor.

Case 02: free signals DoAs In the following example the direction of arrival of the incident signals are not locked to the BCS solver grid.

- Scenario
  - $L = 2$
  - $\underline{\theta} = \{0, 7\}$ $[deg]$
  - BPSK signals
  - $SNR \in [0, 20]$ dB (def. Imaging)
- Array
  - $M = \{10, 25\}$
  - $d = 0.5\lambda$
  - $W = 1$
- Method
  - $\sigma_0^2 = 4.642 \times 10^{-1}$
  - $K = 181$
- Simulation
  - $Q = 200$ independent realizations

More information on the topics of this document can be found in the following list of references.

References

[1] M. Carlin, P. Rocca, G. Oliveri, F. Viani, and A. Massa, “Directions-of-arrival estimation through Bayesian Compressive Sensing strategies,” *IEEE Trans. Antennas Propag.*, vol. 61, no. 7, pp. 3828-3838, Jul. 2013 (DOI: 10.1109/TAP.2013.2256093).

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[3] M. Carlin, P. Rocca, “A Bayesian compressive sensing strategy for direction-of-arrival estimation,” *2012 6th European Conference on Antennas and Propagation (EUCAP)*, Prague, Czech Republic, 26-30 March, 2012, pp. 1508-1509 (DOI: 10.1109/EuCAP.2012.6206667).

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[5] M. Donelli, F. Viani, P. Rocca, and A. Massa, “An innovative multi-resolution approach for DoA estimation based on a support vector classification,” *IEEE Trans. Antennas Propag.*, vol. 57, no. 8, pp. 2279-2292, Aug. 2009 (DOI: 10.1109/TAP.2009.2024485).

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[7] A. Massa, P. Rocca, and G. Oliveri, “Compressive sensing in electromagnetics - A review,” *IEEE Antennas Propag. Mag.*, pp. 224-238, vol. 57, no. 1, Feb. 2015 (DOI: 10.1109/MAP.2015.2397092).

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